On Gelfand-Zetlin modules over $U_q(gl_n)$

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Abstract

We construct and investigate a new large family of simple modules over $U_q(gl_n)$.

1 Introduction and setup

The Gelfand-Zetlin formal construction of simple finite-dimensional modules over the groups of unimodular and orthogonal matrices was developed in the celebrated papers [GZ1, GZ2] in 1950 (see [BR] for more details). Later this construction was studied from different points of view, for example, the main result was reobtained using lowering operators method ([Z]). In the last fifteen years this construction has been generalized on different quantum analogues for Lie algebras, see for example [J1, J2, UTS1, UTS2, C, NT1, NT2, GI, GK]. On the other hand, this method was used to obtain the classification of unitarizable modules for several algebras (see [O1, O2, GK]), to construct and investigate the structure of a large family of simple modules over classical algebras (see [DFO1, DFO2, M1, M3, MO]) or to define and study new classes of algebras (see [M2]). Recently some deep results in this theory were obtained in [O], and an analogue of Gelfan-Zetlin construction for symplectic algebras was obtained in [Mo].

The aim of this paper is to analyze the Gelfand-Zetlin construction of simple finite-dimensional modules over the quantum algebra $U_q(gl_n)$, where q is a non-zero complex non root of unity in order to construct and investigate a new large family of simple $U_q(gl_n)$-modules.

We will work over the complex field and fix q to be a non-zero complex non root of unity. For any complex x we set $[x]_q = (q^x - q^{-x})/(q - q^{-1}) = (e^{xh} - e^{-xh})/(e^h - e^{-h})$, where $q = \exp h$. All the notions that will be used without preliminary definition can be found in [KS].

In Section 2 we recall the Gelfand-Zetlin construction of simple modules over $U_q(gl_n)$. In Section 3 we present a new large family of simple $U_q(gl_n)$-modules. In Section 4 we give an abstract definition of Gelfand-Zetlin modules over $U_q(gl_n)$ and present some examples. Finally, in Section 5 we construct an extension of $U_q(gl_n)$ inspired by modules constructed in Section 3.

2 $U_q(gl_n)$ and Gelfand-Zetlin basis for finite-dimensional modules

We define $U_q(gl_n)$ as a unital associative complex algebra generated by E_i, F_i, $i = 1, 2, \ldots,$ $n-1$, K_j, K_j^{-1}, $j = 1, 2, \ldots, n$ subject to the relations

$$
\begin{gathered}
K_iK_j = K_jK_i, \quad K_iK_i^{-1} = K_i^{-1}K_i = 1, \\
K_iE_jK_i^{-1} = q^{\delta_{ij}/2}q^{-\delta_{i,j+1}/2}E_j, \\
K_iF_jK_i^{-1} = q^{-\delta_{ij}/2}q^{\delta_{i,j+1}/2}F_j, \\
[E_i, F_j] = \delta_{ij}\frac{K_i^2K_{i+1}^{-2} - K_i^{-2}K_{i+1}^2}{q - q^{-1}}, \\
[E_i, E_j] = [F_i, F_j] = 0, \quad |i-j| \geqslant 2, \\
E_i^2E_{i\pm1} - (q+q^{-1})E_iE_{i\pm1}E_i + E_{i\pm1}E_i^2 = 0, \\
F_i^2F_{i\pm1} - (q+q^{-1})F_iF_{i\pm1}F_i + F_{i\pm1}F_i^2 = 0
\end{gathered}
$$

(see, for example [KS, UTS1, UTS2]).

The following Theorem describes the Gelfand-Zetlin approach for simple finite-dimensional $U_q(gl_n)$ modules with a given highest weight. It was obtained in [J1, J2], then reobtained by lowering operators method in [UTS1, UTS2]. We present it in the most general situation (for q which is any non-zero non root of unity), as stated in [KS, Section 7.3.3].

