A solution to the telephone gossip problem*

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Abstract

This paper deals with the "telephone problem", also known as the "gossip problem". It presents a self-contained elementary treatment of the problem, based on simple combinatorics.

Key words: gossiping, telephone, combinatorics.

I Introduction

The telephone gossip problem. Each of n people has his own secret. They communicate via consecutive telephone calls. At each call, the two participants pass to each other all the secrets they have learned up till that time. What is the minimum number of calls needed for everybody to learn all the secrets?

Denote by $f(n)$ the minimum number of calls required. It is easily seen that $f(3) = 3, f(4) = 4$, and after some experimenting a strong conviction arises that $f(n) = 2n - 4$. Surprisingly, the expected elementary proof does not come easily, as the analyses in, e.g., [2] - [5] show.

I learned about the gossip problem from a student of mine, who put it to me as "homework". He knew the answer from Internet and thought the proof must be elementary, and that his teacher might help him with this. This paper is in a way my response to his request. By means of what we shall call the "type description" method, based on simple combinatorics, the following will thus be proved:

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Theorem. The minimum number of talks required in the telephone gossip problem is

$$f(n) = 2n - 4, \quad \text{for} \ \ n \geq 4. \tag{1}$$

Some history. The telephone gossip problem was originally posed by A. Boyd [1] in 1971 in terms of ladies, knowing scandal items and passing them to each other by telephone calls. It seems to have enjoyed a certain popularity with mathematicians since in a relatively short time period it was independently solved in a number of papers, e.g. [2]-[5], by different methods. In some of these papers the authors convey to the reader their specific experiences in encountering the problem. For instance, Hajnal *et al.* [3] refer to the problem as the "telephone decease". Harary and Schwenk [5] write in their introduction that "the problem has so many features of an ailment" and also point out that it is equivalent, although in disguise, to a problem described in the psychological literature as the "common symbol problem".

Most of the earlier solutions to the telephone gossip problem use graph-theoretical techniques. The proof in [5] is based on an assumption called the "four cycle conjecture" introduced there with the following words: "We are so convinced of the next statement that even though it is by definition a conjecture, we shall call it a *True conjecture*" (italics theirs). Despite a prize of US $10 offered in [5] for the first proof or disproof of the True conjecture, it remained unproven for 7 years until Bumby [10] published a paper "devoted to the proof of the "four-cycle" conjecture". At the end of the present paper we will show that our proof yields the existence of the four-cycle in an optimal series of talks.

In the original gossip problem people communicate via telephone talks. One can also consider the analogues problem in other communication models, such as when people communicate via telegraph or via k-party telephone talks. Gossiping in different communication networks is studied in many papers, e.g., [6] - [19], of which [11] is a survey of known results. The main mathematical tools used in these are graph-theoretical. It turns out that the type-description method, that we use here, works as well for gossiping in other communication models, treated earlier by techniques from graph theory. We will give some examples of this at the end of the paper.

II Some notations and definitions. Idea of proof.

Let $A = \{a_1, \dots a_n\}$ be the set of all participants in the exercise. By $[a_i, a_j]$ we will denote a *telephone talk* of the *pair* (a_i, a_j). A *series of talks* will be denoted by

$$\omega = t_1 t_2 \dots t_{m(\omega)}$$

with $t_i = [a_{i_1}, a_{i_2}]$ for $1 \leq i \leq m(\omega)$, where $m(\omega)$ is the length of the series. Let Ω be the set of all series of talks such that at the end of the series for the first time all secrets are known to all participants. The problem is then to find

$$f(n) = \min_{\omega \in \Omega} m(\omega).$$

A series $\omega \in \Omega$ with $m(\omega) = f(n)$ will be called *optimal.*

Types of talks. Let ω be a series of talks. The number of secrets which a participant knows varies from 1 at the beginning of the series to at most n at the end of it. Let us call a participant who has learned all n secrets an *informed person.* If a_k knows all n secrets before the talk $t_i \in \omega$ we will say he is informed at t_i. The couple (a_k, a_l) will be said to be *self-complementary* (SC) at t_i if neither a_k nor a_l are informed at t_i but together a_k and a_l know all n secrets.

