Abstract "Discrete form" means here that we deal with a given time-interval of length 1 which we call unit time and divide it into equal intervals $\Delta t = t_{i+i} - t_i, \Delta t = 2^{-n}, 2^n \Delta t = 1$. Then we introduce physical concepts in the following order, velocity v road s (strata)-distance with the relations $s = vt, \Delta s = v\Delta t$. However, we deal with Δs as vectors $v\Delta t$, i.e., distances with directions. So we get vector-curves of different form. They can be a straight line. But a distance on a line can be uniformly mapped onto a circle as follows. Divide the given distance into four equal parts and form a square with center 0. The corner-points are on a circle. From the square we get a regular 2^3-polygone with sides of the same length and all its corners on the same distance from the center. So we can go on until we have a 2^n-polygon with sides of equal length. Hence there exists a one-to-one mapping of the regular 2^n-polygon onto a distance on a straight line. By uniform convergence we get a one-to-one mapping on a distance onto a line. However, if the distance from the center to the corners of the polygons is 1, then the length of the circle is the irrational, even non-algebraic number 2π, the most interesting number, since the circle is the most important curve in our universe. Circles determine elllipses and the other curves of second degree as conical sections and spheres of any dimension and we get all these curves by uniform convergence of regular 2^n-polygons. Energy is given in potential form and kinetic form and the translation from one form to the other form by the gravitational force, which produces a work determined by an integral, which is the uniform limit of a sum, which is jumps of enerales, i.e., changes of velocities during time-intervals. That is what can be observed.

The most well-known relativity theory is Einstein's special relativity theory. We can visualize it by a cage translated by the velocity of light (in vacuum). The cage could be our whole solarsystem. Information of motions far away are transported by the velocity of light to the stationary observer. The problem is to transform these observations to correct motions far away. The relativity theory gives the solution of this problem.

In Einsteins general relativity theory also gravitational forces in the translated cage may be regarded and movements of planets and other satelites in our solarsystem can be determined.

In the following we let relative time Δt be given by stationary time Δt and velocity of light through a simple formula, given by 2^n-polygons for moving and the stationary coordinate system. Then we must avoid accelerations in points and only deal with changes of velocities for time-intervals. However, we get continuous relations by uniform convergence of discrete systems to continuous systems also in different finite dimensions. This is a form of transformations.

The applications given here are demonstrations of the method, not carried through in all details.

The relativity theory in discrete form

Harald Bergström

1 Time

Consider a timeperiod as a unit (second, hour or year) and divide it into equal intervals of length $\Delta t = 2^{-n}, 2^n \Delta t = 1$. Denote the set of these points on all intervals $[i, i+1], i = 0, 1, \ldots$ by $C(n)$ for given n and i. We say that Δt belongs to $C(n)$ if $\Delta t = 2^{-n}$ and t and $t + \Delta t$ belong to $C(n)$ and call $C(n)$ a partition of order 2^{-n}, and the endpoints of the intervals a chain in $C(n)$ on any given interval. We claim that it is sufficient for our problem to deal only with the t in a finite set $C(n)$.

Usually we consider time on a time-interval, but we may as well consider points on a circle, exactly as the points $t_i, t_{i+1} = t_i + \Delta t, \Delta t = 2^{-n}$, all these points with the distance $1/2\pi$ from the central point. So we get time-points t_i on a clock, whose pointer jumps $\Delta t = 2^{-n}$ and turns round during the time-unit. The points t_i on the circle form a regular 2^n-polygon. When n tends to infinity, the circumference of the 2^n-polygon tends to 1, but it should be an impossible clock if we tried to use all limit points of the polygons on the circle as time-points though these points are countable, but then in a strange order (if the distance to the corners from the central point is 1, then the circumference of the circle is in the closure of succesive squareroots, i.e., as limits.

The time introuduced here is a form of what is called space-time. But man has an experience of duration, time equal to duration of movement. The mathematical relation is

$$s = v \cdot t, \tag{1}$$

s distance, v=velocity, t duration of the movement. Here s, v and t are numbers, but represent physical concepts for the movement, say of a particle from a point $P(t)$ at time t to a point $P(t + \Delta t)$ during Δt. Denote the distance between the points by Δs. Then by definition

$$\Delta s = v \Delta t,\ v = \Delta s / \Delta t \tag{2}$$

and (2) implies (1) for any t, provided that v is constnat. Otherwise Δs represents a change Δv of the velocity during Δt. If this change is constant $= a > 0$, we have

$$v = at,\ s = at^2/2,\ s = v^2/2a \tag{3}$$

so that s and v are determined by t for given a. What we usually can observe are distance and time.

2 Points and vectors determined by time in a space

The elementary concepts in the space, that we live in, are points lines (straight lines) and planes. A line has a direction. An interval on the line is determined by two points P and 0 and between these points is a distance (length). Thus in our space we have an interval determined by its direction and length. It is called a vector. For it we use the notation $P \to 0$.

