Equilibrium Temperature of Small Body in Shearing Gas Flow

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Bell and Schaaf calculated the heat transfer to an infinite circular cylinder in the free molecular flow approximation. They took into account the non-equilibrium nature of the flow. They found a contribution to the heat transfer from the shearing of the gas. The equilibrium temperature of the gas was also shown to be affected by the shearing of the gas.

We consider a body, fixed in space where the macroscopic velocity of the gas vanishes. The body is small compared to the mean free path. We assume free molecular flow. We use the Maxwell model for the surface. α_τ is the tangential momentum accomodation coefficient.

¿From the first order Chapman-Enskog distribution function the net incoming heat flux in a surface element with outward normal $\mathbf{n}$ is

$$\alpha_e \alpha_\tau Q_M[(1-\frac{T_w}{T})+(\frac{3}{4}\beta_Q-\frac{T_w}{2T}\beta_N)n_{<i}n_{j>}\frac{p_{<ij>}}{p}].$$

The subscript M stands for the value for a Maxwellian, β_N, β_Q, β can in a first approximation be put equal to 1, $p_{<ij>}$ is the viscous pressure and p the ordinary pressure. α_e, finally, is the energy accomodation coefficient)

We see that also for $T_w = T$, there is a net influx.

We consider a high thermal conductivity body in a stationary situation. To a first approximation in the shearing, the temperature of the body then has to be

$$\frac{T_w}{T} = 1 + \frac{\beta}{4}\frac{p_{<ij>}}{p}\overline{n_{<i}n_{j>}},$$

where $\beta = 3\beta_Q - 2\beta_N$. The overbar in $\overline{n_{<i}n_{j>}}$ denotes surface average. Note that $\overline{n_{<i}n_{j>}}$ vanishes for a sphere.

For an axially symmetric body, with axis $\mathbf{N}$,

$$\overline{n_{<i}n_{j>}} = aN_{<i}N_{j>}.$$

For a flat body $a = 1$. For a needle shaped body $a = -1/2$.

Let us consider plane shear flow, with $v_{x,y}$ as the non-vanishing component of the velocity gradient . Then (the polar angles (z is the polar axis) of the axial direction of the body by ϑ, φ)

$$\frac{T_w}{T} = 1 + \frac{\beta a}{8} \frac{p_{xy}}{p} \sin^2 \vartheta \sin(2\varphi).$$

A needle-shaped body will settle down to a temperature, higher (lower) than the gas, if the projection onto the plane of shearing of the axis of the

body is in the first (second) quadrant.