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Abstract

Methods of analysing different kinds of variability in an engineering computational process are presented. The approach consists of two main steps. The first step is modelling the different kinds of variabilities and using computational methods to examine the effect on the result. The second step is reducing the uncertainty in the computed result by use of in-service data.

The different kinds of variability sources are identified and suitable ways to describe them are presented and a terminology is established that clarifies the concepts and facilitates the understanding of this topic. Definitions of the concepts scatter, error and uncertainty are made.

It is shown that separating scatter and uncertainty is not only clarifying, but provides a possibility for reduction of uncertainty in engineering computations. The developed methods are applied on several life predictions of mechanical jet engine components.

Keywords
Fatigue, Jet engine, Life prediction, Reliability, Scatter, Uncertainty, Variability

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Introduction

1. Background

The jet engine industry is exposed to an ever-increasing demand for higher performance, reduced weight and extended service lives, at the same time as the safety level must be maintained. Failure in a jet engine component would in most cases have catastrophic and extremely expensive consequences. Therefore, reliable methods to determine the maximum allowed time in-service for the components are needed. In order to be reliable, the methods must consider the scatter in the governing physical parameters and the uncertainty in the computational models should be reduced to a minimum given the available information.

Traditionally, three different methods to determine the life of an engine component have been available. The first one is an analytical prediction based on an understanding of the failure modes. The second method is testing the component in a rig. The third one is making a statistical analysis of in-service data.

The analytical approach is deterministic, which means that each variable in the equations governing the operation of the system is given a deterministic, conservative value, and a conservative value of life is predicted. This value is then divided by a factor of safety, which gives a "safe" maximum life in operation. Neither the scatter nor the uncertainty is considered in this deterministic prediction, except through the conservatism and by the rather arbitrary safety factors. The approach is probably unrealistically conservative, and there is a growing demand to quantify the true safety of the structure.

Rig testing is a reliable method of determining the inspection intervals needed, but it is expensive and cannot be done every time the life of a component should be determined. Furthermore, it could be difficult to obtain real conditions in the test rig.

The statistical risk analysis can be made if field experience is available. In this approach, historical data from past failures and non-failures are used to estimate a probability distribution of the life, and a low fractile in this distribution determines the maximum time in service. This method also involves a certain degree of uncertainty due to the limited number of observations that is normally available. Another problem with this approach is that it only considers specific structures subjected to a specific load. Thus, the effect of a change in one of the physical key parameters cannot be predicted.

For these reasons, a new analytical approach that takes the different kinds of variability into account has been sought. It should include probabilistic modelling of the inherent variability in the governing physical parameters as well as methods of modelling and propagation of the uncertainties. Uncertainties exist both in the input parameters and in the model itself. In an earlier Phd thesis at Volvo Aero Corporation, [10], the different activities in life predictions were studied and a generic life prediction process was developed. This process view can be utilised for keeping track of any uncertainty and scatter introduced in the different stages of calculation.

This process is a starting point for this work, and the idea was that the process view should help to manage the variability in a systematic way.

Furthermore, there is a demand for methods to use the information from inspections in the analytical prediction, i.e. a combination of the analytical and the statistical approach.

The variability sources have different characteristics and there is a need to make a survey of what the differences are and find suitable ways to describe the different kinds of variability. In the literature terminology varies and there is a classic discussion about the difference between random

and systematic errors. Another interesting discussion is the difference between the concepts scatter, uncertainty and error. A consistent use of these concepts cannot be found in the literature.

2. Purpose and scope

From the industrial point of view, the purpose of this licentiate thesis is to develop methods to manage the variability in the life prediction process for jet engine components, and to reduce the uncertainty in this process by use of in-service data. The project should be broad and include the entire computational process. This has been accomplished in the sense that methods applicable for all analysis steps have been studied, and will be presented in this introduction. However, the probabilistic finite element method has not been applied.

Obtaining overarching results has been more important than examining details thoroughly. Quantifications of single variability sources have therefore not been given priority. Thus, the thesis mainly deals with modelling techniques and computational methods to examine the effect of different kinds of variability sources on the response. The response is most often the life of a component.

No initial restrictions were made regarding which kind of methods to use and a broad literature study has therefore been done [14]. The primary aim of the literature survey was to cover the area as completely as possible regarding possible methods to manage variations in a computational process like the life prediction process.

From the scientific point of view, the purpose is to identify the different kinds of variability sources and find suitable ways to describe them and to establish a terminology that clarifies the concepts and facilitates the understanding of this topic. Definitions of the concepts scatter, error and uncertainty will be made, and, except in the first paper, these definitions are followed strictly throughout the thesis.

Furthermore, different modelling techniques for the different kinds of variability sources are discussed. A survey of recommendations in the literature is presented and a suitable approach for our application is chosen.

In the first appended paper, [16], a method for rough uncertainty propagation is presented and in the second, [15], and third paper, [17], more sophisticated techniques of uncertainty modelling, and methods to compare and reduce uncertainties in computational processes are presented.

3. Terminology

3.1 Survey of the terminology used in literature

In [1], comprehensive definitions of uncertainty and errors that are appropriate for modelling and simulation are presented.

”Uncertainty is a potential deficiency in any phase or activity of the modelling process that is due to lack of knowledge”. Note the word ”potential”. It is not certain that the deficiency occurs.
“Error is a recognisable deficiency in any phase or activity of modelling that is not due to lack of knowledge”. An error is thus, according to this definition, identifiable and quantifiable upon examination. When the error is quantified it is either corrected or allowed to remain. The errors can be divided into acknowledged and unacknowledged errors. Acknowledged errors are, e.g., physical approximations made to simplify the modelling of a physical process or a specified level of iterative convergence in a numerical scheme, i.e. model errors. Unacknowledged errors are blunders or mistakes, i.e. human errors.

In [11], it is stated that uncertainty means doubt, and thus, in its broadest sense, “uncertainty of measurement” means doubt about the result of a measurement. Uncertainty in life predictions thus means doubt about the result of a life prediction analysis. But the word “uncertainty” is also used for the quantitative measures of this doubt. Consequently, the word is used in two senses, which is also emphasised in [11].

