Counting and accounting in the proto-literate Middle East.

Examples from two new volumes of proto-cuneiform texts[1] [2]

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1. The corpus of proto-cuneiform text. The series ATU and MSVO

The early history of writing is also *the early history of counting and accounting.* Our knowledge of how, when, and where, writing and counting were first invented has taken enormous strides forward during the last ten or twenty years. Most well-known to the general public at the present time is the role of small *clay tokens* of various shapes and sizes as the precursors in the prehistoric Middle East of written words and number signs[3]. Manipulation with tokens seems to have been a relatively sophisticated, although cumbersome, method of counting and recording. The method was in use for several millennia before the invention of writing with *proto-cuneiform signs* on clay tablets, which in its turn took place somewhere in Mesopotamia in the Late Uruk period.[4] (The period is named after Uruk, one of the largest and oldest cities in ancient southern Mesopotamia.) Not much later, an independent form of writing on clay tablets was invented in the region immediately east of Mesopotamia. This is the short-lived *proto-Elamite script*, still essentially undeciphered, with the exception of its number signs which to a large part are identical with the number signs of the proto-cuneiform script.[5]

Great numbers of proto-cunieform texts have been excavated in Uruk by a series of German archaeological expeditions to the site since 1928. Some of these texts were first published as photographs by A. Falkenstein in *ATU 1= Archaische Texte aus Uruk* (Archaic Texts from Uruk), Berlin 1936. After a long interval

[1] **ATU 5=Archaic Administrative Texts from Uruk. The Early Campaigns**. By Robert K. Englund, with a contribution by Rainer M. Boehmer (Berlin: Gebr. Mann Verlag, 1994), 1-232, plates 1-143, I-X.

[2] **MSVO 4 = Proto-Cuneiform Texts from Diverse Collections**. By Robert K. Englund, with a contribution by Roger J. Matthews (Berlin: Gebr. Mann Verlag, 1996), 1-110, plates 1-41, I-V.

[3] See the present reviewer's paper "Preliterate counting and accounting in the Middle East. A constructively critical review of Schmandt-Besserat's 'Before Writing"', *Orientalistische Literaturzeitung* 89 (1994) 477-502.

[4] An exellent account of the early stages of the development of the proto-cuneiform and cuneiform scripts, with many beautiful illustrations, can be found in H. Nissen, P. Damerow, and R. Englund, *Archaic Bookkeeping. Writing and Techniques of Economic Administration in the Ancient Near East* (Chicago and London: The University of Chicago Press, 1993).

[5] See the research report *ERBM 1* (1989)= J. Friberg, *The Third Millennium Roots of Babylonian Mathematics*, circulated in a limited edition, and P. Damerow and R. Englund, *The Proto-Elamite Texts from Tepe Yahya* (Cambridge, MA: American School of Prehistoric Research Bulletin 39, 1989).

of inactivity, partly caused by World War II, Falkenstein's work was revived and continued in Berlin (lately also in Los Angeles) by a new interdisciplinary research project still using the name *Archaische Texte aus Uruk*.[6] As a result of the energetic activities of this research project, many of the archaic (that is proto-cuneiform) texts from Uruk are now available for study in several Berlin publications. In addition to *ATU 5* (1994), these include *ATU 2*=M. Green and H. Nissen, *Zeichenliste der archaischen Texte aus Uruk* (Berlin: Gebr. Mann Verlag, 1987), a sign list for the more than 900 signs of the proto-cuneiform scripts, and *ATU 3* (1993)= R. Englund and H. Nissen, Die lexikalischen Listen der archaischen Texten aus *Uruk* (Berlin: Gebr. Mann Verlag, 1993), an account of various kinds of systematically organized "lexical lists" of proto-cuneiform or cuneiform signs used in the Mesopotamian scribal schools from the Late Uruk period on, throughout the whole third millenium BC. A separate chapter in *ATU 2* contains a systematic and detailed discussion by P. Damerow and R. K. Englund of all proto-cuneiform systems of numbers.

The data base of the research project *Archaische Texte aus Uruk* currently comprises some 5820 items representing as many texts on excavated clay tablets, records of administrative transactions or writing exercises, from the earliest proto-historic levels of southern Mesopotamia.[7] In addition to fully 5000 such documents from Uruk, ascribed to the earliest script phases IV (3200-3100 B.C.) and III (3100-3000 B.C.), the data base contains 820 texts derving from regular or irregular excavations of a small number of other ancient Mesopotamian sites. These 820 texts are now in the process of being published by the Berlin research project in a separate series of publications with the name *MSVO=Materialien zu den frühen Schriftzeugnissen des Vorderen Orients* (Materials to the Early Written Documents of the Near East). Among these additional documents, there are 245 texts from the northern Mesopotamian site Jemdet Nasr, script phase III,[8] as well as 85 extraordinarily well preserved tablets from the formerly private Erlenmeyer collection,[9] and 410 texts from Early Dynastic levels I-II, mainly from

[6]The scholars in charge of this research project have also been actively involved (together with the present reviewer) in a number of sessions of the *Berlin Workshops on the Development of Concepts in Babylonian Mathematics.* Informal proceedings of these workshops are about to appear as J. Høyrup and P. Damerow (eds.), *Changing Views on Ancient Near Eastern Mathematics* (Berlin: Reimer, 1997).

[7] *The entire data base* is accessible to both PC and Mcintosh users in the form of a diskette inserted in a pocket on the back cover of the volume *ATU 5.* The diskette contains, in three separate data files prepared to be loaded into a common data base program, a *catalog* of all known proto-cuneiform texts, *transliterations* (but not translations) of the texts, and a complete *sign list* with references to the texts. Uppdated versions of the data base, including digitized images of all accessible tablets, will be made available via the internet under the URL http://early-cuneiform.humnet.ucla.edu/archaic/index.htm.

[8]Originally published to some extent in *OECT 7* = S. Langdon, *Pictographic Inscriptions from Jemdet Nasr* (1928), and now to their full extent in **MSVO 1= R. Englund and J.-P. Grégoire,** *The Proto-Cuneiform Texts from Jemdet Nasr* (Berlin: Gebr. Mann Verlag, 1991). See the extensive review of *MSVO 1* by the present reviewer in *Zeitschrift für Assyriologie* 84 (1994)130-135.

[9]See **MSVO 3= P. Damerow and R. Englund, *The Proto-Cuneiform Texts from***

Ur.[10]

The remaining 80 tablets, in most cases well preserved texts of script phase III, deriving from irregular excavations and bought in the antiquities market, are found today in various small collections.[11] These 80 tablets are now being published together, in the volume *MSVO 4* being reviewed here. In the Introduction to *MSVO 4* it is shown that the majority of these texts can be divided into two groups, with texts 1-40 probably deriving from the northerly site *Uqair*, located near Jemdet Nasr, and with texts 41-67 possibly from the southern site *Larsa*, near Uruk, (or from *Uruk* itself). The two groups are easily distinguished from each other, since they were acquired in separate lots, since they make use of separate sets of names of officials, supervisors, and administrative units, and since the texts of the two groups belong to only slightly overlapping text categories.

MSVO 4 is structured according to the general text format used for all the volumes of the two closely related series *MSVO* and *ATU*. Thus, after the introduction there follows a *catalog* of the texts, with brief information in each case about the linear dimensions of the text, its museum number, its content, and previous publications of it. A separate catalog (by R. J. Matthews) gives information about the seal impressions present on a minority of the texts. Then follow *transliterations* of all the texts, giving precise identifications of all the proto-cuneiform signs appearing in the individual texts. An extensive *sign glossary* indicates all appearances, in context, of each proto-cuneiform sign in texts of the volume. Finally, there are *hand copies* of all the texts, carefully drawn by use of computer graphics, and photographs of some representative examples. In the case of tablets with seal impressions, hand copies of these are given, too.

What is seemingly missing in this very elaborate presentation of all the texts treated in the volume is *translations and interpretations.* The primary reason for this shortcoming is simply that in many cases the proper translations are not known. It is not even clear if those who wrote the proto-cuneiform inscriptions spoke Sumerian or some other (today totally unknown) language. Although many of the proto-cuneiform signs are assumed to have the same meaning as the Sumerian cuneiform signs ultimately developed from them, it has not been possible to identify a single phonetic element in the proto-cuneiform script. (For practical reasons, the sign names used for the proto-cuneiform signs are to the largest extent possible the readings in Sumerian of the corresponding cuneiform signs.)

Generally, the proto-cuneiform texts seem to be stereotyped accounts, receipts, or list, containing nothing but enumerations, either of numbers and objects counted by those numbers, or of names and titles. Only in the most favorable

the Erlenmeyer Collection (to appear)

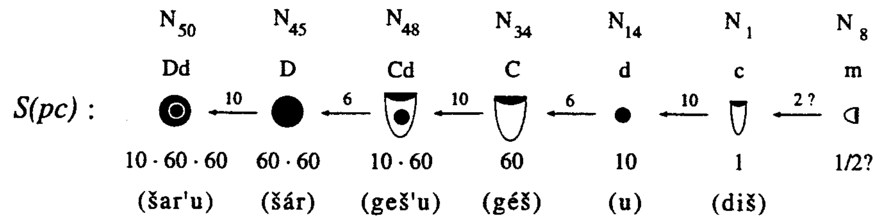
[10]See *UET 2* = E. Burrows, *Ur Excavations. Texts 2* (1935).

[11]Of these 80 texts, 35 were published by A. Falkenstein in *ATU 1* (1936), 17 by P. van der Meer in *Revue d'assyriologie 33* (1936), and 6 by A. FAlkenstein in *Orientalistische Literaturzeinung 40* (1937). Several of these texts were used as illuminating examples by the present reviewer in the research report *ERBM 2* = J. Friberg, *The Early Roots of Babylonian Mathematics 2* (1979).

cases can one get a precise idea about the probable meaning of such an inscription by analyzing and interpreting the more or less complicated numerical relations that may exist between the recorded numbers. According to present publication plans, future volumes of the series *MSVO* will contain such interpretations and discussions.

2. The proto-cuneiform system S(pc) of sexagesimal numbers. Examples.

The first prerequisite for the interpretation of the proto-cuneiform texts, through a detailed analysis of the computations they may contain, is that the proto-cuneiform number signs occurring in the texts can be identified, and that the values of the number signs can be established. To achieve this goal is not an easy task, since there are about *sixty different number signs* appearing in the proto-cuneiform texts, and since many of these number signs have *different values depending on context.* Yet, the establishment of the various values of nearly all the proto-cuneiform number signs has now been achieved, as an essential first step in the still not completed decipherment of the proto-cuneiform script. It is standard practice to let any discussion of proto-cuneiform texts be preceded by a presentation of the *factor diagrams*[12] for the five *major proto-cuneiform systems of number signs.* First of all there is, of course, the factor diagram for the *proto-cuneiform (and proto-Elamite) sexagesimal system S(pc)*:



In this factor diagram are indicated primarily the *sign forms* of all the proto-cuneiform sexagesimal number signs and the numerical relations between them, expressed in terms of a *chain of replacement factors.* The arbitrary names given to the various sign forms, such as 'c' and 'C' for small and large *cup-shaped* signs, 'd' and 'D' for small and large *disk*-shaped signs, 'Dd' for a composite sign, etc., are the ones repeatedly proposed by the reviewer. They are short and easy to remember. The factor diagram shows that here

$$2m=c=1,\ 10c=d,\ 6d=C,\ 10C=Cd,\ 6\ Cd=D, \text{ and } 10D=Dd.$$

Therefore (in anachronistic place value notation)

$$m=1/2,\ c=1,\ d=10,\ C=1.00=60,\ Cd=10.00=10 \cdot 60, D = 1.00.00 = 60 \cdot 60, \text{ etc.}$$

[12] The use of factor diagrams was initiated by the present reviewer in *ERBM I* (1978). The idea was further developed by P. Damerow and R. Englund in the mentioned important survey *ATU 2* (1987), Ch. 3: "Die Zahlzeichensysteme der Archaischen Texte aus Uruk".

The notations N_1, N_2, etc., up to N_{60}, are the names given to the number signs in the general *sign list* for the proto-cuneiform script in *ATU 2* (1987). The names in brackets are the Sumerican names for the sexagesimal number signs (in the cuniform script).

Sexagesimal numbers are used in the proto-cuneiform script, as could be expected, in order to count several kinds of discrete objects (people, animals, non-edible objects, etc.) A few examples taken from *MSVO 4* will make this clear.[13] Consider, for instance, the text *MSVO 4*, 8 (from Uqair), of which the *obverse* (the first side) and the *reverse* (the second side) are shown in Fig. 2.1 below.

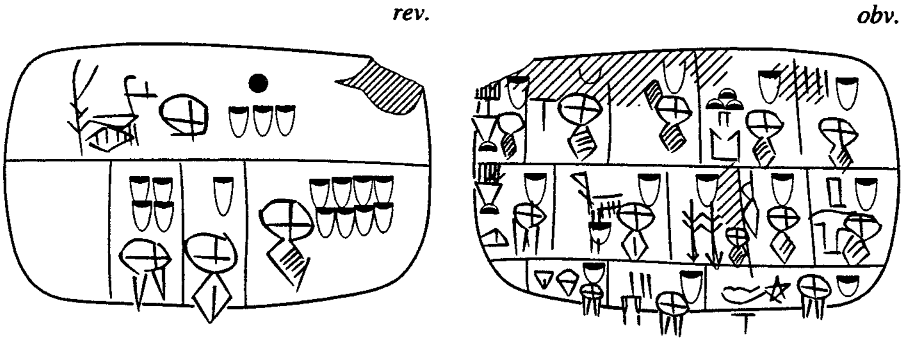


Fig. 2.1. MSVO 4, 8, an account of small cattle. System S(pc), small numbers.

On the *obserse* of a proto-cuneiform text, the *horizontal rows* are ordered *from top to bottom*, and in each row the individual *text cases* are ordered from right to left.[14] Each text case contains, as a rule, a single entry, consisting of a number and a specification of the kind of objects that are being counted, in certain cases also the name and/or title of the one responsible for those objects. In *MSVO 4*, 8, the counted objects are three kinds of male or female small cattle (sheep or goats).

On the *reverse* of a proto-cuneiform text, the *bottom row* is often the *first row*, containing in several text cases, ordered from right to left, the *sub-totals* of the various entries on the obverse. The *top row* on the reverse contains the *grand total*, that is the sum of the sub-totals. In the case of *MSVO 4*, 8, there are three sub-totals listing 8, 1, and 4 animals, respectively. The grand total is 13 (=8 + 1 + 4) animals (small cattle), with 13 written as 1d 3c=10+3.

In *MSVO 4*, 11 (from Uqair), there are only one row of 5 text cases on the

[13]The discussion in this review of the meaning of some selected texts from *MSVO 4, MSVO 1*, and *ATU 5*, is to some extent an updated version of the reviewer's own (inadequately published) research report *ERBM 2* (1979). Cf. the many text interpretations presented in *Archaic Bookkeeping* (fn. 4 above).

[14]In the volumes of *ATU* and *MSVO*, as in many other publications intended to be read by professional assyriologists, the copies of clay tablets are traditionally rotated to the left in such a way that the texts appear to be written in *vertical columns*, ordered from left to right, with the text cases in each column ordered from top to bottom. The text copies appearing as illustrations in the present paper have often been taken directly from *MSVO4, MSVO1*, or *ATU 5*, with the authors' consent, but all have been *rotated back to their original upright position.* This has been done with the objective in mind of emphasizing the considerable visual appeal of the originals. The copies have also been reduced to 75%, in order to save space.

obverse and two sub-totals on the reverse (plus three non-numerical text cases). The objects counted are, apparently, fish-baskets and birds. The first sub-total is 2.25 (= 15+30+1.00+40), written as 2C 2d 5c. The second sub-total is simply 15. There is no grand total.