Let $t_i = [a_{i_1}, a_{i_2}]$. Depending on *how much a_{i_1} and a_{i_2} know before the talk*, t_i can be of *four different types.*

Type T_0: a_{i_1}, a_{i_2} are not informed at t_i and (a_{i_1}, a_{i_2}) is not SC at t_i. In this case t_i does not produce informed people.

Type T_1: exactly one of a_{i_1} or a_{i_2} is informed at t_i . Then t_i produces one new informed person.

Type T_2: the pair (a_{i_1}, a_{i_2}) is SC at t_i. Now t_i produces two new informed people.

Type T^-: both a_{i_1} and a_{i_2} are informed at t_i. The talk t_i then cannot produce new informed people.

Clearly, in any series of talks the first two talks are of type T_0. (Recall that $n \geq 4$). If $\omega \in \Omega$, then the very first talk that produces informed people, is a T_2-talk. Also, the last talk in the series is either a T_1 or a T_2-talk.

Generating talks. Let ω be a series of talks. If (a_k, a_l) is SC at $t_{g+1} \in \omega$ but not at $t_g \in \omega$ we will say that t_g generates the SC pair (a_k, a_l) and call t_g its *generating talk* (GT). Note that then either a_k or a_l necessarily participate in t_g. If the SC pair (a_k, a_l) "closes" in a T_2-talk $t_j = [a_k, a_l]$ of ω, where $j > g$, then we will say that t_g is the *generating talk* of t_j, too. Note that one T_0-talk in ω can generate at most two T_2-talks in ω.

Type description of series of talks. Let $\omega = t_1 t_2 \ldots t_{m(\omega)}$. The *type description* of ω is a series ω^T listing the types of the talks in ω in the same order.

Example. The series of talks

$$\begin{aligned}\omega_0 = &[a_1, a_n]\ldots[a_{n-4}, a_n][a_{n-3}, a_n][a_{n-2}, a_{n-1}][a_{n-1}, a_n][a_{n-3}, a_{n-2}]\\ &[a_1, a_n]\ldots[a_{n-4}, a_n]\end{aligned} \tag{2}$$

has type description

$$\omega_0^T = \underbrace{T_0 \ldots T_0}_{n-2} T_2 T_2 \underbrace{T_1 \ldots T_1}_{n-4}. \tag{3}$$

In words, $a_1, a_2, \ldots a_{n-3}$ talk to a_n, then a_{n-2} talks to a_{n-1}. This latter talk generates the self-complementary pairs (a_{n-1}, a_n) and (a_{n-3}, a_{n-2}) and also the two following T_2-talks, in which a_{n-1} talks to a_n and a_{n-3} talks to a_{n-2}. The now informed a_n then talks to $a_1, \ldots a_{n-4}$. In this way all participants become informed at the end of the series. Hence $m(\omega_0) \in \Omega$, and since $m(\omega_0) = 2n - 4$, ω_0 should then be optimal, by the Theorem.

The series of talks

$$\begin{aligned}\omega_* = &[a_1, a_n]\ldots[a_{n-3}, a_n][a_{n-2}, a_n][a_{n-1}, a_n]\\ &[a_1, a_n]\ldots[a_{n-3}, a_n]\end{aligned} \tag{4}$$

is also in Ω and has type description

$$\omega_*^T = \underbrace{T_0 \ldots T_0}_{n-2} T_2 \underbrace{T_1 \ldots T_1}_{n-2}. \tag{5}$$

This time $a_1, a_2, \ldots a_{n-3}$ talk to a_n, as they did in ω_0 but now a_{n-2} talks to a_n, not to a_{n-1}, as was the case in ω_0. The talk $[a_{n-2}, a_n]$ generates the self-complementary pair (a_{n-1}, a_n), in which a_{n-1} knows only his own secret (since he has not talked before) and a_n knows the rest. This SC pair then performs the first T_2-talk of the series. Note that after this talk the secret of a_{n-1} is only known to him and to a_n, and both of them are informed. Hence the rest of the people may become informed only in T_1-talks with a_n or a_{n-1} or with someone who has already talked to one of them, as is done in ω_*, where $a_1, \ldots a_{n-2}$ talk to a_n. Comparing (3) and (5) we see that ω_* is not optimal.