Other definitions of the concept “uncertainty” are:

- a measure of the possible error in the estimated value of a parameter
- an estimate characterising the range of values within which the true value of a parameter lies.

This definitions are consistent with “a quantitative measure of doubt”.

In [24], the authors divide the variability sources into irreducible and reducible uncertainties. The irreducible uncertainties include the inherent variations in physical parameters while the reducible uncertainties include uncertainty due to estimation, modelling and human errors. No distinction is made between uncertainty and error in this paper. One could argue that the reducibility is completely dependent of the situation one is in, e.g. which parts of the industrial process one could influence. However, there is a fundamental difference, since the irreducible variabilities not can be reduced without physically changing the component, while the reducible can be reduced by learning more about the process and developing more accurate models.

In [3], it is stated that uncertainties in measurements arise both from random and systematic errors. In [11], a clearer distinction is made between uncertainty and error. It is stated that uncertainty of the result of a measurement reflects the lack of knowledge of the value of the measurand, while the error is the deviation from the true value in a specific measurement. The error has two components, one random and one systematic. However, since the exact value of the error is never known, these concepts are not used. Instead, the authors of [11] divide the uncertainties into Type A and Type B uncertainties. Type A is uncertainties that are evaluated by statistical analysis of observations. Type B is uncertainties that are evaluated by other means than by statistical analysis of observations, e.g. by an estimation based on knowledge about the underlying physical process.

This is consistent with [3] in the sense that the errors cause the uncertainty, but the difference between error and uncertainty is not clearly stated. Moreover, the authors of [11] call Type A and Type B uncertainties random and systematic errors, respectively.

A comparison with the classification in [24] shows that the irreducible uncertainties and the estimation uncertainties belong to Type A uncertainties, and model and human uncertainties belong to Type B uncertainties.

According to the definitions in [1] presented above, errors are effects of conscious simplifications and mistakes, and the authors of [1] call an estimation of the possible magnitude of such an effect “error”. This is not in accordance with [11].

In [2] it is stated that uncertainty is uncontrolled inherent stochastic variations, which is irreducible uncertainties according to [24], and furthermore, it is not in accordance with the definitions of uncertainty in [1] and [11]. The reducible uncertainties are called imprecision in this paper.

It can be concluded that the terminology differs in the literature and we need to make our own definitions of the key concepts in this area.

3.2 Terminology in this thesis

From a practical standpoint, the important separation is that between irreducible and reducible uncertainties.

In a computational process, the irreducible variabilities are the inherent randomness in the governing physical parameters. It is assumed that we have no influence on the physical properties of the component, such as the choice of material and the manufacturing process. Given this situation, this kind of variability is irreducible. According to the definition of uncertainty above, this is not an uncertainty, since it really exists and is not due to lack of knowledge. Scatter is suggested as a name for this kind of variability. A distinction is also made between uncertainty and error, as in [11].

Four key concepts in this thesis are now defined. These definitions are not followed consistently in the literature, and contradicting definitions of these concepts can also be found, as was illustrated above.

Definition 1: *Scatter* is inherent random variation in physical parameters. It is variation between components and is observed in real data. Scatter is irreducible in the context considered.

Definition 2: *Error* is the difference between the predicted and the true value of a parameter.

Definition 3: *Uncertainty* is the possible error in a parameter and includes estimation errors, errors due to approximations and errors due to human mistakes. It is reducible.

Variability is used as a collective name for all sources of variation.

In agreement with these definitions, a random error is the difference between the predicted and the true value of a parameter, where the predicted value is different in each prediction. The estimation uncertainties and the human uncertainties fulfil that. The scatter is not associated with any error at all, since there is no true value to compare with.

A systematic error is the difference between the predicted and the true value of a parameter, when the predicted value is the same in each prediction. The model uncertainties fulfil that.

4. The life prediction process

The life prediction process is shown schematically in Figure 1. This process consists of a series of linked activities, and requires specific methods and data depending on the situation. In each of these activities, scatter and uncertainty are introduced through the data, and uncertainties are introduced due to approximations in the methods used.

The generic activities in the middle column use methods (to the right) and data (to the left). Which methods and data are used depend on the situation. The required accuracy together with available resources and data are the most important parameters in deciding which approach to choose.

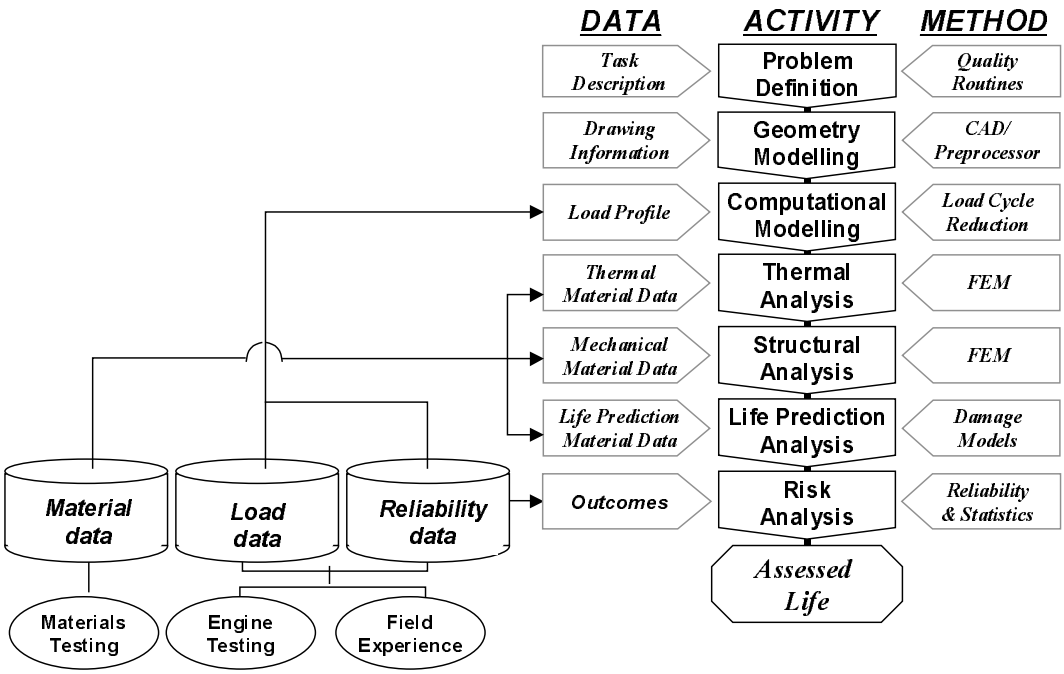


Figure 1: Generic description of the life prediction process at Volvo Aero Corporation, from [10].

