SPACE HOMOGENEOUS SOLUTIONS OF THE BOLTZMANN EQUATION FOR AN ELECTRON GAS IN A SEMICONDUCTOR

A. Majorana and C. Milazzo

Dipartimento di Matematica, Università di Catania

Viale A. Doria 6, 95125 Catania - Italy -

The Boltzmann equation, which is used to describe the evolution of an electron gas in a semiconductor device, is considered. The unknown function $(t, \mathbf{x}, \mathbf{k}) \to f(t, \mathbf{x}, \mathbf{k})$ of the Boltzmann Equation is the existence probability to find an electron at the position $\mathbf{x} \in \mathbb{R}^3$, with the wave vector $\mathbf{k} \in \mathbb{R}^3$ at the time $t \in \mathbb{R}^+$.

The Cauchy problem for space homogeneous solutions of the Boltzmann equation is given by

$$\frac{\partial f}{\partial t} = C(f) \ , \ f(0, \mathbf{k}) = \phi(\mathbf{k}). \tag{1}$$

The collisional operator $C(f)$ is given by

$$C(f) = \int_{\mathbb{R}^3} [S(\mathbf{k}, \mathbf{k}') f(t, \mathbf{k}') - S(\mathbf{k}', \mathbf{k}) f(t, \mathbf{k})] d\mathbf{k}'$$

where the kernel S is defined by

$$S(\mathbf{k}, \mathbf{k}') = G(\mathbf{k}, \mathbf{k}')[(n_q + 1)\delta(\varepsilon' - \varepsilon + \hbar\omega) + n_q \delta(\varepsilon' - \varepsilon - \hbar\omega)] + \tilde{G}(\mathbf{k}, \mathbf{k}')\delta(\varepsilon' - \varepsilon)$$

The function G and $\tilde{G}$ belongs to $C^0(\mathbb{R}^3 \times \mathbb{R}^3)$, $\hbar$ is the Planck constant divided by 2π, n_q is the constant thermal equilibrium number of phonons, ω is the constant phonon frequency and δ denotes the Dirac distribution. The function $\varepsilon(\mathbf{k})$ is the electron energy, which depends on the band structure of the semiconductor. We assume that

$$\varepsilon(\mathbf{k}) = \frac{\hbar^2 \mid \mathbf{k} \mid^2}{m^* \left(1 + \sqrt{1 + 2\frac{\alpha}{m^*}\hbar^2 \mid \mathbf{k} \mid^2}\right)} \qquad \text{(Kane model)}$$

m^* being the effective mass of the electron and α a parameter (vanishing in the parabolic band approximation).

We prove that the Cauchy problem (1) admits a unique global solution in a Lebesgue space, without assuming the boundness of the collision frequency

$$\nu(\varepsilon) = \int_{\mathbb{R}^3} S(\mathbf{k}, \mathbf{k}') d\mathbf{k}'.$$