Clearly all observations are discrete with a time-interval between them. We let t in $C(n)$ determine points $P(t)$ and connected vectors $P(t_i) \to P(t_{i+1})$, {read $P(t_i)$ to $P(t_{i+1})$} of length $\leq a\Delta t, t_{i+1} - t_1 = \Delta t, a > 0$ a constant, this is a 3-dimensional Cartesian space R^3. The coordinates determine the points

$$P(t) = \{x(t), y(t), z(t)\} \tag{1}$$

Then for the unit vectors e_1, e_2, e_3 on the axis in R^3,

$$e_i e_j = 0 \text{ for } i \neq j, e_i e_i = 1, i < j \leq 3,\ P(t) = x(t)e_1, y(t)e_2, z(t)e_3 \tag{2}$$

$$P(t_{i+1}) - P(t_i) = (\Delta x)e_1(\Delta y)e_2(\Delta z)e_3 \tag{3}$$

where $\Delta x = x(t_{i+1}) - x(t_i)$ and so on. The squared length $|\Delta s|$ of the vector (3) is

$$|P(t_{i+1}) - P(t_i)|^2 = |\Delta s|^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 \tag{4}$$

A chain t_i in $C(n), t_i = t_i + \Delta t$, determines a chain of connected vectors, $P(t_i + \Delta t \to P(t_i)$. They represent movements and thus curves in the stationary system R^3 from t_i to $t_i + \Delta t$. We may choose our clock such that $\Delta t = 2^{-n}$, and thus time is a distance. A vector can have different length and thus be a number d of time-distances 2^{-n} so that $\Delta x = (2^{-n})d$, and Δy and Δz in the same way in (4). Do observe that these relations hold for any vector and hence for any configuration of connected vectors. Further the relations (2)-(5) hold for any Cartesian coordinate system and thus for orthogonal transformations. A vector is determined by its length and ints direction, both determined by the coordinates

$\Delta x, \Delta y$, and Δz, according to (1)-(5). Note that Δt is a positive constant, which we, however, may choose. Further we may represent the coordinates in (3) by

$$\Delta x = (\Delta t) tg\beta_1,\ \Delta y - (\Delta t) tg\beta_2 \Delta z = tg\beta 3 \tag{5}$$

all *beta* bounded, not all zero. So t has almost lost its meaning of time when we deal with vectors.

We extend the R^3 to a four-dimensional R^4 by introducing a fourth coordinate axis u with unit vector e_4 and thus represent the point $P(t)$ by

$$P(t) = x(t)e_1 + y(t)e_2 + z(t)e_3 + u(t)e_4 \tag{6}$$

and have the corresponding relations (1)-(4) added by Δu and $(\Delta u)e_4$. Note that we get a three-dimensional space when we put one of the coordinates $x(t), y(t), z(t)$, $u(t)$ equal to 0, a line if we put two of them equal to 0. So we visualize our relations. In the same way we introduce Cartesian coordinate systems R^k of any finite dimension k.

Any two connected vectors $P_1 \to P_0$ and $P_0 \to P_1$ of the same length but different directions determine a circle, in which the vectors are chords, and thus determine a circle, in which the vectors are chords, and thus determine the center 0 of the circle and the length r of its radius, r=distance 0 to P_1 and 0 to P_2. In the circle we may inscribe a regular 2^n-polygon of length d_n where its sides have the length $d_n 2^{-n}$ and P_1 is a corner of the polygon. As n tends to infinity d_n tends uniformly to the circumference of the circle. By the mapping $r \to 1/2\pi$ this circle is mapped onto a circle with center 0 and circumference=1. The inscribed polygon is correspondingly mapped. Connected configurations of connected vectors represent movements in R^3 and in $R^k, k > 3$. They may be complicated particularly for large k.

3 The relativity theory

We shall deal with vectors and they remain the same, even when they are translated. As a matter of fact our principle for introduction of relative time in a moving system (we call it cage) is that any configuration of connected vectors, also called vector-curves, in the moving cage should satisfy the conditions that the length and the direction between connected vectors in the stationary cage remain unchanged in the moving cage, transported by the velocity c of light as in Einsteins relativity theory. We may have different Cartesian coordinate systems for a cage. But the vectors and vector-configurations remain the same. The observer in the stationary cage wants information about motions and this information is transported by the velocity c of light. A motion some time ago is determined by a vector Δs and a timeinterval $\Delta t'$, which may not be the same as Δt. The Δs can have different directions, but the $\Delta t'$ and thus t' must be independent of the vector-configurations in order that information about these to the stationary observer be correct. Then we let our cage be translated by the velocity c of light in a direction that is orthogonal to the coordinate system R^3, i.e, we extend R^3 to a four-dimensional Cartesian coordinate system R^4 by adding a time-axis for t' orthogonal to R^3, and let the cage have the same R'^3 when it is stationary and when it is translated by the velocity c of light, but we add a ($'$) to mark that we deal with this translated coordinate system. Note that if the axist t' is orthogonal to R^3 then is orthogonal to any straight line in R^3. Any two connected vectors are in a plane Pl and there keep their length and the angle between them when the plane is translated in the direction of the axis for t'. Let us now consider one vector Δs in the stationary cage. Let it have the length $v\Delta t$. Note that v then is a velocity by definition. We may choose R^3 such that the vector is along the x-axis and then put Δs equal to Δx. For the stationary observer, a point P at t has moved the distance $c\Delta t$ during Δt to a point P'. But a motion in the moving cage is a vector Δx and this during a time $\Delta t'$ which an imagined observer on the moving cage should establish. The esential observation for the stationary observer is the vector Δx, hence in the actual situation the distance $|\Delta x|$. So he concludes that during his observed time Δt a particle has moved $c\Delta t$ from P to P' and thus there has been a motion represented by the vector Δx in the cage, and during $\Delta t'$ the cage has moved the distance $c\Delta t'$ in the direciton t' orthogonal to the x-axis. Hence we have the relation

$$c\Delta t = \{|\Delta x|^2 + (c\Delta'^2\}^{1/2} \tag{1}$$

When $\Delta t'$ is given by this relation the staionary observer get the ocrrect value of the length of the vector Δx. However, we have already found that any vector Δx and the angle between two connected vectors remain unchanged when the cage is translated by constant velocity and in a direction orthogonal to R^3. Hence in order that the observer in the stationary cage shall get correct information about the motions, represented by vector configurations in the moving cage, he must require the fundamental relations

$$(c\Delta t)^2 = (c\Delta t')^2 + (v\Delta t)^2, \Delta t' = (\Delta t)\{1 - (v/c)^2\}^{1/2}, v\Delta t = v'\Delta t' \tag{2}$$

The last relation (2) follows since $|\Delta s|$ is the same in the stationary and the moving cage and $|\Delta s|$ and Δt determine v in the stationary cage and $|\Delta s|$ and $\Delta t'$ determine v in the translated cage.