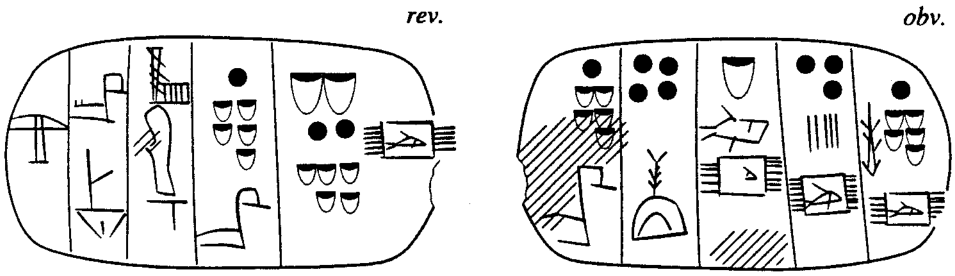


Fig. 2.2. MSVO 4, 11, fish-baskets and birds. System S(pc), intermediate numbers.

The last example mentioned here, *MSVO 4*, 13 (also from Uqair), is just a fragment of a clay tablet, and the meaning of the sign for the objects being counted is not known. It is clear, anyway, that the number recorded in the first text case of this text is a large sexagesimal number, 1D 4Cd=1.40.00$(=60 \cdot 60 + 40 \cdot 60 = 100 \cdot 60 = 6000)$.

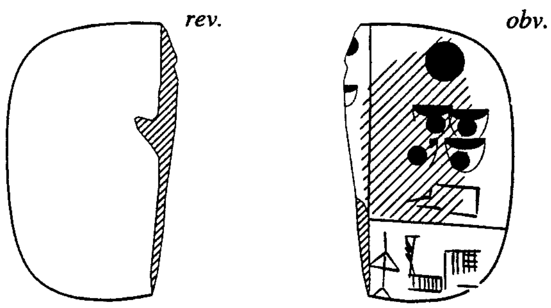
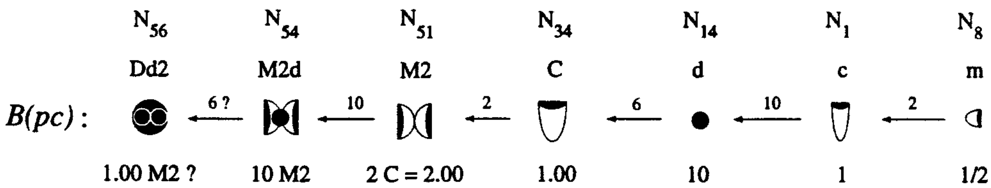


Fig. 2.3. MSVO 4, 13, an account of unidentified objects. System S(pc), large numbers.

3. The proto-cuneiform system B(pc) of bisexagesimal numbers. Examples.

In the proto-cuneiform and proto-Elamite scripts, but (almost) never in the Sumerian cuneiform script, certain kinds of discrete objects often appearing in *ration lists*, in particular various grain products, are counted in the *proto-cuneiform (and proto-Elamite) bisexagesimal system B(pc):*



As shown by this factor diagram, in system B(pc)

$$M2=2C=2.00=2 \cdot 60,\ M2d=10\ M2=20C=20.00=20 \cdot 60, \text{ etc.}$$

A very good example of the use of bisexagesimal numbers is *MSVO 4*, 66 (Fig 3.1), an important so-called *bread-and-beer text* (from Larsa or possibly Uruk). It is a cost account for large quantities of standardized rations of *bread* (or similar products) of various sizes, and of *jars of beer* of various strengths or various quality.

In the first row on the obverse of *MSVO 4*, 66, bread rations of five sizes are counted by the *bisexagesimal* numbers C(1.00), M2 (2.00), M2(2.00), 2M2 C(5.00), and 5M2 (10.00). The corresponding total, specified by a sign possibly meaning 'rations of bread', is given in case 1a of row i, the bottom line, on the reverse. This total is

$$C+M2+M2+2M2\ C+5M2=9M2\ 2C=10\ M2(=10 \cdot 2 \cdot 60=20.00), \text{ written as M2d.}$$

In *obv.* i:6a (that is case 6a of row i on the obverse), a sixth kind of bread ration (specified by a sign meaning 'bread?' and several strokes) is counted by the bisexagesimal number 5M2d=50 M2. The same entry is repeated as a second sub-total in *rev* i:1a-2a is

$$M2d+5M2d=6M2d=60\ M2=60 \cdot 2 \cdot 60, \text{ which is equal to } 20 \cdot 12 \cdot 30.$$

It is not unreasonable, therefore, to interpret the text as an account of *rations of various sizes of bread (and beer) for 20 persons for one whole year of* $12 \cdot 30$ *days.*

In the three text cases *obv.* ii:1a-3a, three kinds of beer are counted by the *sexagesimal* numbers 2C(=2.00), 3C(=3.00), and 5C(=5.00). The corresponding sub-total, recorded in *rev.* i:3a, is

$$2C+3C+5C=10C=10 \cdot 60, \text{ written as 1Cd.}$$

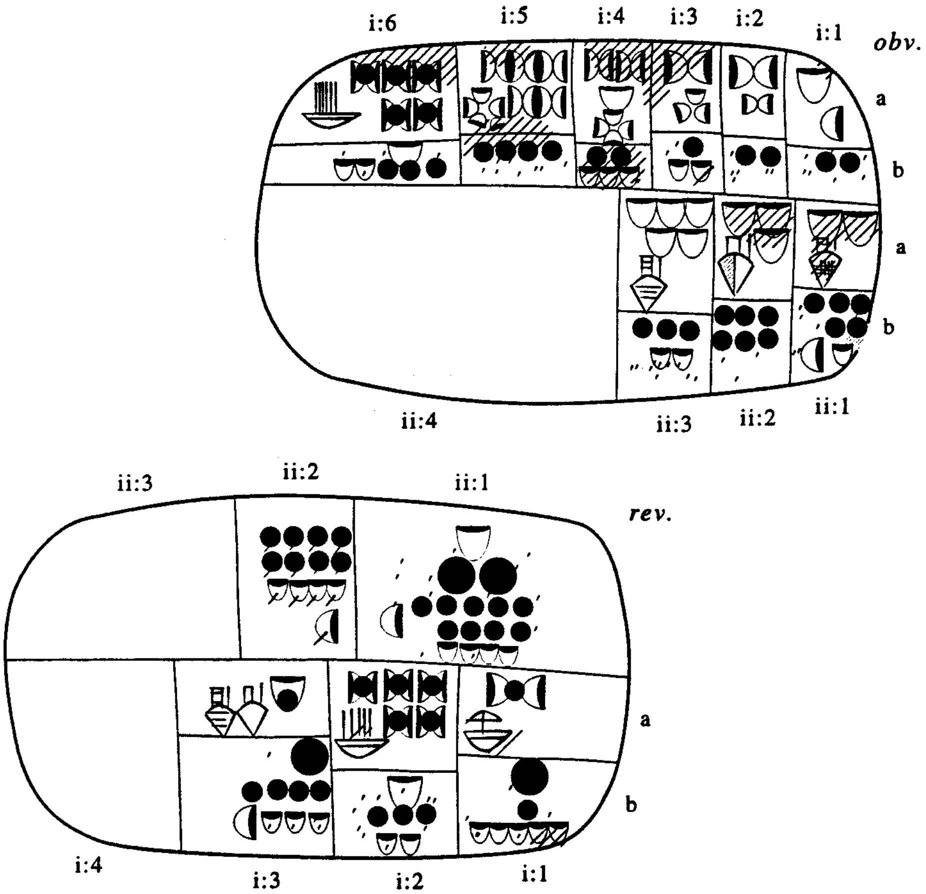
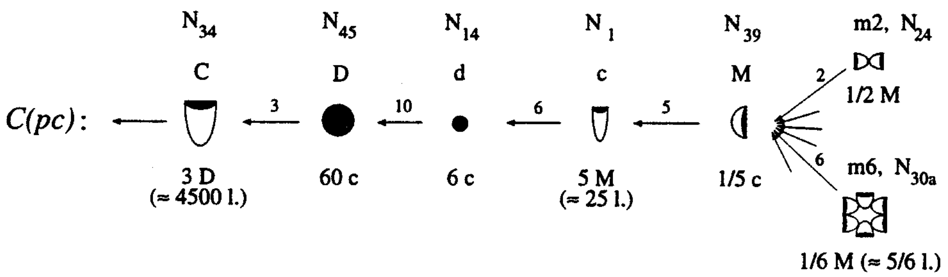


Fig. 3.1. MSVO 4, 66, a bread-and-beer text. Systems S(pc), B(pc), C(pc), C*(pc), and C′(pc).

4. The proto-cuneiform system C(pc) of capacity numbers. Examples.

The remaining numbers in *MSVO 4*, 66 are capacity numbers, written by use of number signs belonging to the *proto-cuneiform (and proto-Elamite) capacity system C(pc)*:



In system *C(pc)*, the *higher* units are multiples of 1 c, while the *fractional* units are sub-multiples of 1M=c/5. (No Sumerian names for the units of this system are known, for the reason that a *different capacity system was used in the Sumerian cuneiform script.*)

It is important to observe that the number signs for the higher units are *not* ordered hierarchically in the same way, and do *not* have the same (relative) values in system $C(pc)$ as in system $S(pc)$. Thus, in system $C(pc)$, the number signs for the higher units are ordered as d, D, C, Cd, and have the values 6c, 60c, 180c, 1800c, while in system $S(pc)$ *same number signs* for the higher units are ordered as d, C, Cd, D, and have the values 10, 60, 600, 3600. The advantage of using different hierarchical orders in the two systems is that it is usually easy to see immediately from the order in which the units in a given proto-cuneiform number follow each other if that number belongs to system $C(pc)$ or to system $S(pc)$ (or to some other system; see below).

In the successive text cases *obv.* i:1a-5a of the bread-and-beer text *MSVO 4*, 66 (Fig. 3.1), that is in the *upper halves* of the five first text cases, the numbers and *sizes* of the different kinds of bread rations are indicated. The sizes are expressed in terms of the corresponding *costs in grain of one ration.* These costs form the decreasing progression

M, m2, m3, m4, m5, that is 1, 1/2, 1/3, 1/4, and 1/5 of 1 M.

In the text cases *obv.* i:1b-5b, that is in the *lower halves* of the five first text cases, the *costs in grain of all the rations of each size* are recorded. Note that these total grain costs are written in terms of capacity numbers sprinkled with *dots.* Such numbers belong to system $C^*(pc)$, one of several so-called *derived* proto-cuneiform capacity systems. It is convenient to mark transliterations of numbers belonging to system $C^*(pc)$ with an asterisk.

It is easy to check that the grain cost is correctly computed in each case. Thus,

C · M=1.00M (=60 c/5)=12c=2d, written as 2d*	(*obv.* i:1b)
M2 · m2=2.00 m2=1.00M=12c=2d, written as 2d*	(*obv.* i:2b)
M2 · m3=2.00 m3=40M=8c=1d 2c, written as (1d 2c)*	(*obv.* i:3b)
2M2 C · m4=5.00 m4=1.15 M=15c=2d 3c, written as (2d 3c)*	(*obv.* i:4b)
5M2 · m5=10.00 m5=2.00 M=24c=4d, written as 4d*	(*obv.* i:5b)

The size of the sixth kind of bread ration, the one recorded in obv. i:6a (quantity 5M2d), can now easily be established. It must be m6=M/6, since

5M2d · m6=50 M2 · m6=50 · 2.00 m6=50 · 20M=50 · 4c=3.20c=(1C 3d 2c)* (*obv.* i:6b)

The sum of the grain costs recorded in *obv.* i:1b-5b is given as the sub-total

2d+2d+1d 2c+2d 3c+4d=11d 5c=(1D 1d 5c)* (*rev.* i:1b)

The grain cost (1C 3d 2c)* in *obv.* i:6b is repeated as a sub-total in *rev.* i:2b.

The costs of the beer rations are computed in the same way as the costs for the bread rations. It is a simple division exercise to find out that the cost *per jar* of the three kinds of beer mentioned in obv. ii: 1a-3a is M+3m10, M, and m3, respectively. Indeed,

$$\mathrm{2C \cdot (M{+}3m10){=}2.00 \cdot 13\ m10{=}12 \cdot 13M{=}2.36M{=}(5d\ 1c\ 1M)^*} \qquad (\textit{obv.}\ \mathrm{ii{:}1b})$$

$$\mathrm{3C \cdot M{=}3.00\ M{=}36c{=}6d^*} \qquad (\textit{obv.}\ \mathrm{ii{:}2b})$$

$$\mathrm{5C \cdot m3{=}5.00\ m3{=}1.40\ M{=}20c{=}(3d\ 2c)^*} \qquad (\textit{obv.}\ \mathrm{ii{:}3b})$$

The sum of the grain costs recorded in *obv.* ii: 1b-3b is given as the sub-total

$$\mathrm{(5d\ 1c\ 1M)^* + (1C\ 3d\ 2c)^* + (1D\ 4d\ 3c\ 1M)^*{=}(1C\ 2D\ 8d\ 10c)^* = (1C\ 2D\ 9d\ 4c\ 1M)^*} \qquad (\textit{rev.}\ \mathrm{i{:}3b})$$

The grand total of the grain costs is the sum of the three sub-totals in *rev.* i:1b-3b:

$$\mathrm{1(D\ 1d\ 5c)^*{+}(1C\ 3d\ 2c)^*{+}(1D\ 4d\ 3c\ 1M)^*{=}(1C\ 2D\ 8d\ 10c)^*{=}(1C\ 2D\ 9d\ 4c\ 1M)^*} \qquad (\textit{rev.}\ \mathrm{ii{:}1})$$

A second derived capacity system is system $C'(pc)$, which is characterized by using number signs tagged with *oblique strokes.* It is convenient to mark transliterations of numbers in system $C'(pc)$ with an oblique stroke. Hence, the two capacity numbers in *rev.* ii of *MSVO 4*, 66 can be transliterated as

$$\mathrm{(1C\ 2D\ 9d\ 4c\ 1M)^*{=}(2C\text{-}9M)^*\ \text{and}\ (8d\ 4c\ 1M)'{=}(9d\text{-}9M)'} \qquad (\textit{rev.}\mathrm{ii{:}\ 1\text{-}2})$$

It is known from other proto-cuneiform texts that cost accounts for beer normally included both grain (barley groats?) and (probably) malt in certain proportions, with the amounts of grain expressed in system $C^*(pc)$ and the corresponding amount of malt expressed in system $C'(pc)$. Therefore, the obliquely stroked number in *rev.* ii:2 ought to be simply related to the dotted number in *rev.*.i:3. This is also the case (see below).

