Thermophoresis of Axially Symmetric Bodies

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Thermophoresis as a phenomenon has been known for a long time, and several authors have approached the problem. For example, Einstein calculated the final velocity of a spherical particle in a heat conducting gas using elementary kinetic theory. A recent review of the phenomenon is given in an article by Sone, [1]. The first systematic attempt to describe the thermophoresis phenomenon using kinetic theory is found in an article by Waldmann is 1959, [2]. That work was made under the assumption that the mean free path of the gas is much larger than the dimension of the body. Further results are found in a variety of articles, but these mostly apply to larger bodies, using asymptotic methods, and often to spheres.

In this work, thermophoresis of axially symmetric bodies is considered. The typical macroscopical length L over which the temperature varies is assumed to be much larger than λ, the mean free path of the gas. The typical diameter R of the body is assumed to be much smaller than λ. R is also assumed to be much larger than d, the diameter of a gas molecule. In calculating the momentum transfer from the gas to the body we therefore use the equations of free molecular flow, [3].

The body shape will here enter into the equations of motion of the body through a set of integrals of the body surface. It is shown that in this case, in contrast to the case of a spherical body, the thermal force will not in general be parallel to the heat current. Moreover, the heat conducting gas will also exert a torque on the body, tending to align the body axis of symmetry with the heat current. Therefore, the body aquires both a velocity and an angular velocity. The gas acts further upon the moving body by means of a friction force and a friction torque. After a time of the order of $\sim (R/d)\tau_o$, where τ_o is the molecular relaxation time, the body will reach an asymptotic state with a constant velocity $\mathbf{v}$, given by

$$\mathbf{v} = \frac{1}{5p}\gamma \frac{1 + \left(3 - \frac{9}{4}\alpha_\tau\right)\mathcal{J}}{1 + \frac{\pi}{8}\alpha_\tau + \left[3 - \left(\frac{9}{4} - \frac{3\pi}{8}\right)\alpha_\tau\right]\mathcal{J}}\mathbf{q}. \tag{1}$$

Here, $p = nk_BT$, $\mathbf{q}$ is the heat current, and γ is a constant depending on the intramolecular forces of the gas molecules. If only the first term of the Sonine polynomial expansion of the heat conducting part of the distribution function is retained, $\gamma = 1$. The parameter α_τ is the accomodation coefficient of tangential momentum, [3]. $\mathcal{J}$ is an integral of the body surface that measures the equatorially symmetric deviation of the body from a sphere. For a coin shaped body, $\mathcal{J} = 1$, and for a needle shaped body, $\mathcal{J} = -1/2$. For a sphere, $\mathcal{J} = 0$. This result generalizes the asymptotic velocity for spherical bodies found by Waldmann, [2]. For a nonspherical but equatorially symmetric body there exist further asymptotic solutions.

References

[1] Sone, Y., *Ann. Rev. Fluid Mech.* Flows induced by temperature fields and their ghost effect on the behavior of a gas in the continuum limit. To be published.

[2] Waldmann, L., *Z. Naturforsch.* **14a**, 589 (1959).

[3] Kogan, N. M., *Rarefied Gas Dynamics*, New York: Plenum Press, 1969.