The cage may contain our whole solar system or only some molecules. But in both cases information is transported by light through the fundamental time-formulas (2) to the real observer in the stationary cage. However, then it turned out that the formulas (2) are not only a communication instrucment but an important physical reality, a carrier of energy such as fission and fusion.

Of interest is here to observe that we simultaneously deal with two motions: a) motion in the cage, b) motion of the cage, the last orthogonal to vectors in the cage. By the relativity theory we separate these two motions.

The only condition for application of this relativity is that we only permit such motions in the stationary cage that are represented by vectors and vector configurations and movements of the cage such that we have the one-to-one mapping, described above, between vector-configurations in the stationary and in the moving cage. We also require that the fundamental relations (2) are satisfied, as they are when the cage is translated by the velocity of light. However, they may hold when the motion of the cage is combined with other motions. Hence we have to do with vector curves and one-to-one mappings between such curves. The cage can be a cartesian R^k of any dimension k. Note that then any vector is on a straight line and a vector configuration may be in one R^3 but also pass over from one R^3 to another R^3. Thus the relativity theory can be used very generally.

In the following sections we apply relativity. There we give applications to well-known dynamics in physics, astronomy, etc., where we can verify the conditions given about connected vectors. However, our main purpose is to point out the use the relativity theory in different situations. We also discuss models for light and other radiation, but most of these are imagined and not founded on experiments.

4 Acceleration, mass, force, and energy relative mass as energy

From the classical mechancics we have so far only introduced the conceptions way (distance) s, time t and velocity v and the relation $s = vt \ldots$ for a moving particle. However, we have also used vectors, which represent distance (length) and direction. Usually the velocity depends on time. Inorder to acclerate the veloctу of a moving particle, a force F is required, where

$$F = ma \tag{1}$$

and m is a factor, called the mass of the moving particle whose velocity is changed by the acceleration a. Here we consider the case when F is uniqeuly determined by the velocity. By the relativity theory we have

$$mv = m'v' \tag{2}$$

for any velocity v and the corresponding v', where m' need not be equal to m. These products, called moments, are then equal in R^3 and R'^3. By 3(2) and 3(3) we get

$$m' = mv/v' = m\{1 - v^2/c^2\}^{1/2} \tag{3}$$
$$m'(v')^2/2 = \{mv^2/2\}\{1 - v^2/c^2\}^{-1/2} \tag{4}$$
$$m'c^2 = mc^2\{1 - v^2/c^2\}^{-1/2} \tag{5}$$

In the next section we show that $mv^2/2$ is a kinetic energy called force live for any velocity v. Hence we may consider also mc^2 as energy and by (5) also $m'c^2$ as energy.

An essential condition in Einsteins relativity theory is that light (in vacuum) has the velocity c. By (5) $m' > m$ and we get a surplus

$$m'c^2 - mc^2 \tag{6}$$

The Taylor expansion gives us the estimation

$$(m' - m)c^2 = mv^2/2 \tag{7}$$

with an error of order $v^2/4c^2$, which is small if v is small compared with c.

The relativity theory is given as a communication instrument by the formulas 3(2), founded on the relative time t' and used to transform information about occurences in earlier time to present time. However, when it changes mass to larger relative mass and so to energy, since mass becomes energy and then the surplus $(m' - m)c^2$ of energy is large even if m is small.

The energy $mv^2/2$ is called "force live" since it forces a vector to change its direction. This change shall be described in the next section. However, this energy also appears when a particle has been accelerated by a force to a certain velocity v in the direction of the force and then has produced an energy called "work". If a force F acts upon a particle and during the time t_0 has moved it a distance s, then by definition F has made a work Fs during t_0. (A car can be accelerated during different time-periods to a given velocity v). Observing that force acting on the particle with mass m is $F = am$ for constant acceleration a of the velocity of the particle, we find that its velocity at t is at and at_0 equal to v by definition. Integrating at from 0 to t_0, we so get by 1(3)

$$Fs = ams = ama(t_0)^2/2 = m(at_0)^2/2 = mv^2/2 \tag{8}$$

We visualize these relations on a right triangle as follows. Let time be on eth x-axis from origin $t = 0$ to $t = t_0, v(t) = at$ on the hypotenuse and s on the third side. The relations are obvious, i.e., the integration is obvious.

5 Gravity, rotation and force live. Movements of satellites and planets

According to Newton's laws a particle moving with given velocity in a given direction, i.e., on a straight line, continues in this direction. In order to change its direction a force is needed. We shall consider the problem when a mass is forced to move on a circle with constant vecocity v. In Section 1 we have pointed out that the circumference of the circle has a given length determined by the radius and that there is a one-to-one-mapping of the circleline onto an interval on a straight line. Thus a point has a constant velocity v on the circle, when there is a constant velocity on this line. Since we in the relativity theory have bounded us to motions defined by vector configurations and time (t'), we should also remember that a circle does not change by translation with the velocity of light since the 2^n-polygons do not change.

