The functional equations

$$f(t+s)=\frac{\sum_{i=1}^{n} y_i(t)u_i(s)}{\sum_{j=1}^{m} z_j(t)v_j(s)} \tag{1}$$

are considered. Such equations arise and work in various problems of mathematical physics. In particular, the case $m=n=2$ contains some equations related to the problem of construction of Lax representation for a system of interactive particles.

It will be proved that "generic" solutions of (1) are ratio of quasi-polynomials. It will be shown that all analytic solutions for the (most important for applications) 2x2-case are expressed by Baker-Akhiezer functions.

The Hyers-Ulam stability ("a function that approximately satisfies the equation is close to a strict solution") will be established for the nx1-case ("Levi-Civita functional equations"). The proof relies on the theory of covariant n-widths, t.i. distances from an invariant convex set to invariant n-dimensional subspaces. This approach gives some quantitative results on the lattices of invariant subspaces.