This text, *MSVO 4*, 66, has the appearance of *a school exercise*, since it lacks all the names of officials, place names, etc., which would normally, in a real account, be recorded in the cases that are now empty, that is in *obv.* ii:4, *rev.* i:4 and *rev.* ii:3. On the other hand, the text may just have been a first draft for a real account. (To judge from a photo of *MSVO 4*, 66, the clay tablet on which the text is written was rather carelessly manufactured, and the writing on it is not very elegant.) Be that as it may, what makes the text really interesting is that it is an unusually clear example of an account, *not of actual costs*, but of costs in a *planned* economy, computed according to certain arbitrarily imposed rules.[15] Indeed, the varius numbers in *MSVO 4*, 66 seem to have been computed in the opposite direction to what might have been expected, *starting with the grand total* in *rev.* ii:1 rather than with the particular expenses (costs) in *obv.* i: 1b-6b and *obv.* ii: 1b-3b.

[15]For another outstanding example, see the discussion of the field division text *MSVO 1*, 2 in J. Friberg, *Archiv für Orientforschung* 44/45 (1997/98) 1-58.

Actually, the numbers recorded in *MSVO 4*, 66 seem to have been computed as follows, with departure from the given grand total and from the apparent rule that

the average cost of a combined bread-and-beer ration is m4* of grain per *man-day*.

(Here *1 man-day* is short for 1 person ·1 day.) From this basic rule it follows that

the cost of M2 average rations is 2.00 · m4*=30M*=1d*.[16]

Consequently,

the cost of average rations for 1 man-year (= 3 M2 man-days) is 6.00 m4*=3d*.

Therefore, ideally,

the cost of average rations for 20 man-years(=60 M2 man-days) is 20· 3d* 2C*.

For some reason (a fee to the accountant or to the disbursing office?) 9M* = 1/200 · 2 C* is subtracted from this ideal figure, so that the total cost for the 20 man-years is reduced to

C(bread-and-beer)=(2C-9M)*=(1C 2D 4c 1M)*, as in *rev.* ii:1.

In the next step of the computation, the ideal grand total 2C* of grain is divided in two parts, one of bread, the other of beer, in the ratio 3:1.[17] The result is the two sub-totals

C(bread)=3/4 · 2C*=(1C 1D 5d)* for bread, and
C(beer)=1/4 · 2C*=(1D 5d)* for beer.

In the third step of the computation, the bread-total, in its turn, is divided in two parts, one with 1 M2d=10 M2 rations of the larger sizes (M to m5), another with 50 M2 rations of the smallest size (m6). The result is that *C(bread)* is split into two new sub-totals

C (small size bread)=50 M2 · m6*=50 · 20 M*=50 · 4c*=(1C 3d 2c)*, as in *rev.* i:2, and
C (larger size bread)=C(bread)-C(small size bread)=(1C 1D 5d · 1C 3d 2c)*=(1D 1d 4c)*.

Next, of the 10 M2=20.00 larger size bread rations, one half, that is 5M2=10.00, are assumed to be of the size m5, and one half of that, that is 2M2 1C=5.00 of the size m4. The corresponding cost in grain is 4d*+(2d 3c)*. There remains (1D 1d 4c-6d 3c)*=(5d 1c)* to divide between 2 M2 1C=5.00 bread rations of

[16] Is this simple *unitary relation* the explanation for the puzzling use of bisexagesimal numbers when counting rations?

[17] This means that the costs of the average bread and beer rations in this text are 3/4 · m4*, and 1/4 · m4* respectively. Cf. the *extended bread-and-beer text MSVO 1*, 93, where in *rev.* i:1a-b the cost for 1M2 1C 3d=3.30 rations of bread is 3.30 · 3/4 · m4*=(1d 1c 4M 1m4 1m8)*=(1d 1c 4M 1m2)*. That text is discussed also in R. Englund, "Grain accounting practices in archaic Mesopotamia", to appear in J. Høyrup and P. Damerow (eds.), *Changing Views on Ancient Near Eastern Mathematics.*

the three sizes m3, m2, and M. The obvious choice is to let there be M2 breads of the size m3, M2 of the size m2, and C of the size M. The corresponding cost in grain is (1d 2c+2d+2d)*=(5d 2c)*, which is a mere 1c* more than the original allotment.

The amount of grain available for *C(beer)* is the ideal amount (1D 5d)*, diminished by the 1c* usurped by *C(bread)* and by the 9M* subtracted as a fee, or whatever. Hence there is available only (1D 5d-1c-9M)*=(1D 4d 3c 1M)* for 1 Cd=10.00 beer rations, as recorded in *rev.* i:3b. (The rule applied here seems to be that the number of beer rations shall be 1/12 of the number of bread rations.) Of the 10.00 beer rations, one half, that is 5C=5.00 are assumed to have a low cost in grain, only m3* per jar, or (3d 2c)* altogether. Of the next 5C=5.00 beer rations, 3C=3.00 have a larger cost in grain, a round 1 M* per jar, or 6d* altogether. For the remaining 2C=2.00 beer rations, the available grain is reduced to (1D 4d 3c 1M-3d 2c-6d)*=(5d 1c 1M)*, as in *obv.* ii:1b. This corresponds to a cost per jar equal to (1M 3 m10)* for the strongest beer. This is an uncharacteristically non-round number, which can be explained by assuming that the author of *MSVO* 4,66 was unable to find a better solution to the complicated partition problem that he was trying to solve.[18]

The last step of this complicated computation was to find the amount of malt that had to be added to (1D 4d 3c 1M)*, the total cost in grain for the beer. Apparently the *ratio of malt to grain* was assumed here to be 3:5, since

$$3/5 \cdot \text{(1D 4d 3c 1M)}^* = 3/5 \cdot 7.16\ \text{M}^* = 4.21\ 3/5\ \text{M}^* \approx \text{(8d 4c 1M)}^*, \text{ as recorded in } \textit{rev.}\ \text{ii:2.}$$

Other known proto-cuneiform ration texts (see, for instance, *MSVO 1*, 83-111) are nearly as complicated as *MSVO 4*, 66, even if not quite as interesting from a mathematical point of view. All of those texts are what may be called *extended* bread-and-beer texts, for the reason that they are accounts of the distribution of several other kinds of food in addition to the rations of bread and beer. In such texts, the cost of bread and beer is always expressed in terms of system $C(pc)^*$.

There are also some quite interesting ration texts of more modest format. Take, for instance, the two small texts in Fig. 4.1-2 below (both from Larsa, or possibly Uruk). In *MSVO 4*, 60 (Fig. 4.1), two sub-accounts are separated from each other by a double dividing line. In the second account the sume of (1+2+1+3+2+1+1)c=11c is given as the number 1d 5c, preceded by the sign for 'barley'. Obviously, the numbers here are proto-cuneiform capacity numbers, for which 1d=6c. The account can be interpreted as specifying the monthly rations of barley for 7 workers of officials of low rank. The monthly rations of 3, 2, or 1c correspond to daily rations of 3, 2, or 1 M/6, in other words to *m2, m3, or m6 of barley per man-day.* (The average daily ration is close to m4, since 7· m4=210 m4=52 1/2 M=10 1/2 c=1d 4 1/2 c.)

[18]In modern, mathematical terms, the text displays a number of *approximate solutions* to several *systems of linear equations!*

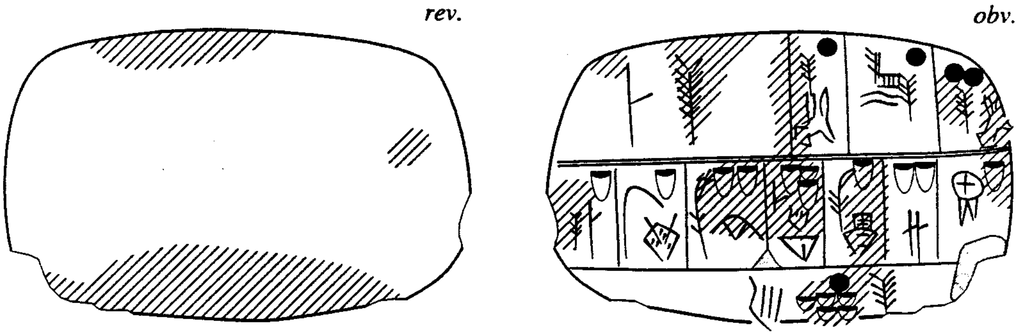


Fig. 4.1. MSVO 4, 60, large and small monthly rations of barley. System C(pc).

This interpretation of the second account on the obverse of *MSVO 4*, 60 makes it likely that the meaning of the first account is similar, namely that the monthly rations of 3 officials are 2, 1 and 1d, respectively, corresponding to 2 *or 1M of barley per man-day.* Thus the largest of the rations is 12 times as big as the smallest.

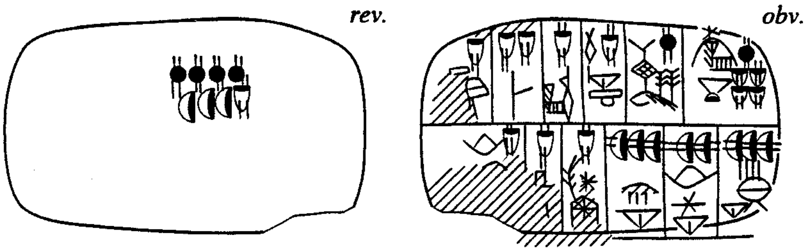


Fig. 4.2. MSVO 4, 65, small monthly rations of emmer wheat. System C"(pc).

In a similar way, *MSVO 4*, 65 (Fig. 4.2), can be interpreted as a record of the monthly rations of 2 officials, 7 male and 3 female workers. The rations are paid out in *emmer (wheat)*, a kind of cereal different from barley. This is shown by the use of numbers belonging to the derived capacity system *C″(pc)*, characterized by number signs tagged with *two parallel strokes.* The monthly rations of the two officials are (1d 4c)″ and 1d″, respectively, corresponding to *1 2/3 and 1 M′ per man-day.* The monthly rations of the male workers are 2 or 1 c″, corresponding to *m3″ or m6″ per man-day.* The monthly rations of the femal workers, however, are only 3 or 2M″, corresponding to *daily rations of no more than 1/2 or 1/3 m5″*. The sum of the monthly rations of the 2 officials and 10 workers is recorded on the reverse of the tablet. It is, correctly, (4d 1c 3M)″.

The next example, *MSVO 4*, 3 (Uqair), is a (probable) ration text, a bit more complicated than *MSVO 4*, 60, but still much less complicated than *MSVO 4*, 66. On the obverse of this text there are two sub-accounts, tagged by the two signs GI and BA, of unknown meaning, but obviously some kind of category markers. In *rev.* i:1-2, two sub-totals tagged by BA and GI are equal to the sums of the capacity numbers recorded in the corresponding sub-accounts on the obverse, separated by a double-drawn dividing line. The grand total in *rev.* ii is 2D 7d 5c of barley, marked by both BA and GI, and attributed to one or two high officials EN ME of an administrative unit with the name KU_6.RAD.UR_2.

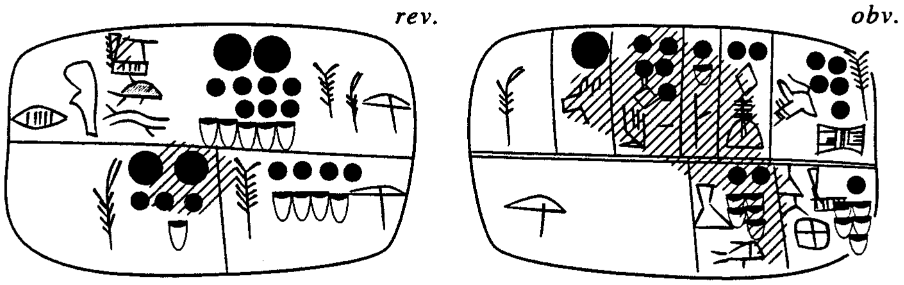


Fig. 4.3. MSVO 4, 3, rations marked BA and GI. System C(pc), large numbers.

The grand total here is equal to 28 d-1c, or nearly 28 d. Although the text itself gives no direct clue to what this number means, it can be interpreted as

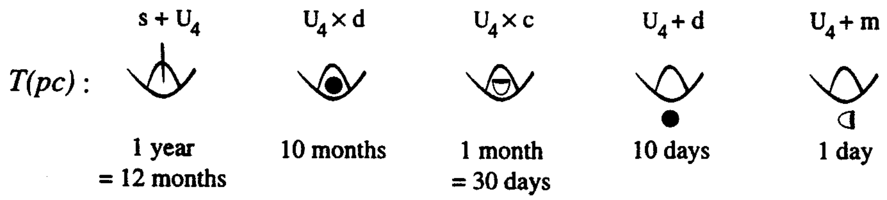
$$28 \text{ d}= 7\cdot 4\text{d}=(1+16)\cdot 24 \text{ d}=(1+1/6) \cdot 10 \cdot 12 \cdot 30 \text{ m5}.$$

Therefore the grand total is an *almost round number*, by which is meant a large and round number plus (or minus) a small fraction of that number. Grand totals equal to almost round numbers are not uncommon in the corpus of proto-cuneiform texts.[19] The significance of the added (or subtracted) fractions is far from clear, however, and may vary from case to case. Anyway, the factorization above of the grand total in *MSVO 4*, 3 suggests that the grand total can be interpreted as rations of barley of the size m5 for 10 man-years, probably for a group of 10 workers for 1 year, alternatively 120=2.00 workers for 1 month, under the supervision of the high officials EN and ME, with an extra 1/6 of the total for the EN and the ME themselves.

5. The proto-cuneiform systems T(pc) and Z(pc) for time. Examples.

One of the unexpected results of the decipherment of the proto-cuneiform number systems was the identification by A. A. Vaiman (1974) and R. K. Englund (1988) of a special system of time numbers, the *proto-cuneiform time system T(pc).* All the number signs in system *T(pc)* are combinations of the sign U_4 'sum, light, day' and a counting number:

[19]Cf. J. Friberg, "Round and almost round numbers in protoliterate metro-mathematical field texts", *Archiv für Orientforschung* 44/45 (1997/98).



The counting numbers are written as ordinary sexagesimal numbers and placed inside the sign U_4 in the case of *months*, they are written as sexagesimal numbers with m instead of c and placed under the U_4 sign in the case of *days*, and they are writen as strokes s at the upper side of the U_4 sign in the case of *years.* The year is clearly an *administrative year* of $12 \cdot 30$ days, not a cultic/agricultural moon-year.

The fragment *MSVO 4*, 27 (Uqair) in Fig. 5.1 below is an interesting example of the use of system $T(pc)$.

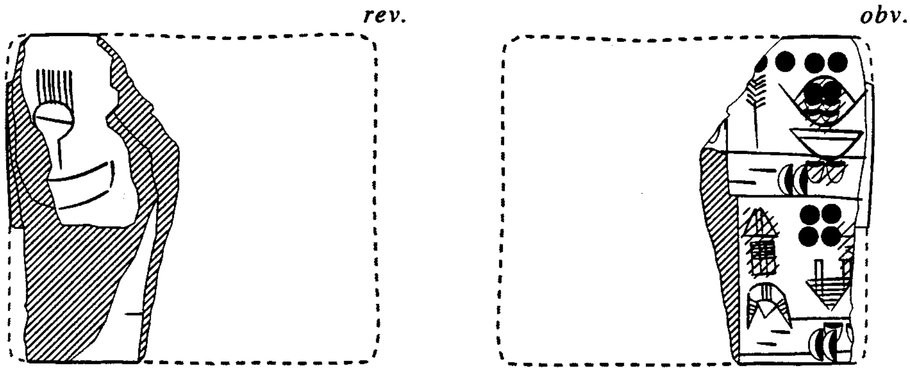


Fig. 5.1. MSVO 4, 27, a ration text with almost round numbers. Systems T(pc) and C(pc).