The gravity force F is given by

$$F = Mm/s^2 \tag{1}$$

where M is at a fixed point Q, m a mass in a point P on the distance s form Q. Usually the right-hand side is multiplied a constant dependent on units, but we let the constant be equal to 1.

Consider the situation when P rotates with constant velocity v. The work done by F to bring P from infinity to the distance s from Q is equal to Mm multiplied by the integral over $1/r^2$ to s. Hence this energy, given to the rotating m, is equal to

$$mv^2/2 = Mm/s \tag{2}$$

The kinetic energy $mv^2/2$ has been called force live since it forces m to move on the circle. Note that s is the distance between M and m and Mm/s is a potential energy, also that the motion of m is bounded by the relation (2) since F is orthogonal to the circle and cannot give any energy to the moving m nor obtain any energy from the moving m.

As an application of the formula we consider satellites. When at the time $t = 0$ a satellite with mass m is shot out from a point $P(0)$ on the surface of the earth and in the direciton $Q \rightarrow P(0)$, where Q is the center of the earth, $P(0)$ in the plane Pl, determined by the line $Q \rightarrow P(0)$, and the axis of rotation of the earth on the distance s from Q. Let the satellite be shot at $t = 0$ in a direction which forms an angle $\beta, 0 < \beta < \pi/2$, with its axis of rotation and in the plane Pl. This plane rotates as the earth. By the shot, the satellite at $t = 0$, obtains a kinetic energy E and it is retarded by the gravity force. On a plane orthogonal to the direction $Q \rightarrow P(0)$ and on the distance s from $P(0)$, the satellite rotates on a circle, where s is the distance belonging to the force Mm/s^2. The satellite has the force live $mv^2/2$ for velocity v, according to (2) and the energy E given the satellite by the shot. Note that the point Q and the circle determine an infinite cone with the axis from Q in the direction $Q \rightarrow P(t)$.

Consider now the situation when the satellite at $t = 0$ is shot in a direction $P(0) \rightarrow P(t)$ so that this direction for $t > 0$ forms an angle $\beta, 0 < \beta < \pi/2$

with the plane Pl, and that it then is given a kenetic energy E. We also assume that the direction of the shot is in such direction that it increases the velocity given the satellite by the rotation of the earth. The force F is independent of the direction of the shot, it only depends on its distance s from Q to $P(t)$ and thus F has the potential mM/s at s. At the point $P(t)$ the satellite is repelled by F in the direction $P(t) \to Q$ and the satellite will loose energy. There F attracts the sattellite. At last it arrives at a point $P(t)$ where F in the direction $P(t) \to Q$ has no component left. But the satellite may have some part of its energy E left and can still move on the plane Pl' orthogonal to the line $Q \to P(t)$.

Further it moves by the motion of the earth as a point in the plane Pl and hence on a circle and there already has the forth live $m(v_0)^2/2$. By surplus of the rest of E the satellite continues on this circle and get a larger velocity v and thus the force live $mv^2/2$. This circle determines a cone from Q as in the first case but its axis forms an angle μ with the axis of that one. This axis determines a plane at $P(0)$ orthogonal to the axis, the horizontal plane. Let this plane through $P(t)$ cut the cone, which $P(t)$ determines. On this plane we then get an ellips and on this one the circle determined by $P(\underline{t})$ is inscribed with the center in one focus of the ellips. The relation (2) holds with constant v and s on the circle, but variable v and s on the ellips. The velocity v changes according to the obvious relation that the radius vector from the common center and focus during the same time intervals Δt of rotation on the circle sweeps over equal areas of the circle and also sweeps over equal areas of the ellips. Keppler did this observation, when he studied the movements of the planets.

We have considered the motion of a satellite under influence of the gravity force from the center of the earth. The earth is satellite of the sun and so are all planets. We may consider the mass of the sun as concentrated to its center and the mass of a planet concentrated to its center. We let the sun together with all its satellites be a cage. Applying relativity theory we let the cage be translated orthogonally to the R^3 of the cage. By the relations given above, the earth moves on an ellips and so do the planets. We also proved that there is a one-to-one mapping of an ellips on a circle. Hence we may only deal with circles. A circle is determined by 2^n-polygons hence of configuratins of vectors. They do not change when R^3 is translated in a direction orthogonal to R^3. The motions in the cage are determined by the vectors and the time. In the satationary cage we have the time t and applying relativity theory we have the time t' in the cage, translated by the velocity$c$ of light, where t' is determined by t according to 3(2). So an observer on the earth in the stationary cage gets information about existence and movements of planet far away.

6 Refraction of light. Models for light

According to Newton's corpuscular theory, light consists of minute particles, corpuscular, shot out from the luminous body. However, it has been argued that this theory cannot give explanation of refraction since the velocity of light by refraction between two media should be in inverse proportion to the refraciton index, not proportional as in the wave theory. Hence the corpuscular theory has been condemned in this case and only the wave theory accepted. It seems to me that the corpuscular theory could be accpted also fo refraction. The essential measurable property of a rang of light is that it has a direction and is a carrier of energy. Its capacity to carry energy may be different for different colours of light and for other rays of the same nature as light.

In the corpuscular theory, the corpuscular are called photons. The fundamental question is what a photon is in this model. It should carry energy, since so do rays related to rays of light, and it It should have a given direction in a given optic media, and its refraction may be different in different optic medias.

