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FATIGUE LIFE PREDICTION FOR A VESSEL SAILING THE NORTH ATLANTIC ROUTE

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Abstract

A method for calculating the wave load induced fatigue damage accumulated by a vessel sailing along the North Atlantic route (NAr) is presented. This method is based on the Palmgren-Miner additive rule and the rainflow cycle (RFC) count. For simplicity, the load the vessel experiences is assumed to be proportional to the encountered significant wave height process, H_s. The asymptotically normal character of the nominal damage is proved and used to derive the probability distribution of the fatigue life prediction. The proposed method improves the already existing ones by making use of the information contained in the variance of the fatigue damage accumulated during the voyages. The method is illustrated through numerical examples.

Key words: Fatigue damage, Gaussian random fields, locally stationary.

1 Introduction

Fatigue design criteria has during the last couple of decades received increasing attention. All major classification societies have introduced fatigue as a specific design criteria. Despite the large amount of attention the fatigue design criteria have received the last years, fatigue life predictions for structural components subjected to random, varying loads continue to be a challenging problem.

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In the present study we consider problems related to fatigue of metals (development and growth of cracks) on vessels due to wave loads. Fatigue is one of the most frequent causes of failure for metallic structures subjected to environmental loads. In our approach, we assume that the damage is defined as the integral

$$D(t) = K \int_0^t H_s(\tau)^\alpha \, d\tau,$$

where $H_s(t)$ is the significant wave height the vessel or structure experiences at time t and obviously depends on the geographic position of the vessel, while K is a material dependent, log-normally distributed, quality factor. Failure is attained when $D(t)$ crosses for the first time a cricital level d^{crt}. This somewhat simplified model is often assumed in the fatigue analysis of vessels since it captures the most important features of the fatigue accumulation process while it is simple enough to preserve transparency of presentation. It can be easily generalized to a more realistic engineering setup under which stresses at hotspots are considered.

For safety analysis purposes, the distribution of H_s is usually taken from wave atlasses. The information contained in these atlasses, although sufficient for evaluation of the average fatigue damage, $E[D(t)]$, is insufficient for evaluation of the corresponding variance, $V[D(t)]$. High space correlation between the different H_s values, results in large variance for the damage that cannot be neglected when safety considerations are of interest. To compute the variance, a complete probabilistic model for the space and time variability of $H_s(\mathbf{s}, t)$ is needed.

Use of satellite and buoy data allow us to obtain a complete probabilistic model for $\log(H_s(\mathbf{s}, t))$ for different small regions along the North Atlantic route (NAr), although further studies, part of future research plans, are needed for verification. Despite its quite simple character, the model is sufficient for studying the correlation structure of the accumulated damage.

2 Review of fatigue damage

2.1 Introduction

Fatigue is the process of development and growth of cracks on the material due to a variable load. In general, the fatigue life for a structural component subjected to a random load is calculated based on experimental data of speciments subjected to constant amplitude load. Therefore it is necessary to define amplitudes of equivalent load cycles A_i, which are functions of the sequence of maxima and minima in the load and assume a method to measure the damage caused by each simple cycle. The commonly used in engineering Palmgren-Miner linear damage accumulation rule, postulates that the total damage due to a random process $X(t)$, is the sum of damages caused by individual load cycles

$$D(t) := \sum_{t_i \le t} \frac{1}{N_{A_i}}, \tag{1}$$

where the sum is extended over all cycles completed by time t and N_A is the cycle life obtained from laboratory experiments with constant amplitude A. It is usual in applications to use the Basquin relation

$$N_A = K^{-1} A^{-\beta},$$

where K and $\beta \ge 1$ are material dependent constants. In practice, β takes values in the interval $[3, 5]$, and the stochastic quality variable K is assumed to be log-normally distributed, i.e. $\log(K) \in N(m_K, \sigma_K^2), \quad m_K < 0$, and independent of the load $X(t)$. Consequently, eq. 1 simplifies to

$$D(t) := K \sum_{t_i \le t} A_i^{\beta} := K D_X(t). \tag{2}$$

We refer to $D_X(t) := \sum_{t_i \le t} A_i^{\beta}$ as *nominal damage*.