In the upper row of the obverse of this fragment, a single text case, divided in two parts, is preserved. In the upper part of the case, there is the sign for barley, the sign for grain rations (a picture of the mass-produced *beveled-rim bowl*, holding about 5/6 liters, which is typical for the period), the capacity number 4d, and the time number meaning 24 months. The interpretation is obvious: *24 man-months of rations* of barley (of the *small size* m6) is as much as 4d=24c=24 · 30 m6 of barley.[20]

In the lower part of the same text case is written the capacity number 2c 2M together with a broken line, a sign clearly signifying an *added fractional part.* As a matter of fact, 2c 2M=12M in the lower part of the case is 1/10 of 4d=24c=120M in the upper part. This is one of the few known cases clearly demonstrating how an almost round number can arise. It is possible, but far from certain, that the fragment is part of a monthly account of rations for a group of 24 workers, and

[20] See R. Englund, "Administrative timekeeping in ancient Mesopotamia", *Journal of the Economic and Social History of the Orient* 31 (1988), 162-164, or *Archaic Bookkepping* (1993) 70-71.

that the added fractional part of the barley total was paid out to the official in charge of the group.[21]

A special kind of (probable) time notation, different from the usual system $T(pc)$, seems to be used in *MSVO 4*, 55 (Larsa, or Uruk; Fig. 5.2).

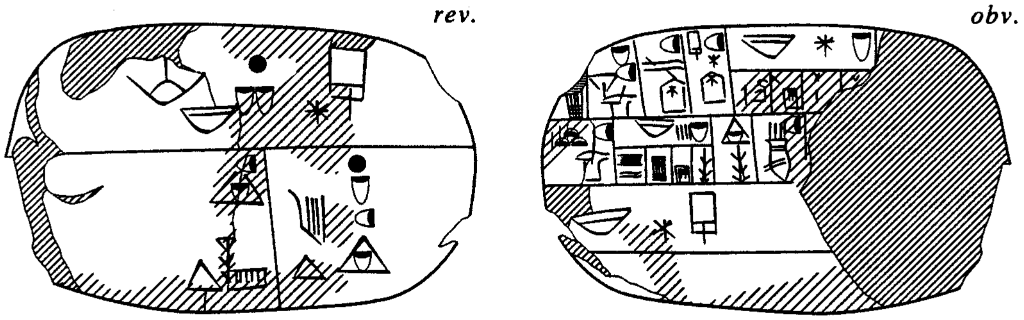


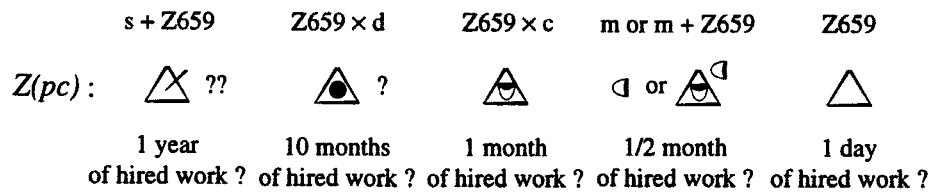
Fig. 5.2. MSVO 4, 55, an account of 12 months of hired labor(?). System Z(pc).

In *rev.* i of *MSVO 4*, 55, two sub-totals are recorded, together with the titles of two officials. The first sub-total is probably to be read as '11 1/2 Z659 $\times$ c, the Z659 of the EN', where Z(ATU) 659 is a sign of unknown meaning (numbered as in the sign list of *ATU 2*). Similarly, the second sub-total is '1/2 Z659 $\times$ c, the Z659 of the ŠU'. In *rev.* 2 is recorded the grand total 12 (meaning 12 Z659 $\times$ c), the name of an administrative unit, the sign representing the ration bowl, and the time notation for 1 year in system $T(pc)$. The interpretation of this text proposed here is that Z659 is a sign meaning 'hired worker' or *a manday of hired work.*[22] Furthermore, the sign combination Z659 $\times$ c may have the meaning *1 man-month of hired work*, in the same way as the sign combination $U_4\times$ c is known to mean 1 man-month (possibly of work by dependent workers). The circumstance that the grand total '12' (man-months of hired work) seems to be "explained" in the text by the sign for '1 year' may serve as supporting evidence for the suggested interpretation.

The scarcity of examples of the use of Z659 and related signs makes it difficult to decide if there ever was a proto-cuneiform system of time numbers for hired work, which could then be called system $Z(pc)$. The following diagram is only tentative but is useful in that it shows the forms of the signs discussed here:

[21] More complete texts of similar type are *MSVO 1*, 121 and 122. In the former text, 1 month and 5 days of barley rations of the relatively large size m2 is as much as 3c 2M 1 m2, to which is added 1/10 of this number. In the latter text, 3 years of barley rations of the same size m2, plus the added 1/10, is equal to the almost round number 1D 8d+1d 4c 4M=1D 9d 4c 4M.

[22] See J. Friberg, *Orientalistische Literaturzeitung* 89 (1994) 495, where it is demonstrated that the protocuneiform sign Z659 and its supposed pre-literate predecessor, the token known as a "small tetrahedron", probably both denoted a man-day of *hired work.*



The first of these signs, s+Z659, is attested only, completely out of context, in Vocabulary 3 (*ATU 3*), a lexical list. The second sign is known only from two very small fragments of clay tablets. No examples are known of texts where the proposed values of the signs can be confirmed. Furthermore, as shown by the reverse of *MSVO 4*, 55 (Fig. 5.2), multiples of Z659 × c are *counted*, in contrast to multiples of $U_4\times$ c in system *T*(*pc*), which are written as $U_4\times$ 2c, $U_4\times$ 3c, etc.

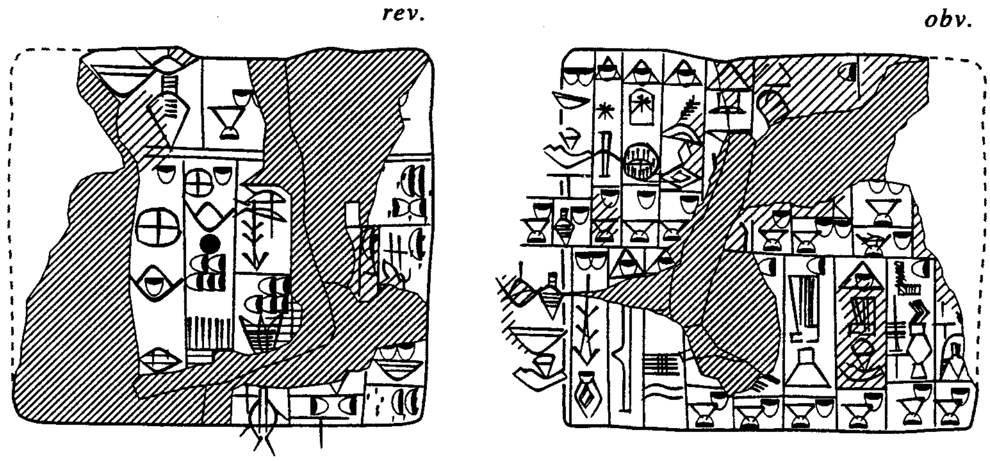


Fig. 5.3. MSVO 1, 146, a large account of monthly(?) rations for hired labor(?)

Anyway, the interpretation of *MSVO 4*, 55 proposed above seems to be confirmed by the texts *MSVO 1*, 146-151 (Jemdet Nasr), which are all concerned with accounts of Z659, that is probably with hired labor. In these texts, months of hired work are counted, apparently, as in *MSVO 4*, 55, that is in multiples and half-multiples of Z659 × c.[23]

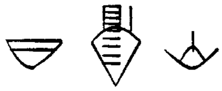
In the very smal fragment *MSVO 1*, 149, the sign Z659 × d(!) may possibly

[23] Cf. the text W 9168,h+(*ATU 5*, pl. 40), script phase III, with apparently two sub-accounts of ' 'bread and beer for 1 year' in about 50 individual text cases. In each text case is written the title of some official and a *bisexagesimal* number, obviously representing the number of rations distributed daily to his subordinates. The bisexagesimal numbers range from 2.56 to 7. Note in particular 2.37 rations for the "children', 2.53 for the highest official, the EN, 1.00 for the BU.PAP.NAM$_2$, and 40 for the PA of the manufacturing unit(?) AN.MAR, 20 or 10 for several groups of male workers, and 11, 11 1/2, or 12 1/2 for three groups of female workers. The total number of rations mentioned in this text seems to be close to 1600.

In other texts, large numbers of people seem to be counted *sexagesimally*. One such text is W 9656,g (*ATU 5*, pl. 86), script phase IV. In that text, which begins with the same names as the canonical lexical text called the Lú A list (see Englund and Nissen, *ATU 3* (1993) 17), the numbers in the individual text cases are 2.00. 1.40, 1.00, 40, 30, 20, or 10. The total is given on the reverse as 31.50 (=1910).

indicate 10 months of hired labor. In the badly preserved fragment *MSVO 1*, 147, a number of officials account for each 1 month of hired labor, with an indicated monthly (?) ration of 1 ŠITA (written as the image of a covered cup; meaning unknown). The EN (the city ruler or chief administrator) accounts for 2 1/2 particularly large monthly (?) rations, each consisting of 1 jar of beer, 1 ŠITA, and 1 SILA$_3$× KU$_6$, written as the image of a cup with a fish inside; meaning unknown. (Cf. W 10736, *ATU 2*, pl. 45, which begins by mentioning 1.16=(76) Z659, and which then goes on to mention various multiples of NINDA (or GAR), ŠITA, SILA$_3$× ŠU, etc.)

MSVO 1, 146 (Fig. 5.3 above) is a similar text, somewhat better preserved. Unfortunately, the number in the total is not preserved, although the specification of the total, 'bread and beer for 1 year', written as



is preserved. Also in this text, both whole and half months of hired labor are mentioned. In a separate sub-account seem to be listed some of the standard ingredients in an extended bread-and-beer text, including '1 sheep every 15th day' (or 1 sheep 15 days old?) for one particularly well-paid worker and '1 sheep every month' for another.

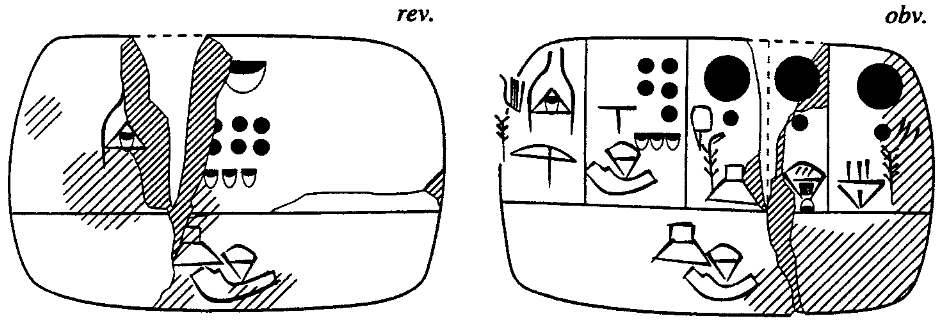


Fig. 5.4. MSVO 1, 26, monthly wages for hired labor? Almost round numbers. System C(pc).

In *obv.* i:1-3 of *MSVO 1*, 26 from Jemdet Nasr (Fig. 5.4), each one of three officials is associated with the almost round capacity number 1D 1d=(1+1/10)· 1D, while in *obv.* i:4, a fourth official is associated with the number 5d 3c = (1+1/10) · 5d. In *obv.*i:5, these four entries are accompanied by the phrase ŠU ŠE 'barley in the hand?', or 'the ŠU of barley', the sign combination NINDA$_2$× Z659 × c, and the tag BA. In Sumerian cuneiform texts, the sign šám=nínda × še (ninda$_2$ together with the sign for 'barley') can mean 'price in barley, barley equivalent, etc'. Therefore, it is possible that the meaning of the specification in *obv.* i:5 is that the recorded numbers are the barley equivalents of certain numbers of man-months of hired work. Assuming, for instance, that the three first officials were in command of 10 hired workers, the fourth official of 5, the resulting conclusion must be that the *wages* of each hired worker was paid to him in the form of extra large rations equal to 1M of barley per day. Indeed,

$$1D=10d=10 \cdot 30 \text{ M, and } 5d=5 \cdot 30M.$$

For his trouble, each official seemingly got for himself 1/10 of the wages of each hired worker under his command.[24]

The interpretation above of the text shown in Fig. 5.4 was partially based on the assumption that teams of hired workers were counted in multiples of 5. This assumption is supported by the evidence of the next example, *MSVO 1*, 84 (Fig. 5.5 below):

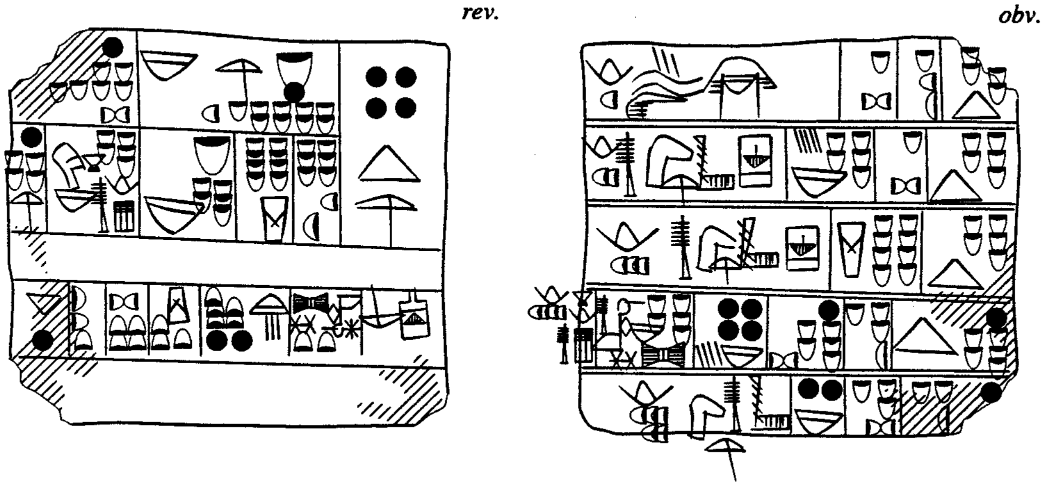


Fig. 5.5. MSVO 1, 84, rations tagged BA, for 40 hired workers. Systems S(pc)?, B(pc), and C(pc).

On the obverse of this text, there are five sub-accounts, labeled 'day 1', 'day 2', etc. Although several details of this text remain difficult to understand, a reasonable interpretation seems to be that the obverse is an account of 1 day's rations distributed to five groups of 5, 5, 5, 15, and 10 hired workers (sign Z659), respectively. The daily rations vary strongly from group to group, yet in such a way that the grand total on the reverse, tagged BA, specifies that 40 hired workers receive together 1.19 1/2 NINDA rations (GAR=NINDA 'bread'?, 'grain ration'?), apparently all of cost M, and 19 ŠITA (?) rations, apparently of cost m2. This makes almost precisely 1 NINDA ration of cost 2 M, and 1/2 ŠITA ration of cost m2 per hired worker and day. This tentative interpretation can be compared with the known fact that in Old Babylonian mathematical texts dealing with, for instance, the maintenance of canals, the daily wages paid to hired workers (*á lú-hun-gá*) were normally 6 barley-corns of silver (Neugebauer and Sachs, *Mathematical Cuneiform Texts* (1945), 81), or, equivalently, 1 bán=10 sìla of barley (*op. cit.*, 76). Recall that 2M is about as much as 10 liters, which in its turn is about the same as 10 sìla.