Restriction of Ω. It is easily seen that the sum of indices of all T_2 and T_1 elements in the type description ω^T of a series of talks ω represents the number of informed persons at the end of the series. Thus if this sum equals n, the series belongs to Ω, as is the case in (3) and (5). We will use this to restrict the set of series under consideration. A series in Ω which contains talks of type T^- is not optimal, since by removing these talks we get a shorter series in Ω. We may therefore assume below that the series in Ω do not contain T^--talks.

Let now $\omega \in \Omega$ and let two consecutive talks in ω have type description T_1T_0 or T_1T_2. One can readily see the two talks cannot have any participant in common.

Thus if we reverse the talks, they will preserve their types and the types of the remaining talks in ω are unchanged. This means that the new series also belongs to Ω. Continuing in this way, in a finite number of steps we will come to a series in Ω with the same length as ω and where all T_1-talks are at the end. Therefore we may assume below that the T_1-talks of any series in Ω take place at the end of the series.

We will thus assume from now on that each $\omega \in \Omega$ has type description of the form

$$\omega^T = T_0 \dots T_0 \underbrace{T_2 \dots \quad \dots}_{\substack{s \quad T_2\text{-talks} \\ \text{and some} \\ T_0\text{-talks}}} \underbrace{T_1 \dots T_1,}_{\substack{n\text{-}2s \\ T_1\text{-} \\ \text{talks}}} \tag{6}$$

where $s \geq 1$ and $2s \leq n$. Denoting by $m_0(\omega)$ the number of T_0-talks in ω, we have

$$m(\omega) = m_0(\omega) + s + (n - 2s) = m_0(\omega) + n - s. \tag{7}$$

Idea of proof. We will show that in any $\omega \in \Omega$ the minimum number of T_0-talks needed to generate one or two T_2-talks is $n-2$. This together with (7) will imply that the length of a series with $s = 1$ is at least $n-2+n-1 = 2n-3$, and as we know from the Example such a series cannot be optimal. Thus $s \geq 2$ in the type description of an optimal series. A series in Ω with more than two T_2-talks in it will have to contain additional T_0-talks to the $n-2$ needed to generate the first two T_2-talks. We will show that such a series cannot be shorter than a series without additional T_0-talks, where after the first two T_2-talks the rest of people get informed in T_1-talks. Such a series has type description (3), and hence the series ω_0 of (2) will turn out to be an example of an optimal series.

III Proof of the Theorem

We now introduce self-complementary k-tuples, generalising the notion of self-complementary pairs.

Let $\omega = t_1 t_2 \dots t_m(\omega)$ be a series of talks. The k-tuple $(a_{\ell_1}, \dots, a_{\ell_k})$, $2 \leq k \leq n$, will be called *self-complementary* (SC) at $t_i \in \omega$, if neither of $a_{\ell_1}, \dots a_{\ell_k}$ is informed at t_i, no two of them know exactly the same secrets and together $a_{\ell_1}, \dots a_{\ell_k}$ know all n secrets. For instance, the n-tuple $(a_1, \dots a_n)$ is SC at t_1 and if $t_1 = [a_1, a_2]$, then the $(n-1)$-tuples $(a_1, a_3, \dots a_n)$ and $(a_2, \dots a_n)$ are SC at t_2. A k-tuple $(a_{\ell_1}, \dots, a_{\ell_k})$ will be called SC at the end of ω if neither of $a_{\ell_1}, \dots a_{\ell_k}$ is informed after $t_{m(\omega)}$, no two of them know exactly the same secrets and together $a_{\ell_1}, \dots a_{\ell_k}$ know all n secrets.