The *problem* has to be defined and an analysis strategy has to be outlined. A *geometry model* is needed as a base for the *computational model* that defines the conditions (loads, topology and boundary conditions). *Thermal analyses* are made as the temperature field is vital for most engine components due to temperature gradients causing thermal stress, and material properties are temperature dependent. The *structural analyses*, which are based on the structural loads and thermal fields, then give information about the stress and strain field. Once the temperatures, stresses and strains are known, various *life prediction* models can be applied to predict the component life. Different models are used depending on the situation, e.g. crack propagation models, crack initiation models and oxidation models. Finally, the scatter and uncertainty in the process should be analysed using probabilistic models, and data from tests and the field should be considered, which is addressed in this thesis.
The scatter is modelled with random variables. In order to analyse the uncertainties in the models and in the estimates of the distribution parameters, a specific modelling technique is needed, see Chapter 6. The feedback of test and field data is considered in Chapter 8.

5. Scatter modelling

Methods for problems with scatter in the input parameters as the only source of variability are presented. The scatter in the input parameters is due to inherent variability in the physical process and is random in its nature. It is therefore suitable to model scatter by random variables and it is also an established modelling technique for scatter.

For structural problems with only scatter the reliability calculation is straightforward and some of the available analysis methods are presented below.

5.1 Reliability methods

The reliability theory and its application to structural problems has been documented in an increasing number of textbooks such as [20], and [19]. The reliability methods can be divided according to the level of description into random variable methods, random process methods and

random field methods. This thesis mainly considers the random variable methods but the random field method will also be mentioned in connection with the stochastic finite element method. The basic problem considered in the reliability theory is the general reliability problem, presented below.

5.1.1 The general reliability problem

Consider a response function, $Z(\underline{X})$, where $\underline{X}$ is a vector of input random variables to the computational process. In our application, Z is the life of the component. Then define a limit state function as:

$$g(\underline{X})=Z(\underline{X})-z_0 ,$$

where z_0 is a value of the response such that $g(\underline{X})=0$ divides the design space into failure and safe regions. The general reliability problem consists in calculating the probability of failure, i.e.:

$$P_f=P\{g<0\}$$

The exact solution requires the solution of the multidimensional integral:

$$P_f = \int_{g<0} f_{\underline{X}}(\underline{x})dx$$

which is generally extremely complicated. An alternative is to use simulation methods, such as Monte Carlo simulation, but that, on the other hand, is prohibitively expensive from a computational point of view. More efficient methods, referred to as Fast Probability Integration methods, FPI, are presented in this chapter. A FPI-method generally consists of three main steps:

1. Transformation of the basic random variables, $\underline{X}$, and the limit state function, $g(\underline{X})$, into the standard normal space, where the random variables are uncorrelated standard normal variables.
2. Determination of the point on the transformed limit state surface, u^*, that is closest to origin. This point is the most likely combination of the input random variables that give the requested response, z_0, and is often called Most Probable Point (MPP). This is the crucial step in the reliability analysis and can be carried out by optimisation algorithms.
3. Determination of the reliability index, i.e. the distance between the MPP and origin, and the associated failure probability.

5.1.2 FORM

Methods based on a first-order approximation of the response function are called First Order Reliability Methods (FORM), [22]. In this method the random variables are transformed into uncorrelated standard normal variables:

$$\underline{X} = (X_1, X_2, ..., X_n) \rightarrow \underline{U} = (U_1, U_2, ..., U_n),$$

and the response function in x-space is mapped on the corresponding response function in u-space. The transformation is particularly simple when the variables are independent:

$$\Phi(u_i) = F_{X_i}(x_i) \rightarrow u_i = \Phi^{-1}\left[F_{X_i}(x_i)\right]$$

For dependent random variables an analogous transformation often referred to as the Rosenblatt transformation can be used, [9]:

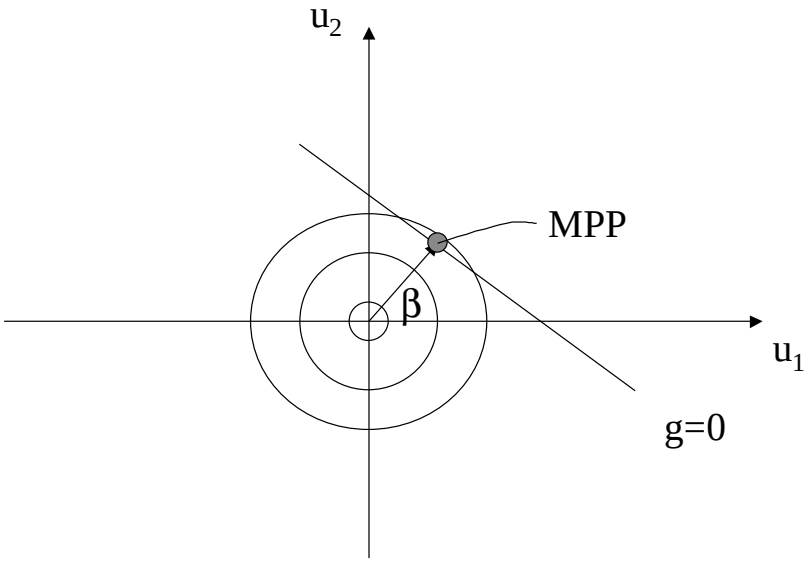
$$\Phi(u_1) = F_{X_1}(x_1)$$
$$\Phi(u_2) = F_{X_2|X_1}(x_2|x_1)$$
$$\Phi(u_3) = F_{X_3|X_2,X_1}(x_3|x_2,x_1)$$
$$\ldots$$

If only the marginal distributions and correlation matrix are given for the basic random variables, the Nataf model can be used [7].