Theorem 1. *Let $V(m)$ be a simple $U_q(gl_n)$-module with a highest weight $m = (m_{n,1}, m_{n,2}, \ldots, m_{n,n})$, $m_{n,i} \geqslant m_{n,i+1}$. Then $V(m)$ possesses a basis consisting of all tableaux $[\,s\,] = (s_{ij})_{i=1,2,\ldots,n}^{j=1,2,\ldots,i}$ such that $s_{n,j} = m_{n,j}$, $j = 1, 2, \ldots, n$ and $s_{i+1,j} \geqslant s_{i,j} \geqslant s_{i+1,j+1}$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, i$ and the action of generators of $U_q(gl_n)$ are given by the following formulae:*

$$
K_k[s] = q^{a_k/2}[s], \quad a_k = \sum_{i=1}^{k} s_{k,i} - \sum_{i=1}^{k-1} s_{k-1,i}, \quad k = 1, 2, \ldots, n,
$$

$$
E_k[s] = \sum_{j=1}^{k} a_{kj}^{+}([s])([s] + [\delta^{k,j}]), \quad F_k[s] = \sum_{j=1}^{k} a_{kj}^{-}([s])([s] - [\delta^{k,j}]),
$$

where $\delta^{k,j}$ is the Kronecker tableau and for $l_{rt} = s_{rt} - t$ we have

$$
a_{kj}^{\pm}([s]) = \mp\frac{\prod\limits_{i}[l_{k\pm1,i} - l_{k,j}]_q}{\prod\limits_{i\neq j}[l_{k,i} - l_{k,j}]_q}.
$$

Remark 1. *It is worth to note, that the highest weight over the corresponding $U_q(sl_n)$ algebra can be expressed as $(m_{n,1} - m_{n,2}, m_{n,2} - m_{n,3}, \ldots, m_{n,n-1} - m_{n,n})$.*

Remark 2. *Originally the Gelfand-Zetlin formulae were obtained for a positive real $q \neq 1$ and had the following form:*

$$K_k[s] = q^{a_k/2}[s], \quad a_k = \sum_{i=1}^{k} s_{k,i} - \sum_{i=1}^{k-1} s_{k-1,i}, \quad k = 1, 2, \ldots, n,$$

$$E_k[s] = \sum_{j=1}^{k} A_{kj}([s])([s] + [\delta^{k,j}]), \quad F_k[s] = \sum_{j=1}^{k} A_{kj}([s] - [\delta^{k,j}])([s] - [\delta^{k,j}])$$

with

$$A_{kj}([s]) = \left(-\frac{\prod_i [l_{k+1,i} - l_{k,i}]_q \prod_i [l_{k-1,i} - l_{k,i} - 1]_q}{\prod_{i \neq j} [l_{k,i} - l_{k,j}]_q \prod_{i \neq j} [l_{k,i} - l_{k,j} - 1]_q} \right)^{1/2}.$$

One can obtain these formulae from those above multiplying the basis elements by appropriate factors.

3 Generic Gelfand-Zetlin modules

Let $1(q)$ be the set of all complex x such that $q^x = 1$. Fix a tableau $[m]$ with complex entries $m_{i,j}$, $1 \leqslant i \leqslant n$ and $1 \leqslant j \leqslant i$ satisfying the following defining condition:

- $2(m_{i,j} - m_{i,k}) \notin 1(q) + 2\mathbb{Z}$ for all $1 \leqslant i \leqslant n-1$ and all $j \neq k$.

We will call such $[m]$ admissible. Consider the set $B([m])$ consisting of all tableaux $[l]$ such that

- $l_{n,j} = m_{n,j}$ for all j;
- $l_{i,j} - m_{i,j}$ is an integer for all $1 \leqslant i \leqslant n-1$ and all j.

Let $V([m])$ be the vector space with a basis $B([m])$. For $[l] \in B([m])$ set

$$K_k[l] = q^{a_k/2}[l], \quad a_k = \sum_{i=1}^{k} l_{k,i} - \sum_{i=1}^{k-1} l_{k-1,i} + k, \quad k = 1, 2, \ldots, n,$$

$$E_k[l] = \sum_{j=1}^{k} a^+_{kj}([l])([l] + [\delta^{k,j}]), \quad F_k[l] = \sum_{j=1}^{k} a^-_{kj}([l])([l] - [\delta^{k,j}]),$$

where

$$a^{\pm}_{kj}([l]) = \mp \frac{\prod_i [l_{k\pm1,i} - l_{k,j}]_q}{\prod_{i \neq j} [l_{k,i} - l_{k,j}]_q}.$$

We will call the formulae above the Gelfand-Zetlin (GZ) formulae.