We now consider the actual problem. Let rays of photons, all with the same direction, hit a plane with the normal $0 \to N$ in a R^3 as in the figure below, where w ehave a coordinate system with 0 as the center and the z-axis along $0 \to N$. Parallel planes to the (x, y)-plane through the points $z = 0, z_1 < 0, z_2 < z_1$ separate R^3 into three parts with different medias. We regard one beam as a point moving in the (x, z)-plane from P to 0 in media 1 and then in media 2 from 0 to P' and then eventually further into media 3. There is a road parallel to $P \to 0$ in media 1 and parallel to $0 \to P'$ in media 2. Here $P \to 0$ makes the angle β_1 and $0 \to P'$ an angle β' with the normal. The quotient $\sin\beta/\sin\beta'$ is called the refraction index. It is this index that is observed.

The neighbour parallel rays behave in the same way, but an essential difference in the situation is given by the two right triangles, both with hypotenuse OP' of length Δx, one triangle in media 1, the other in media 2. We have β_1 and β_2 also in the traingles so that

$$\sin\beta_1 = \Delta s_1/\Delta x, \sin\beta_2 = \Delta s_2/\Delta x, \Delta s_2/\Delta s_1 = \sin\beta_2/\sin\beta_1 \tag{1}$$

Here we consider media 2 as the material (say water) optic denser that media 1 (say air). Δs_2 is shorter than Δs_1 so that velocity v_2 of light in media 2 is smaller than velocity v_1 of light in media 1 and (for light in air, we have

$$v_2/v_1 = \Delta s_2/\Delta s_1 = \sin\beta_1/\sin\beta_2 \tag{2}$$

where $\sin\beta_1/\sin\beta_2$ is the refraction index. It is this that can be oserved. It is different for different coulers, different wave-length according to the wave-theory, different smallest distance between neighbour beams according to the corpuscular theory. However, when we consider β_1 and β_2 as angles between the normal, we get the refraction index $\sin\beta_1/\sin\beta_2$ and so we get a contradiction. The figure, presented here with the essential traingles, is in practice the same used by Christian Huygens in the first wave theory. He is considered as the founder of the wave-theory.

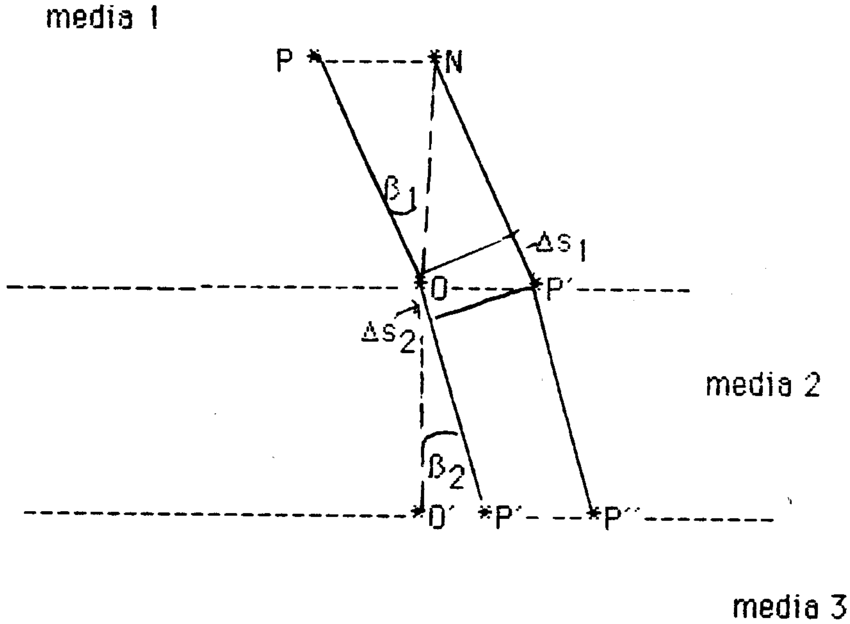

Figure 1.

To explain this contradiciton, it is necessary to find a model for the motion of light. It is not possible to find the real nature of light by observations, since we get information of occurences of light by light. However, we may use the relativity theory. The axis for the relative time t' is orthogonlal R^3 translated by the velocity c of light (in vacuum) along the time-axis for t'. All vectors, hence all directions, and thus the Figure 1 remain unchanged in the translated R'^3. Note that the motions in the moving R'^3, hence velocities v, are determined by vectors and time t'. The relative energy $m'c^2$ and the relations $(4) - 4(6)$ vill not be used here since we now only deal with the beams in refraction. But they are of interest in problems in connection with refraction.

In an experiment, described by the figure, parallel rays are observed. It is required that the rays are parallel both in media 1 and media 2 and that this can be observed. Straight lines orthogonal to the rays in media 1 cuts the rays in media 1, correspondingly in media 2. Thus the traingles used for the relation (2) should be applied. Note that also wave theory determines rays and that a result of an observation often is a vector Δs during a time Δt, reduced to Δs and $\Delta t'$ in the relativity theory.

Since we cannot get the real nature of light, different models, particularly the corpuscular model and the wave model. Both are used but the question has been asked, if there can be only one model. What we observe may depend much on our experiments. Hence we may in some situations find the best explanation by using the one or that one. Can they be joint by relativity theory? Thus apply relativity theory. This means that we use relative time t' in the moving R'^3 and consider only motions determined by vectors and time t'. Hence we first consider such motions in a stationary R^3.