We turn now to the problem of defining amplitudes of cycles. There have been at least eight different counting methods proposed in the literature. Dowling 1972, has found that only the rainflow (RFC) counting method leads to prediction that agrees with actual lives. Hence, the linear deterministic rule defined in eq. 1 together with the rainflow counting method seem to provide with a quite accurate approximation and will be adopted in this study. Many different algorithms for counting RFC-cycles have been introduced in the literature. Here we shall give a definition, introduced in Rychlik 1987, that is more convenient for statistical analysis of RFC-cycles.

Definition 1 *Let $X(\tau), \quad 0 \le \tau \le t$ be a load process and let $\{\tau_i\}, 0 \le \tau_i \le t$ denote the times of the local maxima of $X(\tau)$. The RFC count attaches to each local maximum a cycle as follows: for the i^{th} local maximum with height M_i at time τ_i, let τ_i^+ be the time* *for the first upcrossing after τ_i of the level M_i (or $\tau_i^+ = t$ if no such upcrossing exists), and let τ_i^- be the time for the last downcrossing of M_i before τ_i (or $\tau_i^- = 0$ if no such downcrossing exists). Then determine which of the two, on the right and on the left, paths has the smaller in absolute value global minimum. The smaller in absolute value global minimum attained at time τ_i^{rfc} and denoted by m_i^{rfc}, is the rainflow minimum paired with the i^{th} maximum to give a RFC-cycle with amplitude $A_i = M_i - m_i^{rfc}$, see Rychlik 1987 for a detailed discussion.*

The rainflow method, designed to catch both slow and rapid variations of the load by forming cycles in which high maxima were paired with low minima even if they were separated by intermediate extremes, was first introduced by Endo, the first paper in english being Matsuishi and Endo 1968.

2.2 Expected nominal damage

Having an explicit form for the fatigue damage is preferable, but the rainflow cycle amplitude A is a quite complicated function of the load making it difficult to obtain the exact probability distribution of the damage. Hence, instead we use an upper bound for the expected damage. In the special case of a stationary Gaussian load, this upper bound coincides with the narrow band approximation, see Rychlik 1993. We begin with an introductory definition given in Rychlik 1990.

Definition 2 *Let $X(\tau)$, $\quad 0 \le \tau \le t$ be a load process. For fixed values $u, v, u \ge v$, let $N(u, v; t)$ be the number of RFC-cycles $(X(\tau_i), X(\tau_i^{rfc}))$ with the maximum of the cycle being higher than u and the attached rainflow minimum lower than v. Further let $\mu(u, v; t)$ denote the expected value of $N(u, v; t)$, i.e.*

$$\mu(u, v; t) = E[N(u, v; t)].$$

Assume that $N(u, v; t)$ is a bounded function of u and that at time zero we have zero cycles. Then, as it can be shown after elementary but lengthy computations, the nominal damage is given by

$$D_X(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{u} \beta(\beta - 1)(u - v)^{\beta - 2} N(u, v; t)\, dv\, du. \tag{3}$$

Eq. 3 is a direct consequence of eq. 19 in Rychlik 1993. Taking expectation in eq. 3 and using Fubini's theorem we obtain the first moment of the nominal damage

$$E[D_X(t)] = \int_{-\infty}^{\infty}\int_{-\infty}^{u} \beta(\beta-1)(u-v)^{\beta-2}\mu(u,v;t)\,dv\,du. \tag{4}$$

Note that the counting distribution of $N(u,v;t)$, the nominal damage and the expected nominal damage may be defined for any counting method.