Theorem 2. *GZ formulae define on $V([m])$ the structure of a $U_q(gl_n)$-module of finite length.*

Proof. First we show that GZ formulae define on $V([\,m\,])$ the structure of a $U_q(gl_n)$-module. Let $u=0$ be a relation in $U_q(gl_n)$. It is enough to show that this relation holds in $V([\,m\,])$. For this it is enough to show that $u[l]=0$ for any $[l]\in B([m])$. Clearly, using GZ formulae we can write $u[l]=\sum_{[t]\in I(u,[l])} f([t])[t]$, where the set $I(u,[l])-[l]$ depends only on u and for any fixed u each $f([t])$ is a rational function in $q^{l_{i,j}}$. Thus, it is enough to show that each $f([t])$ is identically zero. Hence, we have only to show that some polynomials in $q^{l_{i,j}}$ are zero. Let p be such a polynomial, k be its degree and s be the degree of u. Clearly, there exists a tableau $[\tilde{l}]$ such that all $\tilde{l}_{i,j}$ are positive integers and for any integer $-k-s\leqslant \tilde{t}_{i,j}\leqslant k+s$ the tableau $[\tilde{l}+\tilde{t}]$ occurs as a basis element in a finite-dimensional $U_q(gl_n)$-module (this means that the entries of it satisfy the conditions presented in Section 2). Taking into account that $q^a\neq q^b$, if $a\neq b$ are positive integers, we conclude that p is identically zero, since GZ formulae really define finite-dimensional $U_q(gl_n)$-modules as in Theorem 1 (thus for tableaux from them $p=0$ holds). This completes the proof of the first part of our theorem.

Let $A([m])$ be a subalgebra of $U_q(gl_n)$ consisting of elements, which are diagonalizable in the basis $B([m])$. It is non-empty, because it contains at least the quantized Cartan subalgebra, generated by K_i. Let $U_q(gl_k)$, $1\leqslant k\leqslant n$ be a subalgebra of $U_q(gl_n)$ generated by K_i, $1\leqslant i\leqslant k$, E_i, F_i, $1\leqslant i\leqslant k-1$. Denote by Z_k the center of this $U_q(gl_k)$. Since Z_k is diagonalizable in the GZ basis of any finite-dimensional $U_q(gl_n)$-module, it follows that it is diagonalizable in the basis $B([m])$. Thus Z_k is a subalgebra in $A([m])$. Let Γ be a subalgebra of $A([m])$ generated by all Z_k. To complete our proof it is enough to show that for any $[l(1)]\neq[l(2)]\in B([m])$ there exists an element $u\in\Gamma$ such that the eigenvalues of u on $[l(1)]$ and $[l(2)]$ are different (see also [M3, Theorem 1]). Indeed, having this we easily obtain that any subquotient of $V([m])$ is determined by the corresponding subset of basis elements from $B([m])$. Form a non-oriented graph with a vertex set $B([m])$ in the following way: we say $[a]$ and $[b]$ to be connected by an edge if $[a]$ occurs with a non-zero multiplicity in $E_i[b]$ and $[b]$ occurs with a non-zero multiplicity in $F_i[a]$ for some i. Now the subquotients of $B([m])$ are determined by the connected components of this graph, and it is trivial, that there are only finitely many of them.

Therefore, we have only to check that Γ separates the elements of $B([m])$. It is easy to see that for $z\in Z_k$ the eigenvalue of z on $[l]$ can be expressed as a rational function on $q^{l_{k,j}}$, where only j varies. Hence, we need only to show that two tableaux in $B([m])$ having different k-th rows can be separated by an element from Z_k. Without loss of generality we can assume $k=n$. Now the last statement is equivalent to the following fact: the central characters of $V([m(1)])$ and $V([m(2)])$, $m(i)_{n,j}-m(i)_{n,s}\notin\mathbb{Z}$ for all $i=1,2$, $j,s=1,2,\dots,n$, where the difference between the upper rows of $[m(1)]$ and $[m(2)]$ is a non-zero vector with integer entries, do not coincide. To prove this we have to compute the central character of $V([m])$. Let $z\in Z_n$ and $[t]$ be a tableau determining the highest weight of a finite-dimensional simple $U_q(gl_n)$-module. Denote by π the Harish-Chandra homomorphism from Z_n to the subalgebra U^0 generated by $K_i^{\pm1}$, $i=1,\dots,n$

([J, Section 6.2]). According to [J, Lemma 6.3] the eigenvalue of z on $[t]$ equals $\lambda(\pi(z))$, where λ is the highest weight of $[t]$. Since we work with $U_q(gl_n)$, its center is generated by a (unique up to inverse) monomial in U^0, whose eigenvalue can be computed directly, and the center Z'_n of $U_q(sl_n)$. Since q is not a root of unity, we can apply [J, Theorem 6.25] and [J, Section 6.26] to obtain the eigenvalues of the elements from Z'_n. According to these results, the eigenvalues are invariant (Laurent) polynomials in $q^{t_{n,i}-t_{n,i+1}}$ under the natural action of the Weyl group (here $t_{n,i}-t_{n,i+1}$ appears as components of the highest weight with respect to $U_q(sl_n)$).