In Section 5 we have found that a satellite with a point mas m can move on a circle round another point mass M by the gravity force Mm/s^2 and then the distance s between M and m is determined by the relation

$$mM/s = mv^2/2 \tag{3}$$

We have this situation in mascrocosmos. We may have the same model in microcosmos.

On the left-hand side of (3) we have the potential energy corresponding to the work that the gravity force has produced to bring m (from infinity) to the distance s. This force is a vector $P(m) \to P(M)$, where we consider m and M as point masses. The relation (3) is only dependent on the length s of the vector, not of its direction. Light has beams in all directions and thus the circle for m's rotation can be a circle on any plane that cuts the sphere with radius s.

At our obsrvations we give the light a direction and sometimes also a plane. Hence when we observe refraction in the plane as in the figure, we can only regard circles on the plane $\{$the (y, z)-plane in the figure$\}$. Note that we here consider photons as corpuscler but as corpuscler complicated as atoms. It is clear that these photons cannot move simultaneously into all directions. However, when we apply relativity theory, we permit motions that do not change vectors and vector configurations. Hence we permit translations and rotations.

At first, we deal with the situation in the figure. The direction is given by a straight line through the centers. We call it the centerline. This line cuts the circle into two points P_1 and P_2 and $P_1 \to P_2$ is a vector. We now introduce relativity theory and the relative time t'. All motions in the translated R'^3 are determined by t' and the vector configurations, and the circle is a vector configuration and they are the same in R^3 and R'^3, so for them we can use with or without the dot ($'$). We get a wave when we let the circle roll as a wheel on a horisontal straight line parallel to the centerline and on the distance s below it. the point $P(t)$ for m on the circle then moves on a wave curve given by

$$P(t) = (2s) \sin 2\pi t/T \tag{4}$$

Here $t = jT/2, j = 1, 3, 5, \ldots$ are the points where the centerline cuts the curve and the tangents have the same direction (the wave has the same phase). The distance $2s$ is called the wave-length.

Let $j(t)$ be the largest uneven integer smaller than t. Then

$$P(t) = (2s) \sin 2\pi(t - j(t)/T \tag{5}$$

describes a standing wave, i.e., the centers do not move.

When we consider a ray in the sapce, we deal with rotation and translation as above and thus tarnslation in the direction of the centerline. But we also permit that m rotates on the circle on a plane orthogonal to the centerline with the same radius s and the same centerline as in the motion considered above. This rotation, called a spin, also satisfies the relation (3) and this independent of the rotation on the circle in the plane through the centerline and orthogonal to the plane orthogonal to the centerline. Indeed, the actual forces in the two rotations are orthogonal two each other and thus the work they perform are independent. When rays are observed in a given direction, it may be as energy $mv^2/2$ in quanta, arriving with time differences. Electrons carry energy transported during time differences. $T(T')$ hence as quanta by velocity v ($v = c$ in vacuum).

In the next section we deal with beams of atoms and molecules taht carry both mass and electrons and also discuss models for light.

7 A relation in Bohr's Atomic Theory

By Rutherford, Bohr and others, an atom consists of a nucleus with mass and an electric charge and of electrons on orbits round the neucleus. Later more complicated nuclei have been considered. Atoms bounded to each other form molecules. The hydrogen atom, has been studied by Nils Bohr in his quantum theory. He has used the formula

$$1/\lambda = R\{1/(n_2(^2 - 1/(n_1)^2\}$$

for the wavelength in his Atomic Theory.

From Encyclopaedia Britanica, 1964, Volum 2, p. 709, I quote: "In seeking to explain such relationships with the help of the quantum hypotheses, Bohr first adopted a quite consciously native and even questionable approach as follows:

Suppose that an electron, initially free and stationary is drawn into a circular orbit of radius r around anucleus of charge $+e$ (e.e., a hydrogen nucleus). Let the mass of the electron be denoted by m and its orbital speed by v. To balance the electrical and centrifugal forces, the relation

$$(e/r)^2 = (mv^2)/r \tag{1}$$

is required".

What seems to be questionable is just this relation. There the left-hand side is the electric force, according ot Coulomb's law. Hence also the right-hand side must be a force. The left-hand side is the force between the positive charge e and of the nucleus (the proton), considered as a sphere with mass M and with the electrical charge e at the center of the sphere and the electron with negative charge e^- and mass m in the same way. By Newton's law there is the force kmM/r^2 between m and M. Here we introduce a constant k since m and M are different. At electron on a straight line from infinity to 0 cuts the circle in a point $P(T)$. The forces kmM/s^2 and e/s^2 then give the velocity v and the potential energy at $P(T)$,

$$kmM/r = mv^2/2 = e^2/r \tag{2}$$

(Compare 5(1) and 5(2) and the corresponding relations for the electric force). Note that the electric force and the gravitational forces are independent of each other. However, when we give them in this simple form the units must be chosen suitablly.

Let now L be the x-axis from infinity through $P(T)$ and consider points $P(t_i)$, on the circle, $t_{i+1} - t_i = \Delta t = 2^{-n}$, time-points. At the points $P(t_i) =$ on the distance $v\Delta t$ from each other, $2^n T$ in number, draw the tangents in these points.

