Fluid-Dynamic Limit for a Gaseous Mixture and the Ghost Effect: Asymptotic Analysis of the Boltzmann Equation

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A binary mixture of a vapor and a noncondensable gas around arbitrarily shaped condensed phases of the vapor is considered. We investigate the steady behavior of the mixture in the continuum limit (i.e., the limit where the Knudsen number, the mean free path of the gas molecules divided by the characteristic length of the system, vanishes) on the basis of kinetic theory in the following situation. (i) the condensed phases are at rest; (ii) the noncondensable gas is present everywhere in the domain; (iii) the mixture is in a state at rest with a uniform pressure at infinity when an infinite domain is considered.

For this purpose, we carry out a formal but systematic asymptotic analysis of the Boltzmann equation for small Knudsen numbers under the assumption that (a) the molecules of both components are hard (or rigid) spheres; (b) the vapor molecules are reemitted from the boundary (surfaces of the condensed phases) according to the complete condensation condition; (c) the noncondensable gas molecules are reflected diffusely on the boundary. More specifically, we first derive, by means of the Hilbert expansion, a set of fluid-dynamic type equations which contains the thermal-stress and concentration-stress terms in addition to the usual Navier–Stokes stress terms. Then, we derive its appropriate set of boundary conditions by analyzing the Knudsen layers in the vicinity of the boundary.

¿From this system of the fluid-dynamic type equations and the boundary conditions, we deduce the following behavior of the mixture in the continuum limit: evaporation and condensation stop and the flow of the mixture vanishes in the continuum limit; however, the vanishing (or nonexisting) flow gives a finite effect on the behavior of the mixture (e.g., the temperature and density distributions) in this limit. This is an example of the ghost effect discovered by Sone and coworkers a few years ago [e.g., Y. Sone *et al.*, Phys. Fluids **8**, 623 and 3403 (1966); Y. Sone, in *Rarefied Gas Dynamics*, edited by C. Shen (Peking Univ. Press, Beijing, 1997), p. 3; Y. Sone, Ann. Rev. Fluid Mech. **32**, 779 (2000)]. It is shown that there are several new source factors of the ghost effect that are peculiar to gas mixtures (concentration stress, diffusion slip, etc.) in addition to the diffusion that was pointed out in our previous study of the two surface problems [S. Takata and K. Aoki, Phys. Fluids **11**, 2743 (1999)]. The present study is the generalization of the latter paper. Some numerical examples showing the ghost effect will be presented at the symposium.