If $(a_{\ell_1},\dots a_{\ell_k})$ is SC at t_{g+1} but not at t_g we will say that t_g is the *generating talk* (GT) of the SC k-tuple $(a_{\ell_1},\dots a_{\ell_k})$. If so, exactly one of $a_{\ell_1},\dots,a_{\ell_k}$ participates in t_g.

We first prove the following:

Lemma 1. *Let $\alpha_n(k)$ be the minimum number of talks needed to get a SC k-tuple from n participants. Then $\alpha_n(k) = n-k$.*

Proof. Let $\omega = t_1 t_2 \dots t_{\alpha_n(k)}$ be optimal in the sense that the k-tuple $(a_1, a_2, \dots a_k)$ is SC at the end of ω and no other k-tuple is SC at some talk in ω.

Assume first that one person, a_1 say, has not talked in ω. Since $(a_1, a_2, \dots a_k)$ is SC at the end of ω but not at $t_{\alpha_n(k)}$, this talk generates the SC k-tuple. This means that one member of the k-tuple, a_2, say, talks in $t_{\alpha_n(k)}$ with someone outside the SC k-tuple, a_{k+1}, say. At $t_{\alpha_n(k)}$ the people in the k-tuple $(a_2, \dots a_{k+1})$ thus know all secrets but the secret of a_1. Then $(a_2, \dots a_{k+1})$ is SC at $t_{\alpha_{n(k)}}$ with respect to the $n-1$ secrets of $a_2, \dots a_n$, and is also the first such k-tuple, since ω is optimal. Therefore $\alpha_{n-1}(k) = \alpha_n(k) - 1$. Continuing in this way we arrive at $\alpha_n(k) = \alpha_k(k) + n - k$. Since $\alpha_k(k) = 0$, the statement follows.

If everybody in $(a_1, a_2, \dots a_k)$ has talked in ω, replace a_1 by a_2 everywhere in ω. At the end of the new series, a_1 will only know his own secret and a_2 will know the secrets which he and a_1 knew together at the end of the old series, exept the secret of a_1. The rest of the people will know the secrets which they knew before, except that of a_1, which is now replaced by the secret of a_2. In this way, $(a_1, a_2, \dots a_k)$ is SC at the end of the new series which has the same length as ω. Since a SC k-tuple cannot be reached in fewer talks, the new series is also optimal in the above sense, and because a_1 has not talked in it, by above $\alpha_n(k) = n-k$. We remark that a_1 and a_2 cannot have talked in ω because then, after the replacement a_2 would have talked to himself, and such a talk could be cancelled without affecting the knowledge of secrets. After this cancellation the new series would be shorter, in contradiction to the optimality of the old series. ☐

Corollary 1. Let ω be as in the proof of Lemma 1 and $2k \le n$. There can be at most two k-tuples which are disjoint and SC at the end of ω.

Proof. If everyone in $(a_1, a_2, \dots a_k)$ has talked at least once in ω (this is possible, since $k \le n-k$), then the people to whom they have last talked form a k-tuple, which is disjoint from $(a_1, a_2, \dots a_k)$ and is SC at the end of ω. Assume now that there exists a third k-tuple which is disjoint from the first two and is SC at the end of ω. Since the people talking in $t_{\alpha_n(k)}$ are not from the third k-tuple, the latter would then have been SC already at $t_{\alpha_n(k)}$. By Lemma 1 this would imply that $\alpha_n(k) - 1 \ge n-k$, which is a contradiction to Lemma 1. ☐

Corollary 2. If $\omega \in \Omega$ is optimal, then it contains at least two T_2-talks.