When the electron is drawn by the proton to the orbit (circle) with the proton in its center along a straight line, the forces e^2/s^2 and kmM/s^2 together produce the potential energy

$$e^2/r + kmM/r = mv^2/2 + mv^2/2 = mv^2 \tag{3}$$

by Coulomb's and Newton's laws, the same potential energy at any point on the circle, but together with the charge e then the mass m of the electron has been drawn by the forces e^2/s^2 and kmM/s^2 from infinity to points on the orbit. Consider the case when $P(T)$ is on the x-axis at origo $(0,0)$ and the proton at $(-r,0)$. Then the y-axis is tangetn to the circle at the point $P(T)$. Let the electron be drawn along a straight line from infinity to $P(T)$ by the force

$$e^2/s^2 + kmM/s^2 \tag{4}$$

These two forces are independent of each other for given s. Hence we let the component e^2/s^2 be along the x-axis and the force kmM/s^2-along the y-axis. It is then the electric force e^2/s^2 that thas drawn the electron to $P(T)$ and thus produced the energy e^2/r at $P(T)$. Then it has also drawn ints mass m from ininity to $P(T)$ and there the gravity force kmM/r has given the electron an additional energy $mv^2/2$ by motion of it on the circle by velocity. In this meaning the drawing of the electron to the orbit by e^2/s has given the energy mv^2 according to (3). Indeed, the potential energy, at any point on the circle, produced by the force (4) when the electron is drawon from infinity to the circle is given by (3). But the component kmM/r^2 is in the directionof the y-axis given the electron the potential energy $mv^2/2$ and thus the velocity v is given the electron at $P(T)$ in the direciton of the y-axis, i.e., along the tangent.

Now according to Newtons laws the electron should go on in the direction of the tangent and this by the velocitty v, if there isn't a force that changes the direction. The only force that can change the direction is the force (4). But a change of (4) should change (3) which is given by the energy mv^2. Here $mv^2/2$

belongs to the movement of the electron. We have the same situation at all points on the circle. (We let $P(T)$ be any point on the circle with the corresponding coordinate system). Thus it is the potential energy $mv^2/2$ that forces the electron to move on the circle by velocity v. Inf act a force is necessary to change this potential energy. Since this energy is determined by a force. It is appropriate to call it "force live" (force vive en francais). The drawing of the electron to the orbit is done by the electric force e^2/s^2 which then has produced the energy e^2/r. But since then the mass of the electron has also been drawn to the orbit and by mM/s has produced the enrrgy $mv^2/2$. It is by the elctric force that the electron has been given the potential energy mv^2 so that we have the relation

$$e^2/r = mv^2,$$

which is Bohr's relation in a slightly changed form. Of course, it is the proton that has done all work, but may be that is the work, produced through the electric forces that are observed.

In order to separate the forces, we could also use relativity theory and consider the orbit in an R^3 in a cage moving with velocity of light in a direction orthogonal to R^3. Then

$$v'\Delta t', e^2/r \text{ not changed }, e^2/r = mv^2/2 = m'(v')/2 = mvv'/2 \tag{5}$$

(Compare 3(2) and 4(2)). So we have separated the electric force and its work form the gravity work, which is the gravity work Mm/r produced by the proton that brings the electron to the orbit, hence to the point $P(T)$ where the electron has the velocity v in the direction of the tangent. But as above we conclude that it is the electric force that has forced the electron to move on the circle.

8 Beams of atoms and molecules

We consider now beams of atoms H, molecules H^2 and along a straight line through the center, also center in the circular orbit. We consider the nucleus as in (7) as spheres with the electrical charges in the centers. The model can be used more generally for molecules and also in macrocosmos.

We shall use Bohr's relation in the form 7(5)

$$e^2/r = mv^2 \tag{1}$$

There the orbit, i.e., the radius of the circle is given, but several orbits are possible and some molecules can have several electrons on orbits with different radii, balanced by suitable mass M and suitable positive electric charge. Thus we must deal with different r and different velocities v. The problem is to determine r by v. Then we use the wave model in 6(4) when we now let the circle with radius r be in the (x,z)-plane and roll on a line in this plane on the distance

$z = -r$ by the velocity v. Then the centers of the moving circle are on the x-axis. An electron at a fix point of the circle describes the sinuswave determined by the formula

$$x(t) = 2r \sin 2\pi t/T \tag{2}$$

where T is the time for the electron to rotate one turn round the circle and thus $vT = 2\pi r$. Here the radius r is given by the distance $2r$ between the electrons in the beam.

It may be possible to observe the number N of atoms that pass a given point $x(t)$ during the time $T > 0$, and then the number of atoms leaving $x(T)$ is also equal to N since it takes the time T for the electron to move the distance $2r$ on the x-axis. Hence the mean velocity of the atoms is

$$v = \{x(NT) - x(t_0)\}/NT \tag{3}$$

This observation gives us a value of v in (2), however we have here an uncertainty. We do not know r or v which satisfy the relation (2), since there can be more than one such relation, i.e., the atom can be in different shapes so that the radius r is different and the electron can move on different orbits it may not be possible to measure both terms in (2) simultaneously. Say that we observe differences

$$\{x(t_{i+1}) - x(t_i)\} = v\Delta t \tag{4}$$

The difference may depend not only on errors in the observations but on uncertainty about the size of the radius (velocity). Then we introduce probability and let the velocity be a random variable, which we denote by v' and hence $v'\Delta t$ is also randomv ariable. (It is close at hand here to change Δt to $\Delta t'$ and use relativity theory but we shall not do so here.) Consider now the translations $2r$ of the electron n the x-axis for the time T. Let v_i be a random variable with mean value v and variance

$$E\{v_i - v\}^2 = \partial^2 \tag{5}$$

Then $v_i T = 2r$ and

$$x(NT) - x(0) = \sum_{0 \leq NT} v_i \tag{6}$$

Remember that a distribution function F for a random variable X has the following implicit definition.