2.3 Upper bound for the damage intensity

In order to compute the integrals in eq. 4, we need to know explicitly the function $\mu(u,v;t)$. By the definition of $\mu(u,v;t)$; $\mu(u,v;t)$ is a decreasing function for u and increasing for v. Consequently, for $u \geq v$ the function μ can be bounded from above as follows

$$\mu(u,v;t) \leq min\{\mu(u,u;t), \mu(v,v;t)\}.$$

After some lengthy derivations we can show that

$$\frac{\partial\mu(u,v;t)}{\partial t} \leq min\{\mu_t^+(u), \mu_t^+(v)\}$$

where $\mu_t^+(u) = \frac{\partial\mu(u,u;t)}{\partial t}$. It is easy to see that $N(u,u;t)$ is equal to the number of upcrossings of the level u, hence $\mu_t^+(u)$ is the upcrossing intensity of the level u at time t which can be obtained by means of the Rice formula,

$$\mu_t^+(u) = \int_0^\infty z f_{X(t),\dot{X}(t)}(u,z)\,dz. \tag{5}$$

Consequently, the damage intensity $d_X(t) = \frac{d(E[D_X(t)])}{dt}$ can be bounded from above

$$d_X(t) \leq \int_{-\infty}^{\infty}\int_{-\infty}^{u} \beta(\beta-1)(u-v)^{\beta-2} min\{\mu_t^+(u), \mu_t^+(v)\}\,dv\,du, \tag{6}$$

with $\mu_t^+(u)$ as defined in eq. 5.

In the special case of a zero-mean, stationary, Gaussian load $X(t)$, with variance $\sigma_X^2 = V(X(0))$ and variance of the derivative $\sigma_{\dot{X}}^2 = V(\dot{X}(0))$, eq. 5 simplifies to

$$\mu^+(u) = \int_0^\infty z f_{X(0),\dot{X}(0)}(u,z)\,dz = \frac{1}{2\pi}\frac{\sigma_{\dot{X}}}{\sigma_X} e^{-\frac{u^2}{2\sigma_X^2}}.$$

Therefore the damage intensity, that is now independent of time, is bounded by

$$\begin{aligned} d_X &\leq \int_{-\infty}^{\infty}\int_{-\infty}^{u} \beta(\beta-1)(u-v)^{\beta-2} min\{\mu^+(u),\mu^+(v)\}\, dv\, du \\ &\leq \frac{1}{2\pi}\beta(\beta-1)\frac{\sigma_{\dot{X}}}{\sigma_X}2\int_0^{\infty}\int_{-u}^{u}(u-v)^{\beta-2}e^{-\frac{u^2}{2\sigma_X^2}}\, dv\, du \\ &= \frac{1}{2\pi}\beta\frac{\sigma_{\dot{X}}}{\sigma_X}2\int_0^{\infty}(2u)^{\beta-1}e^{-\frac{u^2}{2\sigma_X^2}}\, du \\ &= \frac{1}{2\pi}\frac{\sigma_{\dot{X}}}{\sigma_X}2^{\beta}\int_0^{\infty}\beta u^{\beta-1}e^{-\frac{u^2}{2\sigma_X^2}}\, du \\ &= 2^{\frac{3\beta}{2}}\Gamma\left(\frac{\beta}{2}+1\right)\frac{\sigma_{\dot{X}}\sigma_X^{\beta-1}}{2\pi}. \end{aligned}$$

In the derivations we have used the symmetry condition $\mu^+(u) = \mu^+(-u)$.

In the case of a non-stationary Gaussian load, at each time point t the variables $X(t)$ and $\dot{X}(t)$ are dependent and hence evaluation of the integral in eq. 5 and consequently in eq. 6, requires knowledge of the covariance structure of $X(t)$ and $\dot{X}(t)$ at each time point t. Assuming the load is locally stationary, the covariance between $X(t)$ and $\dot{X}(t)$ is a lot smaller than the standard deviations $\sigma_X(t)$ and $\sigma_{\dot{X}}(t)$ and hence can be set to equal zero. Under these assumptions, an upper bound for the damage intensity $d_X(t)$ in eq. 6 is given by

$$d_X(t) \leq 2^{\frac{3\beta}{2}}\Gamma\left(\frac{\beta}{2}+1\right)\frac{\sigma_{\dot{X}}(t)\sigma_X^{\beta-1}(t)}{2\pi}. \tag{7}$$