The transformed response function is approximated by a first-order Taylor series. One possibility is to make the Taylor expansion about the mean values. The method is then called the ”Mean Value Method (MV)”. This approach gives severe errors in most cases and an alternative approach that is much more accurate is to expand the Taylor series about the MPP, u^*. The problem is then to find the MPP, but it can be solved by optimisation methods.

After the transformation, the level curves of the joint density functions become circles with their centres at the origin. The safety index, β, is defined as the minimum distance between the origin and the limit state surface. The limit state surface is a hyperplane in the space of the transformed random variables (u-space), see Figure 2. An approximation of the probability of failure is then given by:

$$p_f = \Phi(-\beta)$$



Figur 2. Illustration of FORM

5.1.3 SORM

In the Second Order Reliability Methods (SORM), the response function in u-space is approximated by a second-order Taylor-series. Since the limit state surface then gets a quadratic form, the failure probability becomes more complicated to calculate. A convenient but approximate expression for the failure probability was derived by Breitung 1984, [26]:

$$p_f = \Phi(-\beta)\prod(1+\beta\kappa_i)^{-0.5},$$

where κ_i is the principal curvature at the most probable point on the limit state surface.

5.1.4 The Mean value method

When the response function is implicitly defined by a computer code, the general reliability methods presented above cannot be applied since they demand a response function in the form of an explicit function. The “Mean value method” (MV), [29], which was mentioned in connection with the FORM, is a first-order method that is suitable in this case. It is based on a first-order approximation of the response function, which is obtained by a Taylor expansion about the mean values:

$$Z(\underline{X}) \approx Z_1(\underline{X}) = Z(\underline{\mu}) + \sum_i \frac{\partial Z}{\partial X_i}(X_i - \mu_i) = a_0 + \sum_i a_i X_i \ ,$$

where a_0 and the a_i:s are coefficients and the derivatives are evaluated at the mean values. These coefficients can in general be computed by numerical differentiation, and the minimum number of Z-function evaluations (computer runs) is (n+1), where n is the number of random variables. When the coefficients are determined, the MPP is located using the linear limit state function, $g(\underline{X}) = Z_1(\underline{X}) - z_0$. When the MPP is located the failure probability can be computed by FORM.

5.1.5 The Advanced mean value method

For non-linear response functions the MV solution is normally not accurate enough. The ”Advanced mean value” method (AMV), [28], takes the MV solution one step further and is capable of establishing full probability distributions even when the response function is non-monotonic, which can result in truncated distributions. In the AMV-method, the response function is written as:

$$Z(\underline{X}) = Z(\underline{\mu}) + \sum_1^n \frac{\partial Z}{\partial X_i} \cdot (X_i - \mu_i) + H(\underline{X}) = Z_1(\underline{X}) + H(\underline{X}) \, ,$$

where Z_1 is the first order approximation and H represents the higher order terms. Initially, the failure probability is approximated by using the linear limit state function, $g(\underline{X}) = Z_1(\underline{X}) - z_0$, i.e. the MV-method. The MPP that is obtained, $\underline{x}^*$, is then inserted in the computer codes and the exact value of the response is computed in that point, i.e. $Z(\underline{x}^*)$. The value of Z that is obtained is used to correct z_0. (If the linear function is correct, $Z(\underline{x}^*)$ should be equal to z_0).

The AMV-method can be summarised as follows:

1. Obtain the linear approximation, $Z_1(\underline{X})$, by pertubations about the mean values.
2. Compute the MPP, $\underline{x}^*$, for a selected value of the response, z_0, based on $Z_1(\underline{X})$.
3. Recompute Z(_*) to correct z_0 .

Many other versions of FPI-methods are available, e.g. [5] and [30].

5.2 Probabilistic finite element methods

The thermal and structural analyses in the life prediction are both based on the finite element method. In order to be able to use random variables as inputs of these analyses a probabilistic finite element method is needed.

The mathematical essence of the problem is the analysis of stochastic partial differential equations with coefficients in the form of random fields. The complete solution is constituted by giving the joint probability distributions of all the random response variables for all sets of time instants. This

is in practise impossible to achieve, and all the *Probabilistic Finite Element Methods* (PFEM) are based on approximations and only parts of the complete solution are obtained.

The probablistic finite elements methods combine the finite element method with statistics and reliability methods and are applicable to linear and non-linear structural mechanics problems as well as fracture mechanics problems. It is considered a milestone in probabilistic mechanics, and several versions are now available. They can be divided into direct and indirect methods. The difference is that in the direct methods, the probabilistic properties of the response is obtained directly by finite element techniques, while in the indirect methods, a set of deterministic solutions is used to compute the probabilistic properties. In the literature, the direct methods are most often referred to as *Stochastic finite element methods* (SFEM), [13], [31], while the indirect methods are often called *Finite element reliability methods*, [6], [23].

5.2.1 The Stochastic finite element method

Several versions of the stochastic finite element method (SFEM) are available. One version is treated in detail in the textbook by Kleiber, [13]. It is a second-order second-moment perturbation, or Taylor expansion method. In this method, all the field variables are expanded by Taylor series about the expectations of the basic random design variables, and terms up to second order are retained. Examples of field variables are elastic moduli, body forces, boundary tractions and displacements. An example of a basic design variable is the thickness of a component.

Three systems of linear ordinary differential equations are obtained. From these equations, the displacement field and its first and second order derivative and their time derivatives can be solved for. When these quantities are obtained, the mean displacement field and its covariance matrix can be computed at each time instant. The drawback of this method is that it requires long derivations and extensive coding and that only two statistical moments are obtained.

Another example of a stochastic finite element method are the Neumann expansion method, [31].