Probability for the occurence $X \leq \mu = F(\mu)$ for any real number μ. A classic Central Limit Theorem of Lindeberg-Lévy (see Lévy Calcul des Probabilités, Paris 1925) states

Theorem 1 *If $z_1, z_2, z_3, = ldots$ are independent random variables with the same distribution and mean value 0 and variance ∂^2, then the distribution function*

$$\{n^{-1/2}\} \sum_{1 \leq i \leq n} z_i \tag{7}$$

as n tends to infinity, converges to the normal distribution $G(u/\partial)$, determined by its derivative

$$\partial^{-1}(2\pi)^{-1/2} \exp -(1/2)(u/\partial)^2 \tag{8}$$

We apply the theorem to the sum (6) but change the terms v_i to $(v_i - v)$ and put

$$z_i = (v_i - v) \tag{9}$$

and observe that the sum of such terms is N in number. It follows by the theorem that the sum (6) of random variables converges in distribution to $G(u/\partial)$ determined by (8).

Remark. The theorem above is only given in the form with $\partial = 1$. But in the paper in my references I have proved it in more general forms. It may have been given in these forms earlier.

Note that there is one term v_i, for any unit time interval, this also in the limit. Thus

$$G\{(v_i - v\mu)/\partial\} \leq G(v_i - v)/\partial\} \leq G\{(v_i - v)/\partial$$

for any $\mu > 0$. By Taylor expansions we get the term

$$\partial^{-1}(2\pi)^{-1/2} \exp -(1/2)\{(v_i - v)\}/\partial\}^2 \tag{10}$$

This is the Bolzmann-Maxwell relation.

The method described here can also be used for beams of molecules. These beams behave as waves when they in certain experiments are influenced by heat or pressure. Their movements, called Brown motions depend on attraction of the molecules in parallel beams. Hence the density of molecules on an interval, say on the interval considered in (6). However some physical constants have to be introduced for applications.

9 A general relativity theory

We have considered realitivy theory above under two conditions, where we called R^3 a cage and let it either be stationary or translated by the velocity of light in a direction orthogonal to R^3. The conditions where:

1. The vectors and the connected vector-configurations remain unchanged in the moving cage.

2. We only permit translation of the cage (by the velocity c). We have found that two kinds of elementary motions are of particular importance: a) motions on a straight line, b) rotations. However, the vectorcurves determine very complicated motions from one point to another point and they are recurrent. We can extend the theory as follows.

We let the cage be R^{k+2} of any finite dimension $k+2$, where R^k, with axis $e_i, i = 1, 2, \dots k$, rotates round the axis e_{k+1} through the center of R^{k+1}, this by constant angular velocity v_1, which means that any plane in R^k rotates round this axis so that vectors and vector-configurations are defined and remain unchanged by this rotation. Then we let R^{k+1} be translated by constant velocity v_2 along the axis e_{k+2}. This means that any R^{k+1} in R^{k+2} is translated along e_{k+1}. At last we introduce a time-axis t' for the relative time.

As in R^3 we deal with vectors $\Delta s = P(t) \to P(t + \Delta t)$ in R^k and configurations (vectorcurves) of consecutive connected vectors. Here we describe these configurations as follows. Two consecutive connected vectors detrmine a plane. The vectors are determined by their length and the angle between them and this angle is determined in their plane. So any vector configuration is given. When R^{k+1} is translated by constant velocity in the direction e_{k+2}, hence orthogonal to R^{k+1}, the configurations do not change. Indeed, any two vectors are in a plane orthogonal to e_{k+2} and the vectors in the plane keep their length and theangle between them when the plane is transported in the direction e_{k+2}. Our cage R^{k+2} is now transported in the direction of the time axis for t' and with the velocity c. The motions in R^{k+2} are given by the vectors and the time t. Hence we may apply relativity theory and introuce the time t', defined in Section 3 and so get the motions in R^{k+2} by the vectors and the time t. Hence we may apply relativity theory and introduce the time t', defined in Section 3 and so get the motions in R^{k+2} by the vectors and the time t'. Already $k = 3$ gives a quite interesting generalization s.

For large k we can have several translations and rotatations in spaces R^k for different order in the cage and so that the movements are independent of each other but have connvected vector congigurations. Note that all motions described in this general form are visualized in some R^3 or even on a plane or on a straight line or simply by a vector.

From Einsteins book, THE MEANING OF RELATIVITY, Forth Edition 1950, I quote:

"A little reflection will show that the theorem of the equatlity of the inert and the gravitational mass is equivalent to the theorem that the acceleration imparted to a body by a gravitational field is independent of the nature of the body. For

Newton's equation of motion in a gravitational field, written out in full, is

$$\begin{aligned}&\text{(Inert mass)(Acceleration)} = \\ &\text{(Intensity of the gravitational field)(Gravitational mass).}\end{aligned}$$

It is only when there is a numerical equality between the inert mass and gravitational mass that the acceleration is independent of the nature.

Here Einstein seems to consider a very general field, even if he disregard masses from bodies far away. Locally we have such relations, as 5(2) for our solar system.

Einstein applies the Riemann geometry. In this theory it is required that any two points on a manifold have transformations of one given point to any other point and that these transformations form a group. I think the vector curves has this property in any R^k. In these manifolds we have only finite many points belonging to the timeintervals $\Delta t = 2^{-n}$ for given n, but when we let n tend to infinity, we so get uniform convergence of movements on straight lines and circles and hence also on ellipses and on parts of these curves, hence for different movemetns between any two points.