Proof. Let $\omega \in \Omega$ and $s = 1$ in (6). Lemma 1 implies that $m_0(\omega) \geq n-2$ in (6) and then (7) gives that $m(\omega) \geq (n-2)+n-1 = 2n-3 = m(\omega_*)$, where ω_* is the series from from (4). Since ω_* is not optimal, neither is ω. □

Corollary 3. If $\omega \in \Omega$ is optimal, then both persons talking in the first T_2-talk of the series must have talked at least once before this talk.

Proof. If only one person from the first T_2-talk in ω has talked before, then necessarily $s = 1$ in the type description (6) of ω, as explained in the case of ω_* of the Example. This is a contradiction to Corollary 2. □

Let $\omega \in \Omega$ and t_i be the first T_2-talk in it. We may assume that $t_i = [a_{n-1}, a_n]$, otherwise we change notations on the participants from the very beginning. Let t_{g_1} be the GT of the SC pair (a_{n-1}, a_n). In t_{g_1} one of a_{n-1} and a_n talks with some other person. Again, by a proper change of notations, we may assume that this person is a_{n-2} and that $t_{g_1} = [a_{n-2}, a_{n-1}]$. The triple (a_{n-2}, a_{n-1}, a_n) is SC at t_{g_1}. Let t_{g_2} be the GT of this SC triple. In t_{g_2} one of a_{n-2}, a_{n-1}, and a_n talks with someone else, say a_{n-3}. The quadruple $(a_{n-3}, a_{n-2}, a_{n-1}, a_n)$ is SC at t_{g_2}. If $n = 4$, t_{g_2} is the first talk in the series. If $n \geq 5$ we look for the GT t_{g_3} of the SC quadruple $(a_{n-3}, a_{n-2}, a_{n-1}, a_n)$, where a_{n-4} talks to some of $a_{n-3}, a_{n-2}, a_{n-1}$ and a_n, and continue in this way until we come to a talk $t_{g_{n-2}} = [a_1, \text{one of } a_2, \ldots a_n]$ generating the SC $(n-1)$-tuple $(a_2, \ldots a_n)$. Since $(a_1, \ldots a_n)$ is SC at $t_{g_{n-2}}$, $t_{g_{n-2}}$ is then the first talk in ω.

In this way we see that ω contains a subsequence of $n-2$ generating talks leading to the first T_2-talk of the series:

$$\omega = t_{g_{n-2}} \underbrace{\ldots}_{other\ talks\ of\ \omega} t_{g_{n-3}} \ldots t_{g_2} \underbrace{\ldots}_{other\ talks\ of\ \omega} t_{g_1} \underbrace{\ldots}_{other\ talks\ of\ \omega} t_i \ldots t_{m(\omega)}. \tag{8}$$

We will study the subsequence ω_{sub} defined by these generating talks.

$$\omega_{sub} = t_{g_{n-2}} t_{g_{n-3}} \ldots t_{g_2} t_{g_1}. \tag{9}$$

Lemma 2. *Let $\omega \in \Omega$ and consider the corresponding ω_{sub} from (9). Then the following is true:*

a) In addition to (a_{n-1}, a_n), there can be at most one pair which is disjoint from (a_{n-1}, a_n) and is SC at the end of ω. Such a pair exists when a_n has participated in some talk of ω_{sub} and consists of a_{n-2} and the person to whom a_n has talked last.

b) In addition to (a_{n-2}, a_{n-1}, a_n), there can be at most one triple which is disjoint from (a_{n-2}, a_{n-1}, a_n) and is SC at t_{g_1}. Such a triple exists if each of a_{n-2}, a_{n-1} and a_n has participated in some talk of ω_{sub} preceding t_{g_1}, and consists of the persons to whom a_{n-2}, a_{n-1} and a_n have talked last.

c) In addition to $(a_{n-3}, a_{n-2}, a_{n-1}, a_n)$, there can be at most one quadruple which is disjoint from $(a_{n-3}, a_{n-2}, a_{n-1}, a_n)$ and is SC at t_{g_2}.

d) There are no SC triples nor quadruples at $t_{g_{n-2}}, \dots, t_{g_3}$.