5.2.2 Finite element reliability methods

The finite element reliability methods (FERM) couple general reliability methods with the finite element method (FEM). A major advantage of these kind of probabilistic finite element methods is that commercial deterministic finite element codes can be used, and also that the whole distribution function can be computed, which is often necessary in reliability and life prediction analyses. This is probably the two most important reasons why the indirect method is more common in industrial applications.

In, [6], the derivatives of the response sensitivities are approximated on the basis of finite differences computed by perturbing each random variable using the deterministic finite element code. The reliability is then computed by means of the general reliability methods. The method becomes computationally expensive when the number of random variables is very large, since it requires one reliability calculation for each response random variable. The number of such variables could be very large if the whole field needs to be solved for. However, it is easy to implement since commercial FE-codes and general reliability methods can be used.

Another approach on the basis of the same basic idea, is to use the theory of design of experiments to estimate the response surface. These methods are called response surface approaches. Each computer run is considered a numerical experiment and the results of these “experiments” are used to fit a polynomial surface.

6. Uncertainty modelling

Methods of modelling different kinds of uncertainty are presented. The simplest way to model uncertainty is to increase the variance of the random parameters, which originally only reflects their scatter. Uncertainty can also be modelled as additional random variables that is incorporated in the

vector of random variables, $\underline{X}$. Both these methods have the drawback that the variance of the response will include uncertainty and the real variance is not obtained. Furthermore, no explicit measure of the uncertainty in the result exists.

The purpose of probabilistic analysis is most often to calculate the probability of failure of a component. The variance of the computed distribution of life will be too large if it includes uncertainty, and the risk will be overestimated, which leads to a conservative design, i.e. either an oversized component or an unnecessarily short inspection interval. Furthermore, since the uncertainty can be reduced with better analysis methods or more information it is interesting to have an explicit measure of it. In order to be able to compare the amount of uncertainty in different analysis methods, and also to reduce the uncertainty by the use of in-service data, it is necessary to separate uncertainty and scatter.

In this thesis, the uncertainties are, just like the scatter, modelled as random variables but they are treated differently in the analysis. We thus have two kinds of random variables: scatter variables and uncertainty variables. The vector of uncertainty variables is denoted $\underline{Y}$.

Two additional concepts need to be defined now:

Definition 4: A *physical parameter* represents a physical property or state of a component. Examples are the Young's modulus and the temperature.

Definition 5: A *statistical parameter* represents a property of a statistical distribution. The most common statistical parameters are the mean and the variance.

A physical parameter can be either random or deterministic. If it is random it is represented by a random variable that has a distribution and this distribution is characterised by statistical parameters.

Statistical parameters can also be random and described by random variables. These random variables are uncertainty variables and included in the vector $\underline{Y}$.

In the Bayesian approach of statistics, [4], a statistical parameter is considered a random variable, and its distribution reflects the investigator's personal belief. This belief can be based on data or entirely subjective. In our application the belief is based is based on several experts gathered knowledge about the physical process.

Since the uncertainty estimations in our application are based on more or less subjective belief, the Bayesian approach is considered suitable to use. In the cases studied, the uncertainties are modelled by treating the statistical parameters of the input random physical parameters as random variables. This is illustrated in Figure 3 and 4, where the physical parameter is normally distributed.

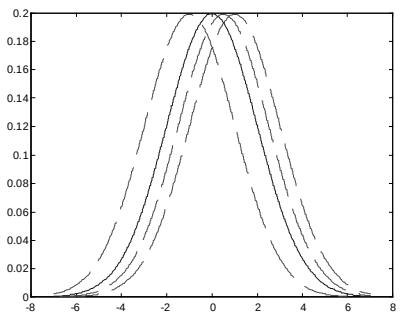


Figure 3. Distribution with uncertain mean

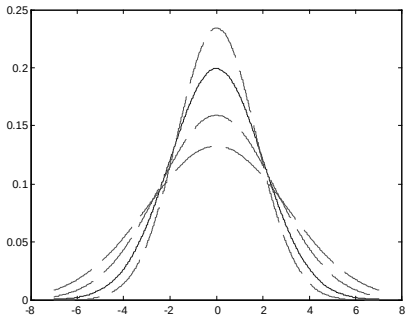


Figure 4. Distribution with uncertain scatter

However, the main reason for using the Bayesian approach is that it makes it possible to use observed data to reduce the uncertainty in the statistical parameters. The observed data can be both test data and in-service data. The uncertain statistical parameters are then updated by Bayes formula and new distributions with less uncertainty are obtained for the statistical parameters.

If the true value of an uncertain parameter is observed, which occurs very seldom in reality, the distribution will vanish since the observations in that case are a deterministic value. In reality, the true value of an uncertain parameter can hardly ever be observed directly. A common situation is that one has an uncertain mean and variance of a random physical parameter. Observations of the physical parameter can, for example, be obtained by measurements. These observations have a scatter, and there is still some uncertainty in the estimations of the mean and variance. By combining the data with the prior belief, the uncertainty is reduced to a minimum given the available information.

Using test and in-service data in the analytical prediction was one of the main aims of this research project, and the updating procedure is presented in some more detail in Chapter 8, and is also applied to the life prediction of a jet engine component in the last appended paper.

Which kind of distribution that should be used for different kind of uncertainties and methods of estimating its parameters are discussed in the two following subchapters.

6.1 Estimation uncertainties

Estimation uncertainties are due to the limited number of observations that is available when a statistical parameter should be estimated. The distribution for the estimate can be derived from the data. Thus, the classic approach to statistics can be applied in this case. The variance of the statistical parameters is inversely proportional to the number of observations, which means that the uncertainty diminishes as the amount of data increases.

6.2 Model and human uncertainties

When a model is used to describe a physical process, approximations have to be made. Therefore, model uncertainties always exist. In a probabilistic analysis, the model uncertainties also include uncertainties in the probabilistic model. If an understanding of the phenomenon is lacking or very simplified models are used, the model uncertainty is great.

Human uncertainties result from mistakes made by the engineers that are involved in the computational process. It can be reduced by quality assurance [12], but is in general hard to define and estimate. Human uncertainties will be treated as model uncertainties.