Other proto-cuneiform texts can be interpreted as implying large rations(?) of 1/2 or 1M per man-day. So, for instance, in the small text *MSVO 1*, 121 (cf.

[24]The text *MSVO 1*, 27 is similar, although there the sign $NINDA_2 \times$ Z659 $\times$ c is replaced by the less specific sign GU_7 (a man's head in front of a ration bowl), which (presumably) has the meaning 'rations'.

Englund, "Timekeeping" (1988), 152), the wages? for 1 month and 5 days (= 35 days, tagged GI, are explicitly stated to be 3c 2M m2 (=17 1/2M) of barley, plus the added fraction 1M m2 m6 (error for m4!). The computation can be explained as referring to

(1+1/10) · 1/2 M per man-day for 35 man-days *MSVO 1*, 121

Similary, in *MVSO 1*, 122 ("Timekeeping", 157), wages? of 'm2 for 3 years' are said to be equal to 1D 9d 4c 4M, where

1D 9d 4c 4M=19d 4c 4M=20d−6M=99 · 6M=(1+1/10)· 12 · 30 · m2.

This means that the recorded total in this text must have been computed as

(1+1/10) · 1/2 M per man=day for 3 man-yers *MSVO 1*, 122

Interestingly, the very same numbers appear also in the extended bread-and-beer text *MSVO 1*, 89, so that the recorded total in that text, too, was computed as

(1+1/10) · 1/2M per man-day for 3 man-years *MSVO 1*, 89

In the grand total of another example, *MSVO 1*, 90, the wages? for *3 man-years* amount to 1C 92 3c 3m, or precisely twice as much as in *MSVO 1*, 89, corresponding to

(1+1/10) · 1M per man-day for 3 man-years *MSVO 1*, 90

Similarly, in *MSVO 1*, 94, an extended bread-and-beer text with two sub-accounts, one of the two grand totals specifies that the wages(?) for *4 man-years* are

1C 2D 2d=52d=(1+1/12) · 48d=(1+1/12)· 4 · 12 · 30M,

corresponding to

(1+1/12) · 1M per man-day for 4 man-years *MSVO 1*, 94a

This result is confirmed by the grand total for the second sub-account which gives the wages(?) for 6(!) man-years as 1 1/2 times as much, or

2C 1D 8d=78d=(1+1/12)· 72d=(1+1/12)· 6· 12· 30M.

Hence, in this case, too, the wages(?) per man-day can be shown to be

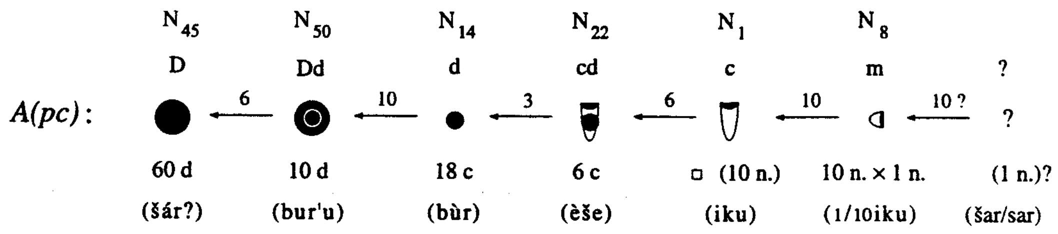
(1+1/12) · 1M per man-day for 6 man-years *MSVO 1*, 94b

It is very annoying that the three interesting texts *MSVO 1*, 89-90 and 94 are all damaged to such an extent that it is impossible to know exactly what they mean.

6. The proto-cuneiform system A(pc) of area numbers. Examples.

The proto-cuneiform (and proto-Elamite) area system *A(pc)* differs only in minor details from the more well-known area system appearing in a great number of Sumerian and Babylonian cuneiform texts. Its factor diagram is shown below.[25]

The basic unit in the proto-cuneiform area system is the iku, 1c, defined as the area of a square of side 10 minda ($\approx$ 60 meters), probably the length of a *measuring rope.*



A particularly interesting and important example of a proto-cuneiform text with an application of this system is *MSVO 1*, 10 below (Jemdet Nasr).

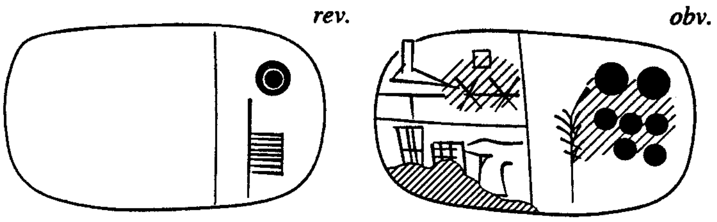


Fig. 6.1. MSVO 1, 10, an area-and-seed text. Systems C(pc) and A(pc).

On the obverse of this text, the capacity number 2D 5d of 'barley' is specified by the tag GI (or a badly written ŠE for 'barley') and is followed by the titles of two officials. On the reverse, the area number 1Dd is accompanied by a sign with the known meaning 'field' or 'area of a field'. The most likely interpretation of the text is that it says that the seed needed for the sowing of a field of area 1Dd (bur'u)=10d=30 cd=3.00 c(iku) is 2D 5d=25d=2.30 c of barley. Since 2.40=5/6 · 3.00, it is clear that in this particular case the implied *sowing rate* must be

5/6 c of barley on 1c (= 1 iku) of field.

It is interesting to observe that if the sowing rate had been instead *1 c of barley on 1 c of field*, then this simple rule could have been reformulated as *1 d of barley on 1 cd of field, or 1C of barley on 1 Dd of field.* Similar simple rules, so-called *unitary relations*, are well known from several categories of Sumerian and Babylonian administrative texts.

[25] The fact that the proto-cuneiform sexagesimal system and the proto-cuneiform area system were both essentially identical with their Sumerian and Babylonian successors was established by F.-M. Allotte de la Fuÿe in a paper in *Revue d'assyriologie* 27 (1930), 65-71, devoted to the study of the *field division text MSVO 1*, 1. See now also J. Friberg, "Round and almost round numbers in protoliterate metro-mathematical field texts" *AfO* 44/45 (1997/98).

MSVO 4, 57 is a *field division text* from Larsa/Uruk, in which 1D 4Dd 1d 1cd=30.24 iku, a large tract of land, is divided between 5 high officials. Similar field division texts, deriving from Jemdet Nasr, are *MSVO 1*, 1-6, where huge tracts of land are divided between 5 high officials. These and many other proto-literate "field texts" of various categories are discussed in great detail in Friberg, "Round and almost round numbers".

The next text, *MSVO 4*, 45 (Larsa/Uruk), is apparently concerned only with capacity numbers but will be shown below to be related to the preceding area-and-seed text. On the obverse of this text, two sub-accounts are separated by a double dividing line. The first of these contains 7 cases, *obv.* i:1-7, each with the title of an official and one or two capacity numbers, for barley and emmer (a kind of wheat), respectively. In *obv.* 8 is noted the name of the responsible high official, the PA of AN.MAR. In the second sub-account there is only one case with the title of some offcial and two numbers for barley and emmer. Here the responsible high official is the BU. PAP.NAM$_2$ of AN.MAR.

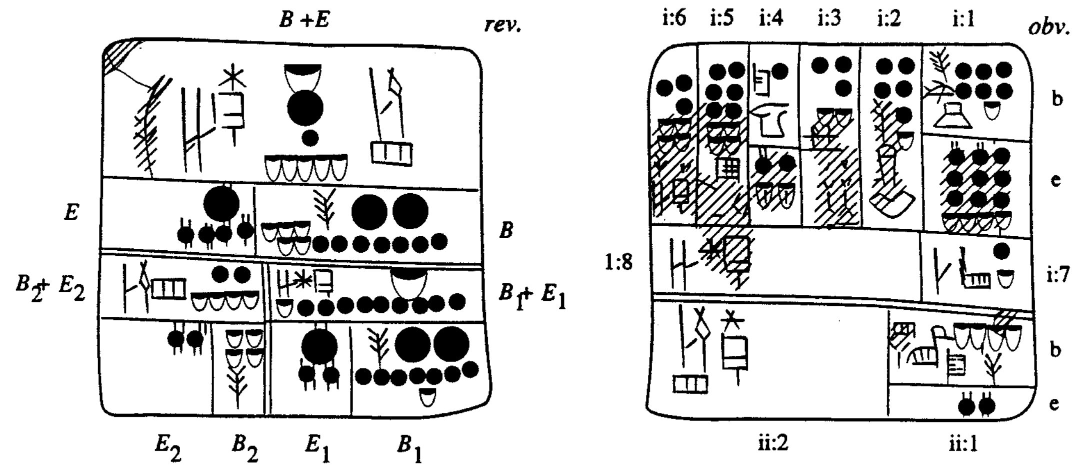


Fig. 6.2. MSVO 4, 45, a complex summation of barley and emmer. Systems C(pc) and C"(pc)

Two sub-totals, one for the PA of AN.MAR, the other for the BU.PAP.NAM$_2$ of AN.MAR, are recorded in rev. i:1-2, two cases framed by double dividing lines. The grand total is recorded in rev. ii. The sub-totals and the grand total are all *complex*, in the sense that they record first *separate totals* for barley and emmer, respectively, then the *combined total* for barley and emmer together. To be more precise, if the separate sub-totals in rev. i: 1b1 and i:1b2 are called B_1 and E_1, respectively, then the combined sub-total in *rev.* i:1a is equal to the sum $B_1 + E_1$. Similarly, if the separate sub-totals in *rev.* i:2b1 and i:2b2 are called B_2 and E_2, respectively, then the combined sub-total in *rev.* i:2a is equal to the sum $B_2 + E_2$. Consequently, the separate grand totals in *rev.* ii: b1 and *rev.* ii:b2 are the sums $B = B_1 + B_2$, and $E = E_1 + E_2$, respectively. The combined grand total in *rev.* ii:a, finally, is equal to the sum $B + E = B_1 + B_2 + E_1 + E_2$.

The tentative interpretation suggested here of *MSVO 4*, 45 is that this text is a record of the *barley and emmer seed*, respectively, for each of 7 fields managed by the PA of AN.MAR and 1 field managed by the $BU.RAP.NAM_2$. The sowing rate may be assumed to be the same as in the Jemdet Nasr text *MSVO 1*, 10 (Fig. 6.1), that is 5/6 c per iku, or 5c per èše (6 iku). Sure enough, it is easy to see that the recorded grand total $B + E$ =1C 1D 1d 5c=51 d 5c=251 c=5· (50 c 1 M)≈ 5 · 50 c. Therefore, $B + E \approx 5/6 \cdot 50$ d, and a reasonable tentative conclusion is that

> *MSVO 4* 45 is record of 1C 1D 1d 5c=41 d 5c=251 c of barley and emmer, the seed(?) for 7+1 fields with a total area of approximately 50 èše, at a sowing rate of 5 c per èše.

Cf. the survey in the Conclusion of "Round and almost round numbers", where it is shown that in proto-cuneiform field texts the total area is usually *close to either a multiple of 5 èše, or such a multiple increased by a simple ractional part (1/6, 1/9, or 1/10) of itself.*

MSVO 4, 45 is not the only example of its kind. There are quite a few proto-cuneiform *barley-and-emmer texts like MSVO 4*, 45, from Larsa/Uruk or Jemdet Nasr, characterized by having a grand total that is very close to an integral multiple of 25 c. Thus, for instance,

> *MSVO 4*, 43 is a record of 8C 2d 1c=1453 c of barley and emmer, the seed(?) for 8+2 fields with a total area of approximately 4.50 (=290) èše, at a sowing rate of 5 c per èše.

Similarly,

> *MSVO 4*, 48 is a record of 8C 1d 2c=4.01 d 2c=1448 c of barley and emmer, the seed(?) for 7 or 8 fields with a total area of approximately 4.50 (=290) èše, at a sowing rate of 5c per èše.

MSVO 4, 43 and 45 are *complex accounts* in the sense that their grand totals are obtained through complex summation of the sub-totals of barley and emmer wheat for two sub-accounts. *MSVO 4*, 48 is simpler since there is only one account, although with several entries, on the obverse.[26] *MSVO 4*, 51 is even simpler since its obverse records just one amount of barley and one of emmer wheat. Except for this more modest format, *MSVO 4*, 51 is of the same type as the *MSVO 4*, 43 and 45. Indeed,

> *MSVO 4*, 51 is a record of 2C 6d 5c=1.06 d 5c=401 c of barley and emmer, the seed(?) for 1 or 2 fields with a total area of approximately 1.20 (=80) èše, at a sowing rate of 5c per èše.

Actually, in addition to the proto-cuneiform barley-and-emmer texts with grand totals close to a multiple of 25c, tentatively interpreted here as seed texts corresponding to a sowing rate of 5c per èše, there are also other proto-cuneiform

[26]Interestingly, the former two, which are very similar to each other, are tagged GI, while the latter is tagged BA.

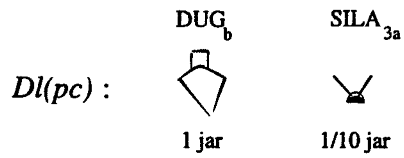
barley-and-emmer texts with grand totals close to a multiple of 20c. They can be interpreted as seed texts corresponding to a sowing rate of only 4c per èše. Thus, for example,

> *MSVO 4*, 46 is a record of 2C 3d 2c=1.03 d 2c=380 c of barley and emmer, the seed(?) for 1 field with a total area of precisely 1.35 (=95) èše, at a sowing rate of 4 c per èše.

All the mentioned suspected seed texts are from Larsa/Uruk. Several other texts of very much the same kind are from Jemdet Nasr, published in *MSVO 1*.

7. The proto-cuneiform systems Df(pc) and Dl(pc) for dairy products.

In a recently published paper dealing with "Late Uruk cattle and dairy products" (*Bulletin on Sumerian Agriculture* 8 (1995)), R. K. Englund identified two previously unknown systems of number notations from script phase III, used to denote fractions of 'jars' containing dairy products (liquids or semi-liquids and fats derived from cow milk). One of these two systems, here called system *Dl*(*pc*) for "dairy liquids?", has the following factor diagram:



The jar figuring in this system is the "spoutless jar" DUG_b, denoting dairy products, which is different from the "jar with spout" DUG_a, the basic form of the sign KAŠ denoting beer (see the three examples in *MSVO 4*, 66, *obv.* ii:1-3 (Fig. 3.1). The indicated fraction, 1/10 of a jar, is the SILA_{3b}, apparently a pictographic representation of the massproduced so-called *Blumentopf*, a conical bowl with foot, which followed and for some time in the Late Uruk period co-existed with the beveled-rim bowl NINDA. The capacity of the *Blumentopf* was about the same as that of the beveled-rim bowl, around 5/6 liter. This may be one of the reasons why in the Sumerian cuneiform the sìla (= SILA_{3b}) and is sexagesimal multiples, the bán=10 sìla, and the barig= $6 \cdot 10$ sìla, came to form the basis of a new system of capacity measure, which replaced the proto-cuneiform systems *C*(*pc*). At the same time, the proto-cuneiform system *Df*(*pc*) itself lived on in the form of a Sumerian system of numbers for liquid capacity measure in general, with the basic units dug (= DUG_a) and sìla (= SILA_{3b}).