Proof. The statements *a*), *b*) and *c*) follow from Corollary 1. By Lemma 1, at least $n-3$ talks are needed to obtain a SC triple, and at least $n-4$ talks are needed to obtain a SC quadruple. Since t_{g_3} is preceded by $n-5$ talks, no triples nor quadruples are SC at t_{g_3} or earlier. This proofs *d*). □

The number of T_0-talks in any $\omega \in \Omega$ is thus at least $n-2$, the number of the talks in the corresponding ω_{sub}. Let $m_1(\omega) = m_0(\omega) - (n-2)$ be the number of possible additional T_0-talks in ω. Then from (7)

$$m(\omega) = n - 2 + m_1(\omega) + n - s = (2n-2) - (s - m_1(\omega)). \tag{10}$$

Lemma 3. *Let $\omega \in \Omega$ be optimal, with type description ω^T as in (6). Then $s = 2$ if $4 \le n \le 7$ and either $s = 2$ or $s = 4$ if $n \ge 8$.*

Proof. Again, without loss of generality we may assume that $t_i = [a_{n-1}, a_n]$ is the first T_2-talk in ω, and that the corresponding ω_{sub} is the one in (9). Since ω is optimal, the value of $s - m_1(\omega)$ in (10) is the largest possible. By Corollary 2 we know that $s \ge 2$.

Assume first that $s = 2$ in ω. By *a*) of Lemma 2, in addition to the SC pair (a_{n-1}, a_n), a second pair which is SC at the end of ω_{sub} can exist without additional T_0-talks if a_n talks in ω_{sub}. If a_j is the last person to whom a_n has talked in ω_{sub}, then the pair (a_{n-2}, a_j) is SC at the end of ω_{sub} and may result in another T_2-talk after t_i. In this case, we have $m_1(\omega) = 0$ and then $s - m_1(\omega) = 2$.

Assume now that $s = 3$. One T_2-talk in addition to $t_i = [a_{n-1}, a_n]$ may exist without further T_0-talks as explained above. In it a_{n-2} talks to a_j, the last person to whom a_n has talked to in ω_{sub}. Where may the SC pair talking in the third T_2-talk come from? It does not contain any of a_j, a_{n-2}, a_{n-1}, and a_n and then by *a*) of Lemma 2, this pair is not SC at the end of ω_{sub}. Neither is the SC pair obtained from a triple which is SC at some talk of ω_{sub}, because such triples do not exist according to *b*) and *d*) of Lemma 2. Then by Lemma 1, the additional T_0-talks needed to get the third SC pair are at least two. This implies that $s - m_1(\omega) \le 3 - 2 = 1$ and then $m(\omega) \ge 2n - 3$. Such an ω cannot be optimal and hence the assumption $s = 3$ is false. Since $s \le 3$ when $n \le 7$ (because $2s \le n$) we must then have $s = 2$ in an optimal series with $n \le 7$ participants.

Assume now that $n \ge 8$ and that $s \ge 4$ in ω^T. Again, another pair than (a_{n-1}, a_n), which is SC at the end of ω_{sub} can be obtained without using additional T_0-talks, and by *c*) of Lemma 2, two further SC pairs can result from a quadruple which is SC at t_{g_2} in two additional T_0-talks. In this case, by *c*) and *d*) of Lemma 2, there will be no quadruple left which is SC at some talk in ω_{sub}. Then further