Note that eq. 7 depends on time and is valid for t inside each period of stationarity, that is $\sigma_X(t)$ and $\sigma_{\dot{X}}(t)$ are different for different stationarity periods. Using eq. 7, we may obtain the following upper bound for the expected nominal damage, denoted by $\tilde{E}[D_X(T)]$, accumulated during the time period $[0,t]$,

$$\tilde{E}[D_X(t)] = 2^{\frac{3\beta}{2}}\Gamma\left(\frac{\beta}{2}+1\right)\int_0^t \frac{\sigma_{\dot{X}}(\tau)\sigma_X^{\beta-1}(\tau)}{2\pi}\, d\tau. \tag{8}$$

Suppose that $\sigma_X(t)$ and $\sigma_{\dot{X}}(t)$ are known for the whole period $[0,t]$. Then, for t large enough the difference

$$D_X(t) - \tilde{E}[D_X(t)]$$

is often assumed to be negligible relatively to the uncertainty in the material properties as represented by the variance σ_K^2. Furthermore, since the constant $2^{\frac{3\beta}{2}}\Gamma\left(\frac{\beta}{2}+1\right)$ is material dependent, it can be included in the parameter m_K. Concluding, for a zero-mean, locally stationary, Gaussian load, the damage accumulated by the material during the time period $[0, t]$ with t large enough, can be approximated using the following conservative bound

$$D(t) \approx K \int_0^t \frac{\sigma_{\dot{X}}(\tau)\sigma_X^{\beta-1}(\tau)}{2\pi} \, d\tau. \tag{9}$$

However, although future values of σ_X and $\sigma_{\dot{X}}$ are known, it is common to model both standard deviations by means of random processes.

2.4 Fatigue life distribution

The failure time T^f is defined as the first time the damage $D(t)$ crosses a critical level d^{crt}. Usually $d^{crt} = 1$ although sometimes it is modelled as a random variable independent of the process $X(t)$, depending on the quality of the material. Obviously

$$P(T^f \leq t) = P(D(t) \geq d^{crt}),$$

which makes the distribution of $D(t)$ of primary interest in reliability analysis.

Assume now the nominal damage is approximately normally distributed, i.e. $D_X(t) \in N(m(t), s^2(t))$, then the distribution of the failure time T^f can be computed as follows

$$\begin{aligned} P[T^f \leq t] &= \int_{-\infty}^{\infty} P\left[\frac{1}{K} \leq \frac{m(t)+s(t)z}{d^{crt}}\right] \phi(z)dz = \\ &= \int_{-\infty}^{\infty} \Phi\left(\frac{m_K + \log(m(t)) + \log(1+\frac{s(t)}{m(t)}z) - \log(d^{crt})}{\sigma_K}\right) \phi(z)dz. \end{aligned} \tag{10}$$

Remember that for a log-normal variable K, the following relation is valid: $\log(K) \in N(m_K, \sigma_K^2) \Rightarrow \log(\frac{1}{K}) \in N(-m_K, \sigma_K^2)$. Also, since $D_X(t)$ was assumed to be approximately normally distributed, $D_X(t) \approx m(t)+s(t)Z$, where Z is a standard normal random variable.

3 Fatigue damage accumulated by a vessel

Let $X(t)$ be a random load, for example the stress a vessel may experience at some critical location. As it is argued in section 6.3, after certain considerations and simplifications it is possible to express the approximation to the fatigue damage accumulated by the vessel during time $[0, t]$ defined in eq. 9, as

$$D(t) = K \int_0^t H_s(\tau)^\alpha \, d\tau, \tag{11}$$

where $H_s(t)$ is the significant wave height process encountered by the vessel at time t, $\alpha = \beta - 1/2$ and K is a log-normally distributed random variable with $\log(K) \in N(m_K, \sigma_K^2)$. Note that m_K is used as a generic constant, which should cause no confusion. The integral $\int H_s(t)^\alpha \, dt$ will also be called *nominal* damage. In the next subsections we provide with formulas for the first two moments of the nominal damage. First we study the case of a single voyage before we turn to the general case of n voyages.