The only known attestation, at present, of the use of system *Dl*(*pc*) is the following text from script phase III (Englund, *BSA* 8 (1995), 43):

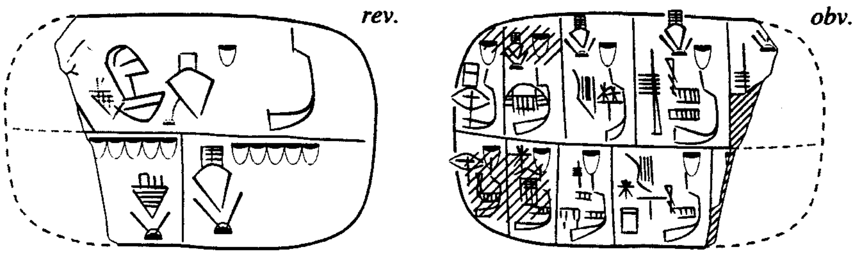
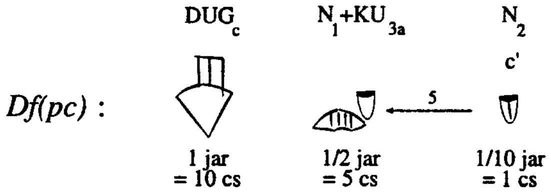


Fig. 7.1. W 21682, two kinds of dairy liquids, GU_7 rations. System Dl(pc).

The reverse of this text contains two sub-totals and a total. The sub-totals are 5 $SILA_{3b}$ of each of two kinds of dairy liquids (milk?). The total mentions the sum of the two subtotals (= 10 $SILA_{3b}$) as 1 DUG_b (1 spoutless jar), with the tag GU_7 'rations'.

The second system of number notations for dairy products identified by Englund, here called system $Df(pc)$ for "dairy fats?", is attested by a large number of accounts belonging to script phase III, almost all from the same find place (locus W 20274). The factor diagram for this system is the following:



This factor diagram should be interpreted as saying that the "stroked spoutless jar", DUG_c=$DUG_b\times$ s is equal to 10 "stroked units" N_2 =c'=c$\times$ s, and that 5 such stroked units can be replaced by KU_{3b} (meaning in this connection unknown). Example:

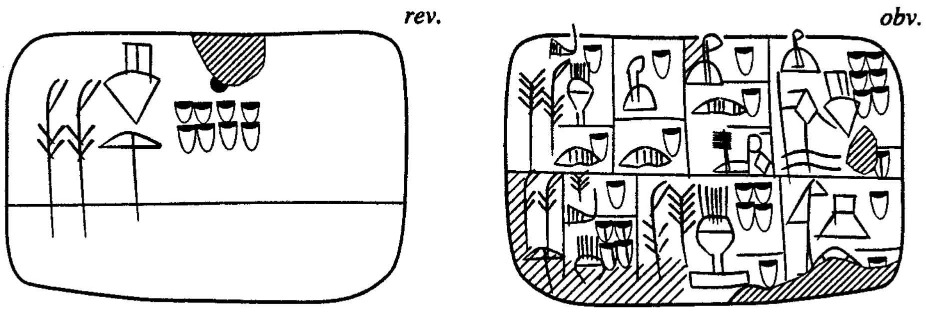


Fig. 7.2. W 20274,35, an account tagged BA. System Df(pc).

For some reason, in each text case of this (and other) texts making use of system $Df(pc)$, fractions of a jar are separated from whole multiples of a jar by a horizontal line. Nevertheless, all the number entries on the obverse of this text are added together, with the resulting total written on the reverse. The correctly executed computation of the total is easily reconstructed. It must have run as follows:

(6 1/10 + 1 1/2 + 1 1/2 + 1 1/2 + 1 1/2 + 4 1/2+ 14$\cdot$ 1/10) jars=18 jars. W20274,35

8. Additional examples. Texts from script phase IV.

In the discussion so far in this paper, interesting applications of the various protocuneiform number systems in administrative texts have been illustrated by examples drawn from *MSVO 4*, in three cases (Figs. 5.3-5 and 6.1) from *MSVO 1*. All these examples are texts from script phase III (3100-3000 B.C.), presumably from either Uqair or Larsa/Uruk, or Jemdet Nasr in the case of the texts taken from *MSVO 1*. In the remainder of the paper, additional examples will be picked from *ATU 5*. This new volume contains carefully drawn hand copies of nearly 900 clay tablets or fragments, of which about 600 were published in less readable form by Falkenstein in *ATU 1*. Most of the texts are from script phase IV (3200-3100 B.C.), that is from the very beginning of the literate period.

One might have expected that so early texts would be of a particularly primitive appearance, with a relatively undeveloped form of the script and of the number systems used. This is not at all the case. The repertory of proto-cunieform signs is, if anything, larger in the texts from script phase IV, and *all the number systems* used in script phase III seem to have been used also in the preceding script phase IV. It is easy, for instance, to find examples of the application of systems *S, B, C, T, A*, as well as of the derived systems C^*, C'', C' among the texts in *ATU 5*. There is even, as will be shown below, one number system that is applied *only* in texts from script phase IV (system E). It is further true that the various *text categories* that can be identified in texts from script phase III have representatives also among the more ancient texts in *ATU 5*. The only difference is that there are more large and complex texts from script phase III texts than there are from script phase IV, but possibly only because of different find circumstances.[27]

The claim that all the number systems of script phase III were known in the preceding script phase IV is nicely supported by the example below (Fig. 8.1) from *ATU 5*, pl. 57. In this text (almost certainly just a writing exercise), *sexagesimal* numbers (system $S(pc)$) is used in *obv.* ii:1 to count some kind of food(?), *capacity* numbers for grain costs and malt (systems $C^*(pc)$, and $C'(pc)$) appear in *obv.* ii:2-3, and *bisexagesimal* numbers (system B^*) are used to count a fourth kind of food(?) in *obv.* ii:4.

[27] The texts in *ATU* 5 are all copies from clay tablets which after having been discarded in antiquity were deposited in trash holes or used as fill in constructions of walls and floors. The tablets copied in *MSVO 4*, on the other hand, were bought from antiquities dealers and therefore represent the "top of the crop". The Jemdet Nasr tablets copied in *MSVO 1* were found where they had been left five thousand years earlier, in a building complex ravaged by fire.

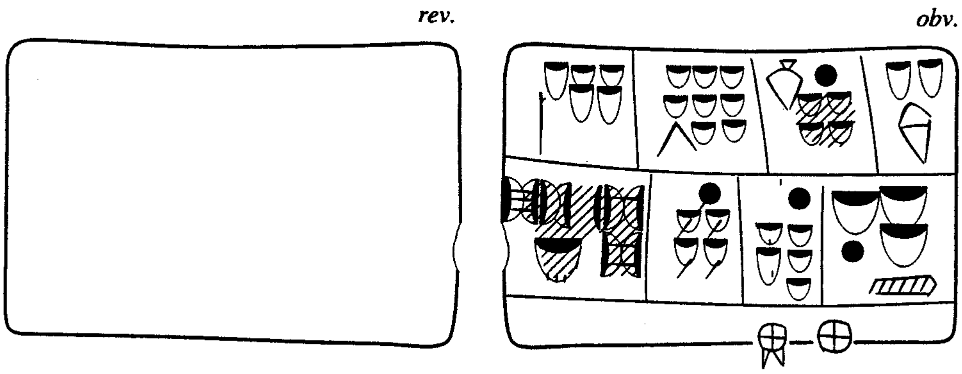


Fig. 8.1. W 9393,e, a writing exercise(?). Systems S(pc), B*(pc), C*(pc), and C′(pc).

Similarly, W 8273 (*ATU 5*, pl. 32) is a text with, apparently, 4 ration totals expressed in the three systems $B(pc), S(pc)$, and $B^*(pc)$. The recorded numbers can be interpreted as the grand totals of monthly rations for a group of 20 workers obtaining 5 M2=20· 30 (barely) *rations of size m3*, 5C=20 · 30 rations of 1/2 each of two kinds of food (unknown which), and 5M2* = 20 · 30 rations some other kind of food. (The 5M2=10.00 rations of size m3 in W 8273 can be compared with the 2 M2 1 C5? d=5.50 *rations of size m5* in W 21021 (*ATU 2*, pl. 55), probably the monthly rations for a group of 10 dependent workers, plus 1/6 of the sum of their rations for the official in charge of them.

The text in Fig. 8.2 below, W 5233,a (*ATU 5*, pl. 1), demonstrates that the fractional units of system $C(pc)$ were in use already in script phase IV. It is one of several probable bread-and-beer texts in *ATU 5* reminding of the more complex text from script phase III in Fig. 3.1. The text begins by mentioning certain numbers of rations (bread?) of progressively smaller sizes: 4(?)M, 2 m2, and 2 m3, together

4M+2 m2+2m3=5 M 2m3=1c m2 m6.

The total cost (in barley?), 1c 4M m2 m6, is recorded on the reverse. This means that the total cost of the 17 units of 7 kinds of rations recorded in *obv.* i: 4-7 and ii:1-3 ought to be equal to

$$4p + 2q + 1r + 1s + 2t + 2u + 5v = \text{1c 4M m2 m6-1c m2 m6=4M.}$$

If the cost of 1 unit of each one of the 7 kinds of rations mentioned had been precisely m4=1/4M, then the total cost of the 17 units would have been 17 m4=4M m4, which is very close to the correct total 4M. Therefore, the average cost is close to m4, and a reasonable conjecture is that the cost of 1 unit in the 7 different cases varies from perhaps m3 to m6. If a sufficient number of texts of this kind had been available, it would have been possible to find out exactly what these costs per unit were (the coefficients p, q, r, s, t, u, v in the equation above).

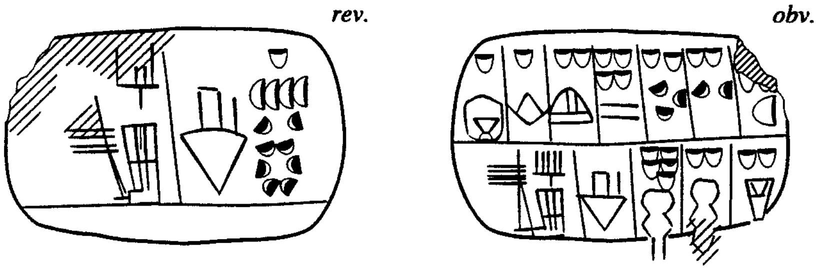


Fig. 8.2. W 5233,a, a bread-and-beer text with fractional units in system C(pc).

A particularly interesting example from *ATU 5* (pl. 65) is the following text with large numbers for barley and emmer:

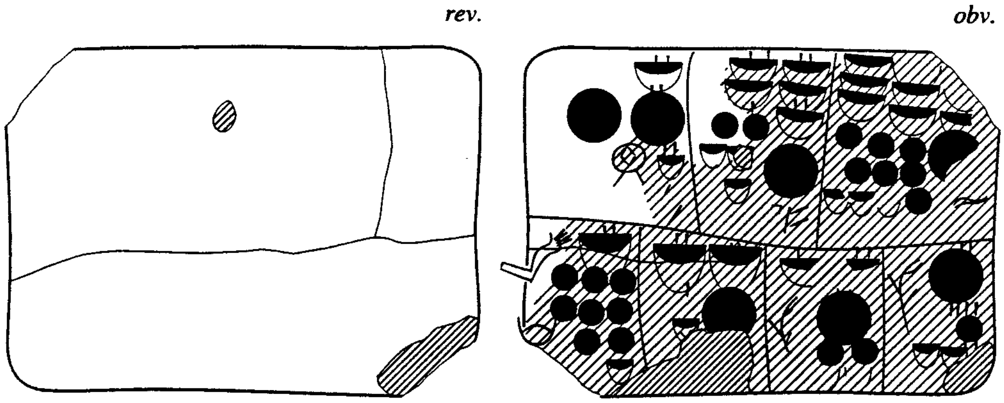


Fig. 8.3. W 9579,w, a seed(?) text for barley and emmer. Mixed systems C(pc) and C"(pc).

On the obverse of this text there are 7 text cases with large capacity numbers for combined amounts of barley and emmer, probably the totals from 7 individual accounts on separate clay tablets. Some of the text cases display the tag GI. The grand total of the 7 numbers is not given (the reverse is empty), so the text appears to be unfinished. However, if the missing grand total is computed, it is found to be

$$20 \text{ C } 7\text{D } 20 \text{ d } 11\text{c} = 23\text{C } 1\text{d } 5\text{c}{=}1.09.11\text{c}{=}4151\text{c}{=}5\cdot\ (830 \text{ c } 1\text{M}){\approx}\ 5 \cdot 830 \text{ c.}$$

The most likely interpretation, therefore, is that

> W 9579, w is a record of 23 C 1 d 5 c = 1.09.11 (4151) c of barley and emmer, the seed(?) for 7 fields with an area total of approximately 13.50 (830) èše, at a sowing rate of 5 c per èše.

Large capacity numbers appear also in what may be the grand totals of ration texts. An interesting, though simple, example is the little text below (W 9656, k; *ATU 5*, pl. 86):

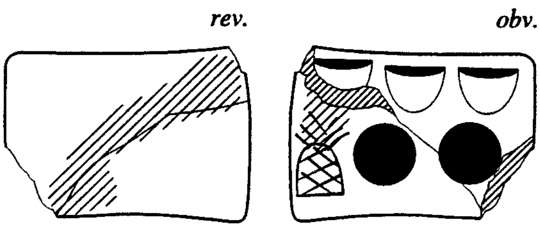


Fig. 8.4. W 9656,k, the grand total of a ration text(?). System C(pc)?.

In this text there is only the sign NESAG (of unknown meaning) and the *almost round capacity number* 3C 2D=11D, which can be interpreted as, for instance,

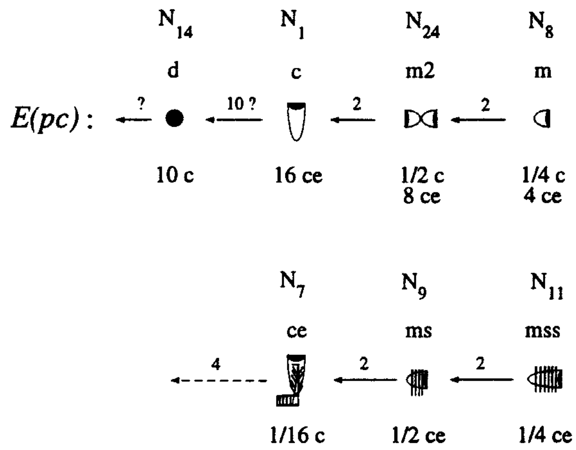
> 3C 2D=11D=11.00c=(1+1/10)· 60 · 12 · 30 m6, that is rations of 1 m6 per day for 50 persons for 1 year, plus an extra 1/10 for the official in charge.[28]

This little text should be compared with the more elaborate text *MSVO 4*, 1 (and the parallel text *MSVO 4*, 2). *MSVO 4*, 1 has the grand total 3C 2D, recorded on the reverse, and 8 individual entries recorded on the obverse. The individual entries are maked from 1s+U_4, probably meaning 'year 1', to 8s+U_4, probably meaning 'year 8'. There are also several signs on the obverse apparently indicating field names. However, the *extreme concisensess* and the resulting ambiguity of the text (in varying degrees a *recurring feature* of all proto-cuneiform inscriptions) makes the interpretation very difficult. The signs 'barley', '8 years', and GU_7 (head+bowl)='rations?' on the reverse of this text seem to indicate that the total 3C 2D here is the total of *grain rations* administered (by a named official) *for 8 successive years.* But why are diffferent field names and/or names of officials recorded in the successive entries? Maybe a better interpretation is that *barley for 1 year's rations* are collected *from 8 different officials in charge of 8 different fields.* Yet another possible interpretation is that the text records the *seed for 8 different fields* which will eventually *yield 1 year's barley rations* for the subordinates of the named officials. Note, in passing, that *only the grand total* on the reverse is an almost round number, not the individual numbers on the obverse. (This phenomenon, too, is a recurring feature of many proto-literate texts.)