Now we see, that the central character of $V([m])$ depends only on entries in the upper row of $[m]$. Moreover, it coincides with the central character of a Verma module with the highest weight $(m_{n,1}+1, m_{n,2}+2, \dots, m_{n,n}+n)$ (with respect to $U_q(gl_n)$). By [J, Claim 6.26] two Verma modules over $U_q(sl_n)$ have the same central character if and only if their highest weights (shifted by a half-sum of all positive roots) lie on the same orbit of the Weyl group. A shift by a half-sum of all positive roots corresponds to the substitution of $(m_{n,1}+1, m_{n,2}+2, \dots, m_{n,n}+n)$ with $(m_{n,1}, m_{n,2}, \dots, m_{n,n})$. The Weyl group acts on $U_q(gl_n)$-space of weights by permutation of the vector entries. Clearly, the restriction of this action on $U_q(sl_n)$-space of weights coincides with the standard action of the Weyl group on it. We remark, that the stable complement of $U_q(sl_n)$-space of weights in $U_q(gl_n)$-space of weights determines the eigenvalue of the additional central element of $U_q(gl_n)$. Hence [J, Claim 6.26] can be extended to $U_q(gl_n)$-case. To complete the proof now we have only to note that under our choice of $[m(i)]$, $i=1,2$ their upper rows can not be conjugated by a permutation. □

Remark 3. *From the discussion above it follows that $A([m])$ does not coincide with Γ. Since Z_n is diagonalizable in $B([m])$ it follows that $V([m])$ has a central character and thus $A([m])$ contains the two-sided ideal of $U_q(gl_n)$ generated by the kernel of this central character. We conjecture that Γ coincides with the intersection of all $A([m])$, where $[m]$ varies. This is equivalent to the fact that Γ is a maximal commutative subalgebra in $U_q(gl_n)$. The last is the case for a non-quantum situation ([O, M3]).*

Corollary 1. *$V([m])$ is simple if and only if $2(m_{i+1,j}-m_{i,k}) \notin 1(q)+2\mathbb{Z}$ for all i,j,k.*

Proof. Clearly this is a necessary and sufficient condition for the graph described in the proof of Theorem 2 to have a unique connected component, which completes the proof. □

Remark 4. *Since $[x]_q \to x$, when $q \to 1$, we obtain that $V([m])$ is a quantum deformation of the generic Gelfan-Zetlin modules constructed in [DFO2, Section 2.3]*

4 Gelfand-Zetlin subalgebra and abstract Gelfand - Zetlin modules

The central arguments used in the proof of Theorem 2 motivate to give the following abstract definition. A $U_q(gl_n)$-module V will be called Gelfand-Zetlin module (GZ-module) if it decomposes into a direct sum of finite-dimensional Γ-modules. Since Γ is comutative, this means that $V = \oplus_{\chi\in\Gamma^*} V_\chi$, where V_χ is a root subspace of V corresponding to the character χ (see [MO, Section 3]).

Clearly, any weight $U_q(gl_n)$-module with finite-dimensional weight spaces is a GZ-module. Thus finite-dimensional modules, Verma modules, highest weight modules are GZ-modules. It follows also from the proof of Theorem 2, that $V([m])$ is a GZ-module. Following [DFO2], one can easily prove the following standard results:

Proposition 1. *Γ is a Harish-Chandra subalgebra in $U_q(gl_n)$.*

Proof. Analogous to that of [DFO2, Corollary 26]. □

Proposition 2. *Let $\chi \in \Gamma^*$, and $[m]$ be such that $V([m])_\chi \neq 0$. Set $S([m])$ to be a submodule of $V([m])$, generated by $V([m])_\chi$. Then there exists a unique maximal submodule $K([m])$ in $S([m])$ and the quotient $S([m])/K([m])$ is the unique simple GZ-module having a non-trivial χ-root subspace. In fact the last is a χ-weight subspace of dimension one.*