3.1 The mean and variance of $\int_{t_0}^{t_0+T} H_s(t)^\alpha \, dt$ during one voyage

Suppose that a vessel departs for a voyage that lasts T hours at time t_0. Suppose also that at any given time $t, t \in [t_0, t_0+T]$ the position of the vessel $\mathbf{s}(t) = (x(t), y(t))$ is known and the route is deterministic, i.e. the velocity $\mathbf{V}(t)$ is determined in advance.

Modelling the logarithmic values of the significant wave height field, $\log(H_s(\mathbf{s}, t))$, where $\mathbf{s}$ denotes position and t time as a locally stationary Gaussian random field, see Baxevani et al. 2004, the expected value and covariance structure of the field are given by

$$\begin{aligned} \mu(\mathbf{s}, t) &= E[\log(H_s(\mathbf{s}, t))], \\ r((\mathbf{s}_1, t_1), (\mathbf{s}_2, t_2)) &= C[\log(H_s(\mathbf{s}_1, t_1)), \log(H_s(\mathbf{s}_2, t_2))]. \end{aligned}$$

The process encountered by the vessel along its deterministic route is defined as $Y(t) = \log(H_s(t)) = \log(H_s(\mathbf{s}(t), t))$ and is also modelled as locally stationary and Gaussian with moments

$$\mu(t) = E[Y(t)] = E[\log(H_s(t))] = \mu(\mathbf{s}(t), t) \tag{12}$$

and

$$r(t_1, t_2) = C[Y(t_1), Y(t_2)] = C[\log(H_s(t_1)), \log(H_s(t_2))] = r((\mathbf{s}(t_1), t_1), (\mathbf{s}(t_2), t_2)). \tag{13}$$

We are ready now to derive formulas for the moments of $\int_{t_0}^{t_0+T} H_s(t)^\alpha \, dt$. The mean value is computed numerically by evaluating the following integral

$$m(T) = E[\int_{t_0}^{t_0+T} H_s(t)^\alpha dt] := \int_{t_0}^{t_0+T} h(t) \, dt \tag{14}$$

with

$$h(t) = \exp\left(\alpha\mu(t) + \alpha^2 \frac{\sigma^2(t)}{2}\right). \tag{15}$$

Similarly, the variance of the integral is given by

$$s^2(T) = V[\int_{t_0}^{t_0+T} H_s(t)^\alpha \, dt] = \int_{t_0}^{t_0+T} \int_{t_0}^{t_0+T} h(t_1)h(t_2) \cdot \left(e^{\alpha^2 r(t_1,t_2)} - 1\right) dt_1 \, dt_2, \qquad (16)$$

with $h(t)$ as defined in eq. 15, and $\mu(t)$ and $r(t_1, t_2)$ as given in eqs. 12 and 13 respectively and $\sigma^2 = r(t,t)$. For the derivation of eqs. 14 and 16, see section 6.1. In practice $m(T)$ is computed using the so-called *long-term statistics* (wave atlases), which are maps of the mean and the variance of H_s indexed by the geographical location and the time of the year.

3.2 Fatigue life distribution for $D(n)$, the damage accumulated during n-voyages

In this section we obtain the distribution of the fatigue life for a vessel that has travelled n voyages during T time. The fatigue failure time T^f is defined as the first time the damage $D(T)$, given in eq. 11, passes a critical threshold d^{crt}, but usually is computed using instead of $D(T)$ the expected damage $m(T)$ given in eq. 14. Hence T^f is defined as the solution to the equation $Km(T^f) = d^{crt}$. In this section, we present a method for improving the prediction of the fatigue life time T^f, by considering additionally in the computations the variance $s^2(T)$. This is because of the approximately normal character of $D(T)$ for large T values. We commence our analysis by motivating the asymptotically normal character of the integral $\int_{t_0}^{t_0+T} H_s(t)^\alpha dt$, with T being measured now in years instead of days.