The model uncertainties are modelled as random variables according to the Bayesian philosophy. In the cases studied, they have been modelled by treating the means of the random physical parameters, i.e. the scatter variables, as random variables. However, any parameter can be used to model a model uncertainty, as long as it is separated from the scatter variables. Which kind of distribution that should be used is discussed frequently in the literature.

According to the ISO-standard [11], the model uncertainties (called Type B uncertainties in the standard) may be assumed to be normally distributed unless otherwise indicated.

In many cases the only possibility is to estimate bounds for the quantity, i.e. the probability that the parameter lies anywhere within the interval is one and the probability that it lies outside is zero.

This corresponds to a uniform distribution with the mean value $\frac{a_- + a_+}{2}$ and the variance $\frac{(a_- - a_+)^2}{12}$, where a_- and a_+ are the lower and upper bound for the quantity, respectively.

The step function discontinuities in the above distribution are often unrealistic. In such cases, it is recommended in [11], that a symmetric trapezoidal distribution with sloping sides, instead of a uniform distribution, should be used. The base of the trapezoid is 2a and the top in 2aβ, where $0 \leq \beta \leq 1$. The variance in such a distribution is $\frac{a^2(1+\beta^2)}{6}$.

In this work, the normal distribution will be used due to its mathematical convenience, mostly with respect to the updating procedure. If data of the parameter are observed, the normal distribution can very easily be updated, and the distribution statistic will remain normal, see Chapter 8. Further, the normal distribution maximises the entropy for given mean and variance. Hence it is the least restricted distribution satisfying these constraints.

6.3 Uncertainty propagation

Because of the uncertainties both in the input parameters and in the physical and probabilistic models, the calculated reliability will also be uncertain. Since the uncertainties are modelled by random variables the reliability will be a random quantity itself, see Figure 5. This randomness can be expressed in terms of a *credible interval*. The term *credible interval* is used in accordance with the Bayesian terminology. However, in the literature this interval is often called *confidence interval*. If the response is calculated at specified probability levels, credibility intervals can be computed for the response.

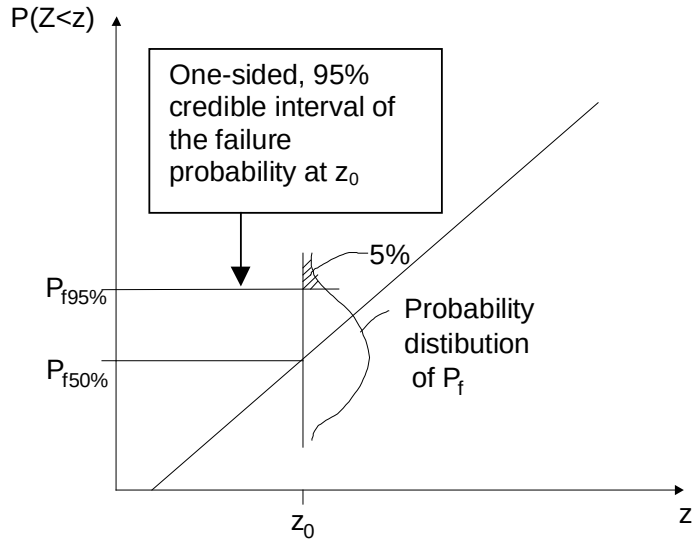


Figure 5. Cumulative distribution function of Z with uncertain probabilities.

The effect of the inherent variability is reflected in the variance of the calculated distribution, and the uncertainty is reflected in the credible intervals. Different methods of computing the credible intervals are available.

The credible intervals can be computed by Monte Carlo simulation. The Monte Carlo technique is described in Chapter 7. A large number of reliability computations with different values of the statistical input parameters are then made. Since each reliability computation can be very time consuming the Monte Carlo simulation can not be based on the exact response function. Tolerable analysis times can be obtained by using the approximate explicit response function obtained in the scatter analysis. A credible interval with credible level α% is then defined as the interval that contains α% of the computed reliability values.

In [24], an analytical approach is proposed for calculating the credible intervals. With uncertainty variables present the response function becomes $Z(\underline{X},\underline{Y})$, where $\underline{X}$ is the scatter variables and $\underline{Y}$ is the uncertainty variables. In order to account for the dependence of $\underline{Y}$, the joint distribution of $\underline{X}$ is expressed as a conditional distribution on $\underline{Y}$, $f_{\underline{X}|\underline{Y}}(\underline{x}|\underline{y})$, and the limit state function becomes:

$$g(\underline{X},\underline{Y}) = Z(\underline{X},\underline{Y}) - z_0 \ ,$$

where z_0 is a specified value of the response, e.g. the life of a component. For a given $\underline{Y} = \underline{y}$, the probability of failure is given by $P_{f|Y=y} = \int_{g<0} f_{X|Y=y}(x)dx$, which can be solved by the reliability methods presented in Chapter 5. For uncertain $\underline{Y}$, the failure probability, $P_{f|Y}(\underline{Y})$, is a random variable. A one-sided credible limit for the failure probability is given by:

$$P(P_{f|Y}(\underline{Y}) \le p_{\max}) = \alpha \ ,$$

where p_{max} is the credible limit and α is the credible level. In order to solve this, a new limit state is defined:

$$G(\underline{Y}) = P_{f|Y}(\underline{Y}) - p_{\max} = 0$$

If p_{max} is specified, the problem of calculating the credible level is analogous to the problem of calculating the probability of failure given a specified response. The opposite problem, i.e. calculating the maximum limit of the failure probability given the credibility level is analogous to calculating the response corresponding to a specified failure probability. These problems can be solved by the reliability methods presented in Chapter 5.

Since each evaluation of the limit state function, $G(\underline{y})$, includes computation of the failure probability, $P_{f|Y=y} = \int_{g<0} f_{X|Y=y}(x)dx$, which itself is a reliability problem, a nested analysis is required. If, for example, the Advanced Mean Value method is used, the problem demands (m+2) computations of the G-function and hence computations of $P_{f|Y}(\underline{Y})$. Here, m is the number of uncertainty variables. Each computation of $P_{f|Y}(\underline{Y})$ is a reliability analysis with different specified values of the uncertainty variables. If the AMV-method is used, each of these reliability analyses demands (n+2) computations of the original limit state function, $g(\underline{x},\underline{y})$, where n is the number of scatter variables. The total number of evaluations of $g(\underline{x},\underline{y})$ is therefore (m+2)*(n+2) in this case. Since each evaluation is often a complete life prediction in our application, the analysis time can be excessive. Thus, an approximate and more efficient method is needed.

