CHALMERS | GÖTEBORGS UNIVERSITET

Multi-Adaptive Galerkin Methods for ODEs

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ABSTRACT

In this thesis we formulate, analyze and implement multi-adaptive Galerkin methods for the numerical solution of initial value problems for systems of ordinary differential equations (ODEs) of the form

$$\begin{cases} \dot{u}(t) &= f(u(t),t), \; t \in (0,T], \\ u(0) &= u_0, \end{cases} \tag{1}$$

where $u : [0,T] \to \mathbb{R}^N$, $f : \mathbb{R}^N \times (0,T] \to \mathbb{R}^N$ is a bounded function that is Lipschitz-continuous in u, $u_0 \in \mathbb{R}^N$ is a given initial condition and $T > 0$ a given final time. We refer to these methods as mcG(q), the multi-adaptive version of the standard continuous Galerkin method for ODEs, and mdG(q), the multi-adaptive version of the standard discontinuous Galerkin method for ODEs.

The multi-adaptive features include in particular individual time-steps for the different components $u_i(t)$ of the solution $u(t)$, with the objective of efficient solution of problems with several time-scales.

In the first of two papers included in this thesis, we introduce the multi-adaptive methods, explore a couple of their basic properties, and prove a priori and a posteriori error estimates. We present adaptive algorithms for global error control and iterative solution methods for the discrete/algebraic equations. We also describe the multi-adaptive ODE-solver *Tanganyika*, implementing mcG(q) and mdG(q) for $q \leq 20$. In the second paper, we apply the multi-adaptive methods to a variety of problems, chosen to illustrate the potential of multi-adaptivity.

THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING

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ABSTRACT

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$$\left\{ \begin{array}{rcl} \dot{u}(t) & = & f(u(t), t), \ t \in (0, T], \\ u(0) & = & u_0, \end{array} \right. \tag{2}$$

where $u : [0, T] \rightarrow \mathbb{R}^N$, $f : \mathbb{R}^N \times (0, T] \rightarrow \mathbb{R}^N$ is a bounded function that is Lipschitz-continuous in u, $u_0 \in \mathbb{R}^N$ is a given initial condition and $T > 0$ a given final time. We refer to these methods as mcG(q), the multi-adaptive version of the standard continuous Galerkin method for ODEs, and mdG(q), the multi-adaptive version of the standard discontinuous Galerkin method for ODEs.

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PREFACE

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Göteborg, April 2001
Anders Logg

INCLUDED PAPERS

This thesis consists of the following two papers:

Paper I *Multi-Adaptive Galerkin Methods for ODEs I: Theory & Algorithms*, A. Logg, Chalmers Finite Element Center Preprint 2001–09. Submitted to SIAM J. Sci. Comput.

Paper II *Multi-Adaptive Galerkin Methods for ODEs II: Applications*, A. Logg, Chalmers Finite Element Center Preprint 2001–10. Submitted to SIAM J. Sci. Comput.

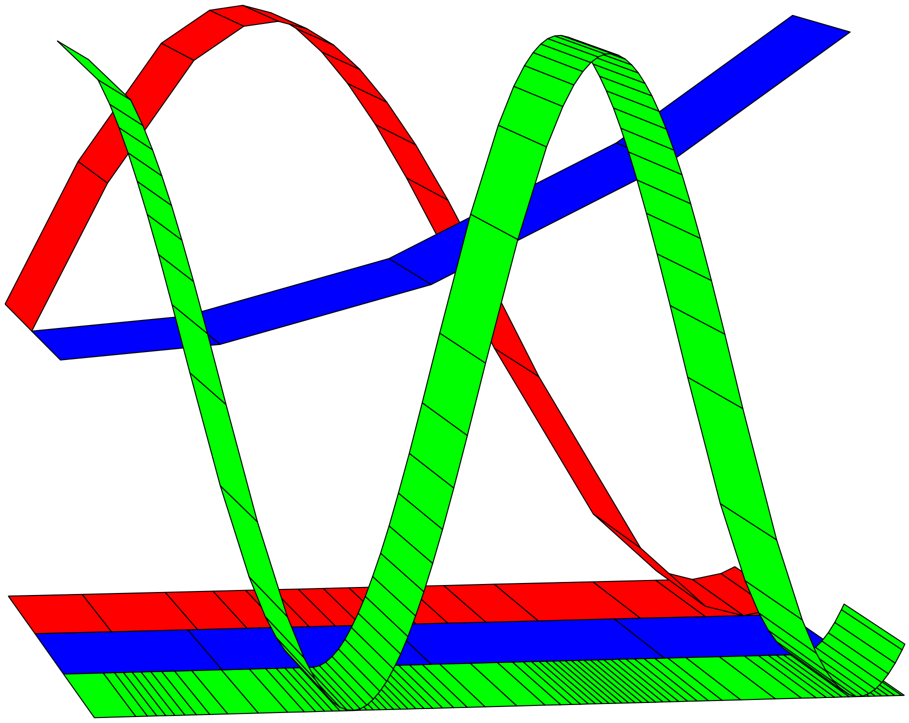
The following two papers are related, but are not included in the thesis:

(i) *A Multi-Adaptive ODE-Solver*, A. Logg, MSc Thesis 1998, Chalmers Finite Element Center Preprint 2000–02.

(ii) *Multi-Adaptive Error Control for ODEs*, A. Logg, Oxford University Computing Laboratory Research Report 1998/20, Chalmers Finite Element Center Preprint 2000–03.

Paper I

Multi-Adaptive Galerkin Methods for ODEs I: Theory & Algorithms



Paper II

Multi-Adaptive Galerkin Methods for ODEs II: Applications