Sharp results on large coupling convergence

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Abstract: let $(A_b)_{b\geq 0}$ be a family of nonnegative selfadjoint operators. Suppose that there exist a linear space D and a mapping $P : D \longrightarrow \mathbb{R}$ satisfying $D(A_0) \subset D \subset D(\sqrt{A_0})$ and

$$\begin{aligned} D(\sqrt{A_b}) &= D, \quad b > 0, \\ \| \sqrt{A_b} f \|^2 &= \| \sqrt{A_0} f \|^2 + bP(f), \quad f \in D, \, b > 0. \end{aligned}$$

We shall show that

$$L(A_0, P) := \lim_{b \longrightarrow \infty} b \, \| \, (A_b + 1)^{-1} - \lim_{b' \longrightarrow \infty} (A_{b'} + 1)^{-1} \, \|$$

exists and determine this limit. In particular, we shall derive a condition which is necessary and sufficient in order that the limit $L(A_0, P)$ is finite. In the special case that A_0 is the free quantum mechanical Hamiltonian and $A_b = -\Delta + b\mu_\Gamma$, $b \geq 0$, for any 1-equilibrium measure $\mu_\Gamma \neq 0$ we shall show that $L(A_0, P) = 1$ provided the interior of Γ is not empty.