If the AMV method is used, a linear limit state function, $g_1(\underline{x},\underline{y})$, is constructed each time $G(\underline{y})$ is evaluated. One approximation is not to reconstruct the g_1-function for each change in $\underline{y}$ but construct it once, based on $\underline{Y} = \underline{\mu}_y$, and then replace the original limit state function with this function.

The uncertainty variables are often the mean and standard deviation of the scatter variables. In that case an approximate function of G can be made based on the probabilistic sensitivity factors.

This method is proposed in [25], where the credible interval is expressed in terms of the safety index, β. In the case of independent scatter variables, the probabilistic sensitivity factors are given by:

$$\frac{\partial \beta}{\partial \mu_{X_i}} = \frac{\partial \beta}{\partial u_i} \cdot \frac{\partial u_i}{\partial \mu_{X_i}} = \alpha \cdot \frac{\partial u_i}{\partial \mu_{X_i}}$$

$$\frac{\partial \beta}{\partial \sigma_{X_i}} = \frac{\partial \beta}{\partial u_i} \cdot \frac{\partial u_i}{\partial \sigma_{X_i}} = \alpha \cdot \frac{\partial u_i}{\partial \sigma_{X_i}}$$

Here, α is the sensitivity factor obtained in the scatter analysis, u_i is the transformed variable, $\frac{\partial u_i}{\partial \mu_{X_i}}$ and $\frac{\partial u_i}{\partial \sigma_{X_i}}$ can be derived analytically since u_i is a function of μ_{X_i} and σ_{X_i}. Here, independence is assumed between μ_{X_i} and σ_{X_i}. The limit state function, G($\underline{Y}$) is then approximated by:

$$G(\underline{Y}) \approx \beta_0 + \sum_{j=1}^{m} \frac{\partial \beta}{\partial \mu_{X_j}} \cdot (\mu_{X_j} - \mu_{0X_j}) + \sum_{j=1}^{m} \frac{\partial \beta}{\partial \sigma_{X_j}} \cdot (\sigma_{X_j} - \sigma_{0X_j}) - \beta_{\min} = 0$$

Here, β_0 is the safety index based on the mean values of the uncertainty variables, i.e. $G(\underline{\mu}_y) = \beta_0$, μ_{0X_j} is the mean of the j:th uncertain mean, i.e. $E(\mu_{X_j}) = \mu_{0X_j}$, and σ_{0X_j} is the mean of the j:th uncertain standard deviation and $\beta_{\min}$ is the lower credible limit of the safety index.

This approximate limit state function can be used to estimate the safety index for different sets of uncertainty variables without re-evaluating the original complex limit state function, $g(\underline{x}, \underline{y})$.

The advantage of the analytical approaches is that they produce sensitivity factors with respect to each uncertainty source, which gives valuable information about which uncertainties have the geatest impact.

In order to reduce the uncertainty in the predicted life by using in-service data, the resulting uncertainty in the distribution of life needs to be expressed in terms of prior distributions of the statistical parameters of the predicted distribution, see Chapter 8. These prior distributions can be derived from the computed credible intervals. An example of that is carried out in the last appended paper.

7. Alternative approaches

The presented method of variability analysis is a probabilistic approach. Deterministic approaches are also available, like for example interval analysis, [18], in which bounds for the parameters are defined and a domain for the response is created in which the worst case is chosen.
In [2] a number of different methods of uncertainty modelling are reviewed. Examples are the method of Fuzzy sets, which is applicable when the parameters have a “fuzzy character” , utility theory and optimisation theory.

However, since the overarching purpose of the project is to quantify the risk of failure, and since the computational methods are well established for random variables, a probabilistic approach is used.

Most of the proposed probabilistic methods are based on Fast Probability Integration techniques. An alternative to that is to use simulation techniques. The most straightforward simulation method is Monte Carlo simulation. One value is sampled from each input random variable and the response is calculated on the basis of these values. This procedure is repeated, and the number of failures is counted. The failure probability is defined as the fraction of failures in the total number of simulations. The method is frequently used in reliability problems due to its simplicity but is computationally far too expensive in the context of life predictions for jet engine components, as each simulation requires a number of complex analyses. In paper 2, the Monte Carlo simulation is used to compute the credible bounds. The simulation is in that case based on the approximated explicit response function and does not involve the complex analyses. Thus, Monte Carlo simulation can be used for uncertainty propagation. It could also be used for scatter propagation if it is combined with a response surface method, but not as a straightforward reliability method.

The computational cost can be reduced significantly by importance sampling. The main idea of importance sampling is to restrict the sampling domain to the tail parts of the joint distribution of the basic random variables. Different sampling strategies exist, see e.g. [8]. In [21], importance sampling is applied on finite element analyses.

Importance sampling can also be combined with analytical reliability methods in order to reduce the uncertainty in the probabilistic model, [29]. This approach has not been used in this project, but it seems to be efficient and is suggested as a topic for further work, see Chapter 10.

8. Uncertainty reduction by use of in-service data

In Bayesian statistical theory, [4], the statistical parameters are considered random variables. Before any data is observed the parameters are estimated by a prior distribution, which is based on experts subjective belief. When data is observed the prior distributions are updated by (1), and the posterior distribution (1) is obtained.

$$f_{\mu|\underline{X}}(m|\underline{x}) = \frac{f_{X|\mu}(x|m) \cdot f_{\mu}(m)}{\int f_{X|\mu}(x|m') \cdot f_{\mu}(m')dm'} \tag{1}$$

Here X is the random variable, μ is the uncertain statistical parameter, f_{μ} is the prior distribution of μ, $f_{X|\mu}$ is the conditional distribution of X given μ, $\underline{X}$ is a vector of observations and, $f_{\mu|\underline{X}}$ is the posterior distribution of μ, i.e. the conditional distribution of μ, given $\underline{X}$.