9. The proto-cuneiform system E(pc) of weight(?) numbers. Examples.

As will be shown below, the *system E(pc)* has the following factor diagram:

[28]Unless, of course, the text is a record of the seed for (1+1/10)·2.00 èše at a sowing rate of 5c per èše.



This system was first studied by A.A. Vaiman in 1974. It is currently attested in only 26 texts, all belonging to script phase IV. Of the 25 texts, 22 are from a find place in Uruk within the excavation square Qa XVI 2. Of these 22 texts, all but one (W 9656,61+dx; Fig. 9.3 below) are *single accounts.* System E is further attested in 2 texts from the nearby square Qa XVI 3, and in 1 text from another nearby square Pe XVI 3 (see the maps in *ATU 5*, 13-14). Of these three texts, one is a single account like the ones from square Qa XVI 2, while two are *combined accounts* (W 9393,b and W 7227,e; Figs. 8.3-4 below).

One tablet (W 19530,b: still unpublished), almost certainly a simple writing exercise (?), was found far away from the others, in square NCXVI 2. With the exception of the doubtful example W 19530,b, all the texts with a single account look like the examples in Figs. 8.1-2 (see *ATU 5*, pls. 61-62, 66-67, 69-71, 73-75, 80-81, 85-87, 93, 110).

Thus, these texts with a single account have a number E_a in system E on the obverse, and a number E_b in the same system on the reverse. Most of the texts have the tag BA on the obverse, although one text has it on the reverse. With this exception, the reverses of all the texts are free from non-numerical signs.

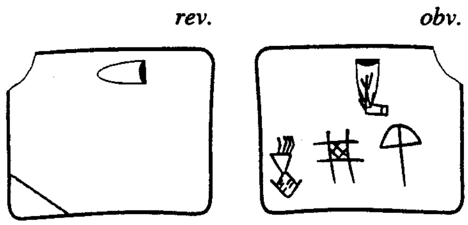


Fig. 9.1. W 9655,m, a single account in system E(pc). Total: 5 ce. A corner line on the reverse.

Instead, in almost all cases, one corner of the reverse is marked by a brief slanting line (see Fig. 9.1 above). The purpose of this line is probably to indicate that the inscription on the reverse is a *continuation* of the inscription on the obverse, not a second account, and not a total. Now, compare the simple example in Fig. 9.1 with the slightly more complicated example in Fig. 9.2 below. A key to the meaning of these strange texts, and to the function of system E, is the observation that in both cases the *sum* of the numbers E_a and E_b is *a multiple of 5 ce.* Indeed (see the factor diagram for system $E(pc)$),

$E_a + E_b$ = 1ce + 1m=(1+4)ce=5 ce Fig. 9.1,

and

$E_a + E_b$ = 1ce 1 ms+ 1c 2 ce 1 ms=(1 1/2 + 18 1/2)ce=20 ce Fig. 9.2.

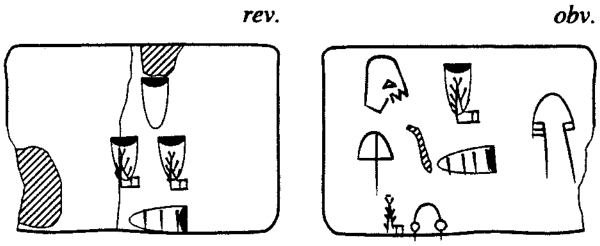


Fig. 9.2. W 9579,ce. System E(pc), a single account. Total: 20 ce.

It is easy to check that, actually, $E_a + E_b = 5$ *ce in 16 known texts* of the same type as the examples in Figs. 8.1-2, and that the sum is *20 ce in 4 such texts.*

There are two exceptions to the observed rule that the sum $E_a + E_b$ must be a multiple of 5 ce. Thus, in W 9579,ae (*ATU 5*, pl. 66) the sum is

$E_a + E_b$ = 1ms+1 ms=(1/2+1/2)ce=1 ce,

and in W 9579, bw (*ATU 5*, pl. 70), the sum is

$E_a + E_b$ = 1*ce* 1mss + 1 c 3 ce 1ms 1mss=(1 1/4+16+3 1/2 1/4) ce=21 ce.

On the other hand, the rule holds for the three single accounts that are joined into one *combined* account in W 9656,bl+dx below (*ATU 5*, pl. 93; see Fig. 9.3). Here, apparently, the numbers in *obv.* i:1-3 are taken from the obverses of three single accounts in system E, and the numbers in *obv.* ii:1-3 from the reverses of the same texts. (Compare *obv.* i:3, ii:3 with the text in Fig. 9.1.) Indeed, it is easy to check that

$(E_a + E_b)_1$=2 ce+1c 2 ce=(2+16+2)ce=20 ce *obv.* a:1+b:1,
$(E_a + E_b)_2$=1 ce 1 mss$^!$ + 3 ce 1 ms 1 mss=(1 1/4 + 3 1/2 1/4)ce=5 ce *obv.* a:2 + b:2,
$(E_a + E_b)_3$=1 ce+1m=(1+4)ce=5ce *obv.* a:3+b:3

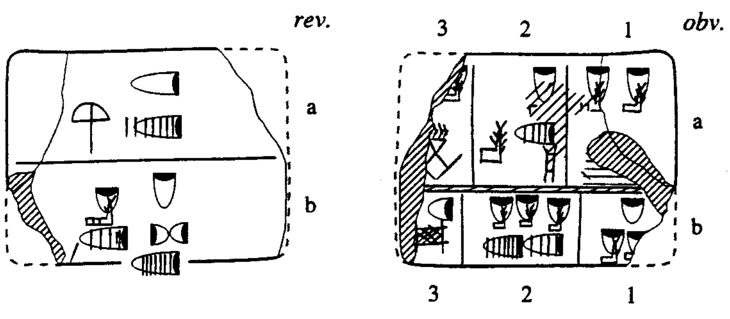


Fig. 9.3. W 9656, bl+dx, three sub-accounts, two sub- totals in system E(pc). Grand total: 30 ce.

The upper sub-total on the reverse is the sum of the numbers in *obv.* 1. Note that this total is marked by the tag BA. The lower sub-total is the sum of the numbers in *obv.* 2. Indeed,

E_a = 2ce+1 ce 1 mss[!] + 1 ce = 4 ce 1 mss=1m 1mss *rev.* a,
E_b = 1c 2ce+3ce 1 ms 1mss+1m=1c 1m 5 ce 1 ms 1 mss=1c 1m2 1 ce 1ms 1mss *rev.* b.

The grand total (not indicated in the text) can be computed as follows:

$$E = (E_a + E_b)_1 + (E_a + E_b)_2 + (E_a + E_b)_3 = (20 + 5 + 5)\text{ce}=30\text{ ce}.$$

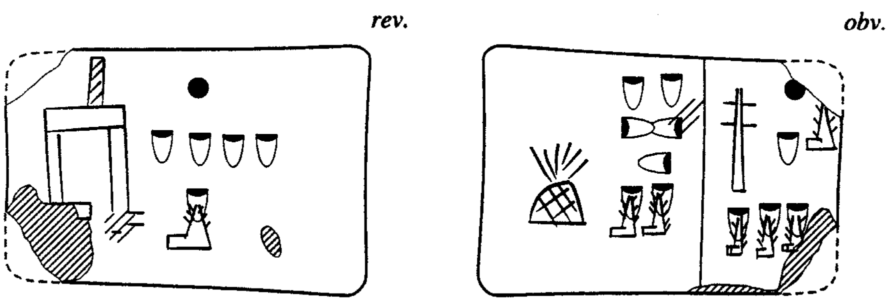


Fig. 9.4. W 9393,b, two sub-totals in system E(pc), and a grand total: 3.45 (225) ce.

On the obverse of the next example, W 9393,b in Fig. 9.4 (*ATU 5*, pl. 56) are recorded, apparently, two sub-totals like the ones on the reverse of the text in Fig. 9.3.

$$\begin{aligned} E_a &= \text{1d 1c 3 ce=(11}\cdot\text{ 16+3)ce=2.59 (179)ce,} \\ E_b &= \text{2c 1m2 1m 2 ce=(2 1/2 1/4 }\cdot\text{ 16+2)ce=46 ce.} \end{aligned}$$

On the reverse is recorded the grand total:

$$E = E_a + E_b = (2.59 + 46)\text{ce}=3.45\ (225)\text{ ce}.$$

Note that this text may very well be just a *writing exercise*, since it has an unusual format and an unusually large total, and since it was found together with some other tablets including the text W 9393,e (Fig. 8.1 above) which looks like a writing exercise.

The last example is W 7227,e (*ATU 5*, pl. 27). It lists three large E-numbers:

$$E_1 = \text{7c 1m2=120 ce, } E_2 = \text{3c 1m2 1m=60 ce, } E_3 = \text{1c 1m2 1m 3 ce=31 ce.}$$

In all the other texts making use of system $E(pc)$, the non-numerical signs in the text contribute nothing to the understanding of the text. They either stand for names or titles, or are badly understood tags like BA. The situation is different in the case of W 9656,bl+dx:

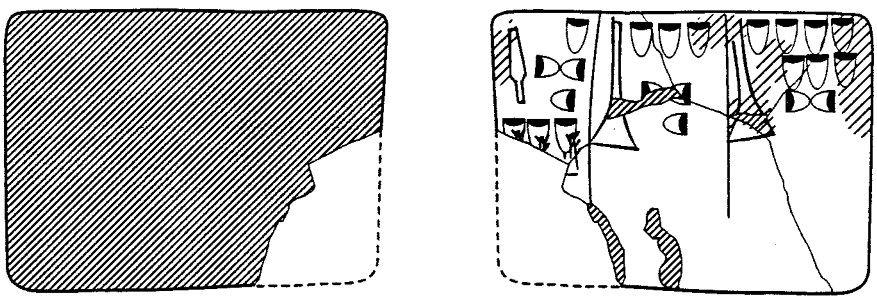


Fig. 9.5. W 7227,e, three large numbers: 2.00 (120), 1.00 (60), and 31 ce.

This text contains two non-numertical signs, Z(ATU)722 in *obv.* 1-2, and Z(ATU)723 in *obv.* 3. (The reverse is totaly effaced.) The meaning of the sign Z722 is not known. The sign appers in only one other proto-cuneiform text, *MSVO 1*, 40, in a non-informative context, as part of the phrase NIMGIR KU_3 Z722, possibly to be read as 'the official in charge of the silver Z722'.

The meaning of the sign Z723 is unknown, too, although it is likely that it is a variant of the sign TAG_{a1}, which appears next to the more well-known sign NAGAR in *obv.* i:6-7 of *MSVO 4*, 22. The form of the sign NAGAR (Sumerian: 'carpenter') suggests that it is a pictogram with the meaning 'drill, chisel'. Similarly, the form of the sign TAG_{a1} suggests that its meaning is 'saw'. The only difference between the two signs TAG_{a1} and Z773 is that the teeth of the saw are not shown in the case of the second sign.

The same variant of TAG_{a1} appears also, apparently, in the "metal text" W 13946,a, *obv.* iii:3. This observation motivates a closer study of all the five known proto-cuneiform metal texts W 13946, a, b, d, n1 and W 14265 (*ATU 2*, pls. 47-49), which were discussed by A. A. Vaiman, in a paper about "Iron in Sumer" (*Archiv für Orientforschung*, 19 (1982)). The structure of all the five texts is the same; they are simple lists of at least seven *tools or other metal objects*, probably counted sexagesimally. The names for the tools, etc., appear in what seems to be a fixed order, as follows:

TUN_3, TAG_{a1}, NAGAR, MAR_b, ŠUM, DIM, Z695,

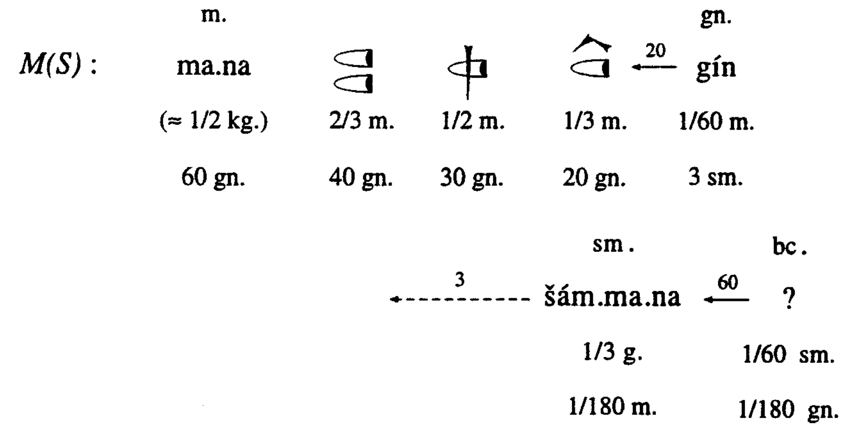
where the first sign, TUN_3 (actually the same sign as the Sumerian gín 'shekel') looks like a pictogram for a tube-ax of the characteristic Bronze age type. Four metals appear to be mentioned, namely:

$KU_3.U_4$ 'white metal', KU_3. NE 'red metal', ŠELU '??', AN 'heaven'

where $KU_3.U_4$ can be assumed to be identical with Sumerian: kù.babbar 'silver', and where AN just possibly may denote 'meteoritic iron'. (The metal texts seem to be incomplete. In particular, they contain no totals and no computations, and therefore have the appearance of simple writing exercises. Besides, it would not make much sense outside the world of the school to fabricate axes, saws, chisels, spades, etc., using a variety of precious metals.) Cf. the proto-cuneiform lexical text known as the "Metal list" (*ATU 3*, 32), which mentions, for instance, TUN_3 GAL, NAGAR, AN NAGAR, ŠELU, and KU_3.NE.

The detailed analysis above of all known texts[29] making use of system $E(pc)$ has brought to light so much new information that it should now be possible to propose at least a tentative interpretation of this enigmatic number system and of the texts in which it appears. The idea that first comes to mind is, of course, to try to find out if any one of the known Sumerian number systems may be the succesor of system $E(pc)$. Some potential candidates can be excluded from consideration at once, namely the Sumerian *sexagesimal* system (the immediate successor of system $S(pc)$ and a replacement for system $B(pc)$), the Sumerian *area* system (the immediate successor of system $A(pc)$), and the Sumerian *capacity* system (which for some entirely obscure reason totally replaced system $C(pc)$).