Suppose we have a vessel that travels for $T_i, i = 1, \ldots, n$ hours and then spends certain time at a harbour. At this point certain assumptions need to be made. First of all, we assume that the encountered process $H_s(t)$ is identically zero during the time spent not sailing and also the vessel spends at the harbour enough time so that the encountered process during the different voyages becomes independent. Additionally, we assume that all the voyages travelled during the same month have the same duration and hence the integrals $\int_{t_i}^{t_i+T_i} H_s(t)^\alpha \, dt$ are approximately independent and identically distributed. Then, the Central Limit Theorem (CLT) suggests that the distribution of the nominal damage is normal. We should also note that the damage accumulated during the different voyages is not an independent process due to the random variable K that is a quality factor constant at the location of the crack growth.

Consider now a vessel sailing between two harbours along the North Atlantic route (NAr). The following method could be applied to different scenarios and this specific case is for demonstration reasons only.

Denote by t_{ijr} and $T_{ijr}, j = 1,\ldots,12, r = 1,2$ and $i = 1,\ldots,n_{rj}$, the departure time and duration of the i^{th} voyage that takes place during the j^{th} month in the r^{th} direction. Note that usually $n_{1j} = n_{2j}$ or they differ by one. Assume also that the passage times T_{ijr} depend only on the month j and direction r. Then, the random variables $Z^i_{jr} = \int_{t_{irj}}^{t_{irj}+T_{irj}} H_s(t)^\alpha\, dt$ are independent with mean m_{jr} and variance s^2_{jr}. Consequently, the CLT asserts that $D_{jr} = \sum_{i=1}^{n_{rj}} Z^i_{jr}$ is approximately normally distributed with mean $n_{rj}m_{jr}$ and variance $n_{rj}s^2_{jr}$.

The total nominal damage accumulated during n voyages is given by

$$D_X(n) = \sum_{j=1}^{12}\sum_{r=1}^{2} D_{jr} = \int_{t_0}^{t_0+T} H_s(t)^\alpha\, dt,$$

where $n = \sum_{j=1}^{12}\sum_{r=1}^{2} n_{rj}$. It was possible to write $D_X(n)$ in the form of an integral since it is assumed that $H_s(t) = 0$ for $t \notin [t_{ijr}, t_{ijr}+T_{ijr}]$. Since D_{jr} are approximately normally distributed and jointly independent, $D_X(n)$ is approximately normally distributed with mean

$$m(n) = \sum_{j=1}^{12}\sum_{r=1}^{2} n_{rj}m_{jr}$$

and variance

$$s^2(n) = \sum_{j=1}^{12}\sum_{r=1}^{2} n_{rj}s^2_{jr}.$$

Hence, if by $D(n)$ we denote the total damage accumulated during n voyages, then $D(n) = K \cdot D_X(n)$. That is $D(n)$ is the product of two independent random variables, the log-normally distributed quality factor K and the normally distributed nominal damage $D_X(n)$, caused by the variable load . Consequently the distribution of the failure time T^f can be computed as follows

$$P[T^f \le n] = \int_{-\infty}^{\infty} \Phi\left(\frac{m_K + \log(m(n)) + \log(1 + \frac{s(n)}{m(n)}z)}{\sigma_K}\right)\phi(z)\, dz. \qquad (17)$$

Remark 3 *If the covariance between two points t_1 and t_2 of the encountered process is unknown although positive, then setting $r(t_1,t_2) = 0$ results to an underestimation of the variance $s^2(T)$. In other words, assuming independence between any values of the* *encountered process for which the covariance structure is unknown, results in a fatigue failure time having a distribution that is more concentrated around its median.*

Remark 4 *Notice also that although most of the damage in a vessel may occur during such operations as loading cargo or supplying with fuel, these loads are not considered here. Obviously though, this damage could also be included in the analysis by allowing the function $h(t)$ defined in eq. 15, to take on suitable values during the times between travelling. In our case $h(t) = 0$ during those times.*

4 Fatigue life prediction for a vessel sailing the NAr

In this section we demonstrate how the results presented previously may be used to obtain the fatigue life for a vessel sailing along the NAr. As we have seen, for the distribution of the nominal damage is necessary to know the distribution of the encountered H_s process. Hence, we start our presentation by providing with the climate along the NAr. A methodology introduced in Baxevani et al. 2004, is extended to obtain the spatio-temporal field of the logarithmic values of significant wave height in three dimensional areas that can be considered homogeneous and isotropic.

