The grazing collision limit for the Boltzmann-Lorentz model

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The Fokker-Planck collision operator is usually considered as an approximation of the Boltzmann collision operator when the collisions become grazing. This has been proved in a series of papers, starting from Arsen'ev and Buryak who show in [1] this convergence result under restrictive assumptions on the scaterring cross section and on the initial data. A mathematical framework has also been given by Desvillettes for more physical situations, but which still exclude the Coulombian case; in [4], the scaterring cross-section is smooth and depends upon a small parameter ε which tends to zero. This parameter however does not have any precise physical meaning. A quite different asymptotics is used in [2] to treat the Coulombian case: here, the scattering cross section has a non integrable singularity when the relative velocity of the colliding particles tends to zero. Moreover, the small parameter involved in this asymptotics has an actual physical meaning: it is clearly identified to the plasma parameter. Recently, Villani [6] obtained a complete rigorous proof of this asymptotic problem in the space homogeneous situation, and for potentials which are not "too soft".

In this paper, we are concerned with a simplified collision operator which is known as the Boltzmann-Lorentz model in plasma physics. It is used, for example, to describe the effects of collisions of electrons with neutral particles. In first approximation, the electrons are assumed to diffuse with a stationary equilibrium distribution of target particles. The simplified Boltzmann-Lorentz model is then derived from the Boltzmann equation in the limit of small electron mass with respect to the mass of atoms; this asymptotics has been completely justified from the theory of kinetic collisional operators in [3].

In the same way, we can define a simplified Fokker-Planck-Lorentz model which can be derived from the Fokker-Planck-Landau equation in the limit of small electron mass with respect to the mass of ions (see again [3]). Physical situations actually exist for which this operator appears as the leading order collision term (see [5] for an example in the context of plasmas). From a probabilistic point of view, the Fokker-Planck-Lorentz operator describes a random walk of the particles on any sphere of constant energy.

In this paper, we are interested with the limit of the Boltzmann-Lorentz operator towards the Fokker-Planck-Lorentz model in the so-called grazing collision limit. Two situations are considered: a first one corresponds to the asymptotics developed by Desvillettes in [4]; a second one is the asymptotics introduced in [2] to treat the Coulombian case. In both cases, we show the convergence of the operators, but also of the solutions of the Cauchy problems associated with these operators: for the last, the convergence is uniform with respect to time, on any finite time interval. But, thanks to a precise spectral analysis (the operators share the same basis of eigenfunctions), we can go further. First, we can show a uniform convergence result when time goes to infinity. Second, we have a precise knowledge of the convergence speed towards equilibrium.

References

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