There remains only one candidate (suggested already in *ATU 1*, 50), the Sumerian *weight* system, system $M(S)$, normally used to measure large or small amounts of more or less precious *metals*. The obvious conclusion is that, unless precious metals were of no great importance in the Late Uruk period, system $E(pc)$ ought to be the predecessor of the Sumerian system $M(S)$. The earliest known evidence for the use of this Sumerian number system can be found in land sales documents from the Fara period (Early Dynastic III, around the middle of the third millenium B.C.). In those documents, the main unit of weight measure is the *mina* (Sum. ma.na; about 500 grams), and there are special notations for 2/3, 1/2, and 1/3 mina.



In land sales documents and other texts from the ensuing Presargonic period one can find also special notations for the *shekel* (Sum. gín; about 8 grams) equal to 1/60 mina, and for fractions of the shekel. Those fractions include, in particular, the šám ma.na 'exchange mina', equal to 1/3 shekel=1/180 of a mina, and 1/60 of the šám ma.na, later explicityly called a 'barley-corn'=1/180 of a

[29] The unpublished text W 19530,b, found far away from the other texts employing system *E*, is of a unique format, probably just a writing exercise. It contains, in two consecutive text cases on the obverse, nothing but the two numbers 1c 1m2=24 ce and 3 ce. Their total, 27 ce, is *not* a multiple of 5 ce.

shekel. In the mentioned land sales documents, amounts of copper and silver are exchangeable at a ratio of 180:1, so that *1 mina of copper was worth as much as 1 exchange mina of silver*. It is fairly safe to conclude that the Sumerian system $M(S)$ arose as the result of a merger between two systems of weight numbers, for *copper* with the main units *mina and shekel*, another for *silver* with the main units *exchange mina and barley-corn*.

It is striking that the factor diagram for system $E(pc)$ exhibits a dichotomy similar to the one of system $M(S)$. Apparently, system $E(pc)$, too, arose as the result of a merger of two number systems, one with the main unit 1c and the fractions 1/2 and 1/4 of 1c, the other with a main unit called c × EN (= ce) 'the EN's unit' and the fractions 1/2 and 1/4 of that.This similarity of system $E(pc)$ with system $M(S)$ may be used as an excuse for suggesting that the *upper* part of system $E(pc)$ was used (originally) to count amounts of some precious metal, while the *lower* part of the same system was used (originally) to measure smaller amounts of another metal, 16 times more precious. The two metals in question were probably not copper and silver, since the ratio 16:1 is so much smaller than the ratio 180:1. Instead, they may have been *silver* and *gold*, the latter worth 16 times more than silver.[30] If the lower part of system $E(pc)$ was primarily associated with gold, then the strange name 'the EN's unit' for the main unit of this part of the system may be an indication that only the EN, the city ruler or chief administrator, was entrusted with the handling of this very precious metal.

In certain Old Babylonian mathematical texts, 'hired workers' are paid wages at a rate of 6 *barley-corns of silver or 10 sìla (about 10 liters) of barley per day*, corresponding to 1 shekel of silver or 5.00 sìla (300 liters) of barley per month. (See above, just after Fig. 5.5). Assume, hypothetically, for the sake of the argument, that

> In Uruk, script phase IV, the wages for 1 day of hired work normally amounted to 1 or 2 ce of silver, with 1 ce of silver worth 1 M of barley (a typical unitary relation).

In that case, 1 ce would presumably be equal to about 3 barley-corns, and its smallest fraction, 1/4 ce, to about 1 1/2 barley-corn. Furthermore,

> 30 or 60 ce of silver, presumably the wages for 1 month of hired work, would then be equal to 90 or 180 barley-corns, that is to 1/2 or 1 shekel of silver.

The hypothesis ventured above, about the probable size of the unit c × EN=ce as a unit of weight, can be used to find a quite reasonable explanation of the meaning of the proto-cuneiform texts employing system $E(pc)$. In particular, the meaning of the single accounts, like the ones shown in Figs. 8.1-2 above, may have been that 1 day's wages for groups of, typically, 5 or 20 hired workers,

[30] According to H. Limet, *Le travail du métal au pays de Sumer* (1060) 103-105, gold was between 15 and 10 times expensive as silver in the late Sumerian Ur III period, at a time when silver was between 100 and 80 times as expensive as copper.

alternatively 5 or 20 days' wages for one hired worker, was 5 or 20 ce, but that one part of these wages was accounted for in one way, the remaining part in some other way. There was no fixed ratio between these two parts. Maybe the first part of the wages, the one specified on the obverse of the single accounts, was spent for food rations, while the second part was counted as credit for the group of hired workers. According to this interpretation, the text in Fig. 9.3, with three sub-accounts on the obverse and two sub-totals on the reverse, can be explained as follows. It is an account of how much of 1 day's wages for 30 man-days of hired work, the sum of 20, 5, and 5 man-days, were spent on food rations (the upper sub-total, marked by BA), and how much remained.

On the obverse of the text in Fig. 9.4 may be recorded two similar sub-totals for 3.45 (225) man-days of hired work, and on the reverse of that text is recorded the corresponding grand total. It is possibly significant that this text with large numbers in system E was found in square Qa XVI 3, a small distance away from the majority of the single accounts.

As mentioned above, the text in Fig. 9.5 was found alone, in square Pe XVI 3, but like the text in Fig. 9.4 only a small distance away from the single accounts of wages in square Qa XVI 2. This text may now tentatively be interpreted as an account of the values, in silver, of three copper objects, two stands and one saw. The values, 2.00 ce, 1.00 ce, and 31 ce (a mistake for 30 ce?), would have been equal to, according to the hypothesis, 2, 1, and 1/2 shekel, respectively. Expressed differently, the hypothesis made above implies that the cost of manufacturing a copper saw, for instance, would have been equal to the wages for 15 days of hired work. Along these lines, one may spculate that the origin of the use of the sign TUN_3 'ax' to denote the Sumerian gín 'skekel' as a unit of weight was simply that the normal cost in silver of a copper axe was 1 gín=180 barley-corns.

10. On rations and wages in the proto-literate period.

Many of the examples considered in the paper are proto-cuneiform texts dealing with *rations or wages* in one form or another. The situation with respect to the *nature* and the *size* of such rations or wages is, at least initially, very confusing. Thus, in the big bread-and-beer text *MSVO 4*, 66 (Fig. 3.1), apparently an account of rations for 20 man-years, the cost of a daily ration of bread? varies from 1 to 1/6M, and the cost of the beer rations varies in a similar way, at the same time as the *average* cost of a ration becomes precisely 1/4 M* per man-day. In *MSVO 4*, 60 (Fig. 4.1), an account of monthly rations of barley, the rations for the dependent workers amount to 1/2, 1/3, or 1/6 M per man-day, while the rations for the supervisors are bigger, 2 or 1 M per man-day. In the similar account *MSVO 4*, 65 (Fig. 4.2), the supervisors get rations of 1 2/3 or 1M" (emmer) per day, while in the case of the workers the daily rations are 1/3 or 1/5 M″ for the men, but only 1/10 or 1/15 M″ for the women. In the small fragment *MSVO 4*, 27 (Fig. 5.1), an account of NINDA rations for 24 man-months, the daily ration is (1+ 1/10)·1/6 M.

In *MSVO 1*, 26 (Fig. 5.4), monthly wages for teams of 10 or 5 *hired* workers correspond to daily wages of 1M per day, tagged BA, while in the five-days account *MSVO 1*, 84 (Fig. 5.5), the average daily wages per person for 40 hired workers is (1+1/8)· 2M, tagged BA. In §5 above are mentioned, but not shown, the following three Jemdet Nasr accounts, tagged GI: *MSVO 1* 89, a complicated account of daily wages of (1+1/10)· 1/2 M of barley for 3 man-years of hired work, *MSVO 1* 90, a similar account with daily wages of (1+1/10)· 1M of barley, also for 3 man-years, and *MSVO 1*, 94-a-b, an equally complicated account of daily wages of (1+1/12)· 1M for 4+6 man-years. Less complicated texts with similar numbers are *MSVO 1*, 121 and *MSVO 1*, 122, with daily wages of texts with similar numbers are *MSVO 1*, 121 and *MSVO 1*, 122, with daily wages of (1+1/10)· 1/2 M of barley for 35 man-days and 3 man-years, respectively.

Taken together, all these examples suggest that there was a considerable economic stratification of the proto-literate Mesopotamian society, with supervisors and hired workers getting considerably larger rations than ordinary workers, even with male workers getting considerably larger rations than women. The *rations for supervisors* and the *wages for hired workers* amounted to, typically, 1/2, 1 or 2M per day, while the *rations for male, dependent workers* ranged from about 1/3 to 1/6 M per day. The rations were paid out once every month, while the wages may have been paid out on a daily basis.

All this agrees remarkably well with what is known about rations and wages in the Neo-Sumerian society of the Ur III period, a thousand years later. See, in particular, the article "Rations, wages and economic trends in the Ur III period" by K. Maekawa in *Altorientalische Forshungen* (1989), 42-50. There, the discussion opens up with the declaration that "A strict distinction should be made between rations and wages in ancient Mesopotamian economy." It is mentioned that the Ur III public administration hired many workers for a short period of the year. The wages for the hired workers (á lú, hun.gá) were calculated per day and normally paid in barley, only rarely in silver. The smallest amount of wages to the daily worker was 5 sìla per day, about as much as 1 M in the proto-cuneiform texts. The personnel of the public household, on the other hand, were given barley rations once a month. Many of them were regularly summoned for a month of mandatory corvée work (for the necessary maintenance of fields and canals, etc.). If they continued to work *after* the corvée month, they were regarded as hired for a period of maximally 20 days and received wages (á.hun.gá erín.bal.tuš.a), up to 5 sìla (1 M) per day.

Maekawa's mention of the 20 days continuation of the corvée month, with wages amounting to 5 sìla (1 M) per day suggest an improved explanation of the proto-cuneiform texts of the type illustrated by the examples in Figs. 9.1-3 above. As pointed out in the discussion of those examples, out of 23 known *single accounts* making use of system $E(pc)$, the sum $E_a + E_b$ of the numbers on obverse and reverse is equal to 5 ce in 16 cases, and to 20 ce in 4 cases. In addition, in the case of the three sub-accounts in Fig. 9.3, the sums $E_a + E_b$ are equal to 20, 5, and 5 ce. Therefore, maybe the meaning of the single accounts is that dependents of the public administration performing extra corvée work for periods from 5 to maximally 20 days were awarded with a credit per day of 1 ce, *nominally* in silver, equal in worth to 1 M (about 5 liters) of barley. (Note that then the weight of 1 ce would have to be only 3 barley-corns or 1/60 of a shekel, about 1/8 gram.) This new idea strongly suports the interpretation of system $E(pc)$ proposed in §9 above, at the same time as it opens up the perspective of an amazing permanence of traditions of administrative practices in the ancient Mesopotamian society.

11. On the polyvalence of the proto-cuneiform number signs.

Finally a few words about the historical reasons for the seemingly strange *abundance* of different proto-cuneiform number systems, and the resultant *polyvalence* of the basic number signs c, d, C, D, etc. Agreed that the invention of the proto-cuneiform number systems was directly inspired by the previous use of tokens for counting and accounting, an explanation is not difficult to find. Indeed, suppose that a batch of "number tokens", kept together in a pouch of cloth or leather, or in a spherical envelope of clay, were employed to represent an *account* of the type 'so and so much of barley is delivered for so and so many rations, or in exchange for so and so much hired labor', etc. There would be no way of knowing *in which order* the individual tokens should be "read", other than, possibly, that the bigger tokens should generally be considered before the smaller ones. Therefore, *polyvalence of number tokens could not be accepted.* More precisely, the *form and size* of each number token must *uniquely* indicate both *kind and value.* Thus, for each kind of numbers (sexagesimal numbers for people, bisexagesimal numbers for rations, capacity numbers for barley, other capacity numbers for jars, or whatever), there must have existed a *specific series* of token types representing successively smaller number units, with no given token type appearing in more than one such series.

When the first "numerical tablets" appeared, the described situation changed in two ways. First, it was then possible to write the units of a number of a given kind *in a definite order*, beginning with the biggest units. Therefore, the order in which the units followed each other could be used to indicate which kind of numbers was being recorded. Secondly, *three-dimensional number tokens* were replaced by *basically two-dimensional impressed number signs.* The polyvalence of the number signs could no longer be avoided, because there are only a limited number of ways of punching a sign into the clay. Thus, what was gained through the introduction of a hierarchic ordering of the number signs within each "system" of numbers could be used to compensate for the confusion caused by the poloyvalence of the impressed signs. There is little reason to doubt that this explanation is correct, at least in the case of the proto-cuneiform systems S and C, possibly also in the case of systems B and A. All four of these systems were *common proto-literate*, that is common to the proto-cuneiform and proto-Elamite scripts.

The proposed proto-cuneiform system Z (if there ever was such a system) may also have had a predecessor in a series of number tokens, as mentioned above (see footnote 22). The same goes for system Df (above, §7), except that the predecessors of the number units in this system (DUG_c=$DUG_b\times$ s, KU_{3b}, and N_2=c $\times$ s) apparently must have been "complex" (that is decorated) tokens.

The situation is different in the case of the proto-cuneiform systems T (above, §5) and E (§9). In these two systems of number notations, the impressed number signs were supplemented in an essential way by "written" word signs, be it U_4 or EN. Therefore these two number systems, which by the way have no known proto-Elamite counterparts, can be suspected of having appeared on the scene first *after* the invention of writing.

Even more special is the case of system Dl (see §7), which is the only one of the protocuneiform number systems where multiples of the lowest unit are *counted* rather than replaced by higher units or expressed through repetition of the sign for the unit. Compare, for instance, W 21682 (Fig. 7.1), *rev.* i:1, where a fraction of 1 jar (of milk?) is written as 5c, followed by the sign for the unit ($SILA_{3b}$), and W 20274, 35 (Fig 7.2), *obv.* ii:3b, where a fraction of 1 jar (of butter?) is written as the sing c′, repeated four times. In the Sumerian cuneiform script, the sìla as the lowest unit of capacity measure for solids or liquids would continue to be counted in the same way. Ultimately, in the Neo-Sumerian Third Dynasty of Ur, around 2000 B.C., this way of *counting* the lowest unit of a given number system would lead to the invention of place value notation for sexagesimal numbers, and to the closely related appearance of cuneiform "metrological tables", in which traditionally written numbers in varius number systems were equated with sexagesimal multiples of the lowest units of the systems.

Conclusion.

The present paper initially set out to be a review of the two new books *ATU 5* and *MSVO 4*, to be published in the journal *Historia Mathematica.* For the review to be palatable to a mathematically inclined public, it had to include a discussion of the various proto-cuneiform number systems, with examples, since the extraordinary nature of these number systems it is not generally known outside a small group of specialists. However, it soon became apparent that this goal could not be achieved in the few pages of a normal review. Instead the paper grew into a survey article, with new interpretations of one or two of the more obscure proto-cuneiform number systems. While writing, the reviewer grew more and more enthusiastic about the decisive advance in the excruciatingly difficult study of the world's oldest written documents resulting from the current publication in Berlin of the two parallel series *ATU* and *MSVO*. The reader should take notice that the present survey is in no way exhaustive. There is enough material in the corpus of protocuneiform texts published in *ATU* and *MSVO* for years of further study. However, for this will be needed the combined efforts of outstanding scholar with expertise in the various branches of assyriology, that is in such diverse fields as the decipherment of forgotten scripts and languages, archeology, the history of agriculture, the history of economy, the history of metrology, and son on.