The posterior distribution is thus based on a combination of prior knowledge and data and the variance of this distribution, which measures the uncertainty in the parameter, will decrease with the amount of data that is provided, and finally approach zero when the number of observations tends to infinity. The difference from classical statistics is that prior knowledge of a parameter is included. The prior distribution should thus be considered an extra piece of information, which will reduce the uncertainty provided the prior belief is reasonable.

This theory is applied to the computed probability distribution of the life of jet engine components. The mean and variance in these distributions are considered random variables and the prior distributions are obtained by propagating the elemental uncertainties through the computational process by the methods presented in Chapter 6. The prior distribution is thus based on the estimated uncertainties in the input parameters and in the computational models, which are based on engineering judgement and knowledge about the physical process. Bayes theorem, (1), is then used to update the mean and variance of the predicted life distribution when data from in-service or

tests are received, and a distribution of life will then be obtained that is based on a combination of analytical prediction and data from reality.

In order to consider the information in the non-failed components, the likelihood function of X given the parameter can be used instead of the density function of X. The likelihood function is the probability density of the observations. In our application, the observations at hand are lives of failed components and times which are below the lives of survivors. The likelihood is then defined as:

$$L(t,s|\mu) = \prod_i f_{X|\mu}(t_i) * \prod_j \left[1 - F_{X|\mu}(s_j)\right]$$

where t_i are the observed failures and s_j are the observed survivors, and $f_{X|\mu}$ is the probability density function of X, given the parameter μ, and, $F_{X|\mu}$ is the corresponding cumulative distribution function.

An important concept in Bayesian statistics is conjugate families of distributions. This concept is developed for tractability in implementing the Bayesian theory. A prior distribution belongs to the conjugate family with respect to the likelihood of the observations, if the posterior distribution obtained belongs to the same family of distributions. In other words, if a smart choice of prior distribution is made, equation (1) will be very easy to evaluate. Since the prior distribution often is based on subjective belief, the tractability can be given high priority.

Consider for example a normally distributed life with a normally distributed mean with the parameters μ and τ, and specified value of the precision (inverted variance), r. Due to the concept of conjugate distributions, the posterior distribution will also be normal with the following parameters:

$\mu' = \frac{\tau\mu + nr\bar{x}}{\tau + nr}$ and $\tau' = \tau + nr$, where μ' and τ' are the mean and precision of the posterior

distribution, respectively, n is the number of observations, and $\bar{x}$ is the mean value of the observations.

It can also be shown that the conjugate distribution for the variance in a normal distribution is the inverted gamma distribution.

Observations in the context of jet engine components are failures, which, fortunately, are rare due to high safety demands. But cracks are often detected at inspections. Detected cracks often means "end of life" for the component and if crack propagation is allowed, failure observations can be generated by adding the time of crack propagation to the observed initiation time. Thus, real failures are not necessary, and using the Bayesian method is of interest in the jet engine industry.

9. Summary of appended papers

9.1 Literature survey

A survey of the methods available in the literature that are relevant for the research project. It is divided into a general and an application part. The general part contains some general statistical tools and methods that are appropriate to use in the life prediction application. The application part contains methods to model different kinds of variability in the life prediction process, computational methods to derive the response given stochastic inputs and a method to incorporate historical data.

9.2 Paper 1

A simplified method of uncertainty propagation is presented. The purpose is mainly to examine the relative importance of different uncertainty sources. The method is based on the Gauss approximation formula, which is based on a Taylor expansion of the response function. Since only two moments of life are considered (mean and variance), only the partial derivatives of the response function are needed. These derivatives are obtained approximately by analytical derivation. All kinds of variability are treated equally. The method is applied to life predictions of three different jet engine components, namely a stationary combustion chamber, a turbine blade and a flameholder. It is found that the final life prediction gives the largest uncertainty contribution among the different steps in the life prediction process.

9.3 Paper 2

The uncertainty in two different analysis approaches is compared. The uncertainty and scatter are separated and treated differently in the analysis, which is necessary in order to compare the uncertainty in the two approaches. The compared life prediction approaches are analytical life prediction and statistical analysis of in-service data. The method of uncertainty comparison is applied to the life prediction of a coverplate, which is a component in the high-pressure turbine in a jet engine. It is shown that statistical analysis of in-service data is more reliable than analytical prediction in this case.

9.4 Paper 3

The analysis approaches that were compared in the previous paper, are instead combined by the Bayesian updating method presented in Chapter 8. In-service data are used to reduce the uncertainty in the analytically obtained probability distribution of life. The updating is carried out both with and without survivors. The updating without survivors underestimated the mean life considerably, but the maximum time allowed in service could still be increased after this updating because of the uncertainty reduction. The updating with survivors enables an even larger increase of the allowed time in service. The coverplate is used as an application example also in this paper.

10. Discussion and further work

The aim of the first part of this work was to develop a modelling approach for scatter and uncertainty. The fact that separation of scatter and uncertainty is both a good principle and necessary in several cases is probably the most important finding in this project. Most of the work has consisted in applying established probability theory to structural problems and the number of possible applications are unlimited. The proposed modelling approach has only been applied to the fatigue analysis of a mechanically loaded component. An interesting extension is to apply the approach to a thermally loaded component and to include both the thermal and the structural analyses in the probabilistic analysis, using some of the presented probabilistic finite element methods. Regarding the probabilistic methods, a natural extension is to use the combination of approximate reliability methods and importance sampling.

However, the greatest potential improvement lies in the feedback methods of test and in-service data. Methods to update the predicted result were presented. It demanded derivation of a joint prior distribution of the mean and variance of life from the computed credibility bounds. It is understandable if this seems to be backwards, but it has been done for simplicity in this project. It would be more natural, however, and also more interesting to update the input parameters to the computational process. This is suggested as a topic for further work as well as the development of methods to evaluate the life prediction theories on the basis of the data from service or tests.

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