Proof. Analogous to that of [DFO2, Theorem 30]. □

Theorem 3. *Suppose that $V([m])$ is simple. Then the category of all GZ-modules decomposes into a direct sum of two full subcategories $A \oplus B$ such that A contains the unique simple object, namely $V([m])$.*

Proof. Analogous to that of [DFO2, Corollary 33]. □

5 Extending $U_q(gl_n)$

Consider a field $\mathbb{F}$ of rational functions with complex coefficients in $n(n+1)/2$ variables $q^{\mathfrak{l}_{i,j}}$, $1 \leqslant i \leqslant n$, $1 \leqslant j \leqslant i$. Let $[T]$ be a tableau (i.e. $n(n+1)$-dimensional doubly-indexed vector) with entries $T_{i,j} = \mathfrak{l}_{i,j}$. Consider a set B consisting of all $[t]$ satisfying the following conditions:

- $t_{n,i} = T_{n,i}$ for all i,
- $T_{k,i} - t_{k,i} \in \mathbb{Z}$ for all k, i.

Let $V(B)$ be an $\mathbb{F}$-vectorspace with the basis B. Define $\mathbb{F}$-linear operators $X^{\pm}_{k,j}$, $H^{\pm}_{k,j}$, $1 \leqslant i \leqslant n-1$, $1 \leqslant j \leqslant i$ as follows: for $[t] \in B$ set

$$X^{\pm}_{k,j}[t] = \mp \frac{\prod_i [t_{k\pm1,i} - t_{k,j}]_q}{\prod_{i\neq j} [t_{k,i} - t_{k,j}]_q} ([t] \pm [\delta^{k,j}]), \quad H^{\pm}_{k,j}[t] = q^{(\pm(t_{k,j}+j)/2)}[t]$$

and extend this action on $V(B)$ by $\mathbb{F}$-linearity. Consider a complex associative algebra $\mathfrak{A} = \mathfrak{A}(q, n)$, generated by all $X^{\pm}_{k,j}$ and $H^{\pm}_{k,j}$.

The following property of $\mathfrak{A}$ follows immediately from the definition.

Proposition 3. *Let $[l]$ be an admissible tableau. Then the specialization $\mathfrak{l}_{i,j} = l_{i,j}$ in the above formulae defines on $V(B)_{[l]}$ a structure of an $\mathfrak{A}$-module.*

Theorem 4. *The subalgebra A of $\mathfrak{A}$, generated by*

$$e_k = \sum_{j=1}^{k} X^+_{k,j}, \quad f_k = \sum_{j=1}^{k} X^-_{k,j}, \quad 1 \leqslant k \leqslant n-1, \quad h_k = \prod_{j=1}^{k} H^+_{k,j} \prod_{s=1}^{k-1} H^-_{k-1,j}, \quad 1 \leqslant k \leqslant n$$

is canonically isomorphic to $U_q(gl_n)$.

Proof. It follows from the proof of Theorem 2, that the natural map $\psi : U_q(gl_n) \to A$, defined by $\psi(E_i) = e_i$, $\psi(F_i) = f_i$ and $\psi(K_i) = h_i$ can be extended to an algebra homomorphism, which should be an epimorphism since the image contains all generators of A. To prove the injectivity of ψ compose it with the specialization $s_{[l]}$, for an admissible tableau $[l]$. From the construction of $V([l])$ we deduce that under the map $s_{[l]} \circ \psi$ the space $V(B)_{[l]}$ becomes an $U_q(gl_n)$-module, isomorphic to $V([l])$. Thus the kernel of $s_{[l]} \circ \psi$ coincides with the annihilator of $V([l])$. To complete the proof we have only to show that the intersection of annihilators of all $V([l])$ is trivial. Suppose not and $u \neq 0$ annihilates all $V([l])$ with admissible $[l]$. Since the set of admissible $[l]$ is large enough, Gelfand-Zetlin formulae defining $V([l])$ are rational and also valid for finite-dimensional modules, we deduce, that u annihilates all finite-dimensional modules, which contradicts [J, Proposition 5.11]. Theorem is proved. □

We outline, that a part of the proof of Theorem 4 can be formulated in the following statement.

Corollary 2. *The restriction of $V(B)_{[l]}$ on A canonically isomorphic to $V([l])$.*

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