SC pairs may only be obtained by using k-tuples with $k \geq 5$ which are SC at some talk in ω_{sub}. By Lemma 1 at most two pairs which are SC at the end of ω_{sub} or later can be obtained from one such k-tuple, and this will require at least $k-2 \geq 3$ more T_0-talks. Thus having formed four SC pairs, no further SC pair can be obtained in only one additional T_0-talk. Summarising we see that two SC pairs are obtained without additional T_0-talks to those in ω_{sub}, another couple of SC pairs may be obtained in two additional T_0-talks and that no further SC pair can be obtained in only one further T_0-talk. This gives that

$$s - m_1(\omega) = (2-0) + (2-2) - \underbrace{((m_1(\omega)-2) - (s-4))}_{\geq 0} \leq 2,$$

where, as explained above, the expression $m_1(\omega) - 2 - (s-4)$ equals 0 when $s=2$ (and $m_1(\omega)=0$) or $s=4$ (and $m_1(\omega)=2$) and is positive for $s \geq 5$. Therefore equality can hold in the above inequality only if $s=2$ or $s=4$. □

It follows from Lemma 3 that a series in Ω with type description

$$\omega^T = \underbrace{T_0 \ldots T_0}_{(n-2)\ \ talks} T_2 T_2 \underbrace{T_1 \ldots T_1}_{(n-4)\ \ talks}$$

is optimal. An example of such a series is provided by ω_0 of (2). The proof of the Theorem is now complete. □

As an illustration in the case $n \geq 8$, in addition to the optimal ω_0 of (2), another optimal series for $n=8$ is

$$\begin{aligned} \omega = \ & [a_1,a_5][a_2,a_6][a_3,a_7][a_4,a_8] \\ & [a_1,a_3][a_2,a_4][a_1,a_2][a_3,a_4]\ \ [a_5,a_7][a_6,a_8][a_5,a_6][a_7,a_8] \end{aligned}$$

with type description

$$\omega^T = T_0T_0T_0T_0\ \ T_0T_0T_2T_2\ \ T_0T_0T_2T_2.$$

In words, in $8-4=4$ T_0-talks we get the SC quadruples (a_1,a_2,a_3,a_4) and (a_5,a_6,a_7,a_8), in each of which people then exchange their secrets in the optimal way, that is, in two T_0 and two T_2-talks. This optimal way of exchange between 4 people is what is referred to as the *four-cycle* in graph-theoretical treatments of the Gossip problem. As was mentioned in the introduction, the existence of a four-cycle in an optimal series of talks was introduced in [4] as a True conjecture. Here the existence of a four-cycle in an optimal series is a consequence of the proof of Lemma 3. Note that according to Lemma 3, an optimal series can contain one or two four-cycles when $n \geq 8$.

Assume now that the communication between any two people is one-way only, such as when they use, e.g., the telegraph. Each talk now produces zero or one new informed people. Let us say that such talks are of types T_0 and T_1, respectively. The first two talks in any series ω are T_0-talks and the first T_1-talk in the series is between two people who constitute a SC pair. The number of the preceding T_0-talks is then at least $n-2$, in accordance with Lemma 1. By letting the person who has become informed in the first T_1-talk of ω talk to everyone else, we find an optimal gossip series which thus has length $(n-2)+n = 2n-2$, the result obtained in [5].

Assume now that people communicate via k-party telephone calls. Introducing types T_j for talks that produce j new informed people, $j = 0, 1 \ldots k$ we can use the same procedure as the one described above for $k = 2$, to arrive at a type description of an optimal series ω. If, for example, $n \geq k^2$, one can show that a series ω with type description

$$\omega^T = \underbrace{T_0 \ldots T_0}_{\lceil \frac{n-k}{k-1} \rceil \ talks} \underbrace{T_k \ldots T_k}_{k \ talks} \underbrace{T_{k-1} \ldots T_{k-1} T_i}_{\lceil \frac{n-k^2}{k-1} \rceil \ talks}$$

is optimal. Here i equals the rest in the division of $n-k^2$ by $k-1$, when this rest is positive, and $k-1$ otherwise. The length of such a series is $\lceil \frac{n-k}{k-1} \rceil + k + \lceil \frac{n-k^2}{k-1} \rceil = 2\lceil \frac{n-k}{k-1} \rceil$, the result obtained in [9].

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