4.1 Wave climate along the NAr - local model

In Baxevani et al. 2004, we have studied the mean and the spatial covariance structure of the Gaussian field $\log(H_s(\mathbf{s}, t))$ in a central part of the NAr extending between -28.5 and -22.5 degrees in longitude and between 48 and 52 degrees in latitude. In this section we propose a full spatio-temporal model for the specific region and apply the proposed methodology to the rest of the NAr.

For the expected value of the field, we have assumed the presence of an annual cycle. To avoid problems with correlated data, the cyclic model was estimated by using only the first observation from each satellite passage. Residual analysis indicated that the observations were independent and normally distributed with constant variance σ^2. The annual cycle was found to dominate the within-year variability of H_s, accounting for almost 42% of the variance in the data at that specific region. The spatial correlation structure was obtained as a solution to a system of temporal equations involving the total variation of the field along the satellite tracks.

We turn now to the problem of modelling the correlation structure of the spatio-temporal field. The proposed model is an extension of the presented spatial correlation model, that additionally includes time. Denote by $\mathbf{s} = \mathbf{s}_2 - \mathbf{s}_1 = (x, y)$ and $t = t_2 - t_1$, the difference between the spatial and temporal coordinates of two points, $(\mathbf{s}_1, t_1)$ and $(\mathbf{s}_2, t_2)$, and let $r(\mathbf{s}, t) = C[\log(H_s(\mathbf{s}_1, t_1)), \log(H_s(\mathbf{s}_2, t_2))]$, denote the covariance between any points at spatial distance $\mathbf{s}$ and temporal distance t. The following simple model is proposed

$$r(\mathbf{s}, t) = \sigma^2[p \exp\left(-\frac{(\mathbf{s}, t)\Lambda^1(\mathbf{s}, t)^T}{2\sigma^2} - c|t|\right) + (1-p)\exp\left(-\frac{(\mathbf{s}, t)\Lambda^2(\mathbf{s}, t)^T}{2\sigma^2} - c|t|\right)] \quad (18)$$

where the matrices Λ^i are defined as:

$$\Lambda^i = \begin{pmatrix} \lambda^i_{200} & \lambda^i_{110} & \lambda^i_{101} \\ \lambda^i_{110} & \lambda^i_{020} & \lambda^i_{011} \\ \lambda^i_{101} & \lambda^i_{011} & \lambda^i_{002} \end{pmatrix}, \quad (19)$$

for $i = 1, 2$, and $\mathbf{s}^T$ denotes the transpose of the matrix $\mathbf{s}$. This covariance structure can be viewed as the covariance function of a sum of two independent fields, say due to covariance on a coarse scale from one source and with the second on a finer scale due to a second independent source. This model was chosen since it is simple and at the same time describes accurately the important covariance properties of the data. An additional reason for choosing the particular shape of covariance function $r(\mathbf{s}, t)$, was the belief that the covariance should always be positive, as one does not expect that larger storms would result in calmer regions (or periods), a fixed short distance (or time) apart.

The spectral moments $\lambda^i_{200}, \lambda^i_{110}$ and λ^i_{020}, i.e. the parameters in the spatial covariance structure, are estimated by means of data from the TOPEX-Poseidon satellite, see Baxevani et al. 2004. The spatial field was found to be isotropic implying $\lambda^i_{200} = \lambda^i_{020}$. Moreover, the two spatial derivatives were also found to be uncorrelated, that is $\lambda^i_{110} = 0$. We will show that λ^i_{002} can be related to some macroscopic meteorological parameters that could be taken either from buoy or hindcast data. One such parameter is the so-called principal velocity which equals the median velocity with which a contour line is moving. Defining as storm a region in which H_s exceeds some fixed level, the principal velocity is the average local velocity the boundary of the storm region moves with, see Baxevani et al. 2003 and Baxevani and Rychlik 2004 for a more detailed discussion. The principal velocity is defined as

$$\mathbf{v}_{max} = (v_x, v_y) = \left(-\frac{\lambda_{101}}{\lambda_{200}}, -\frac{\lambda_{011}}{\lambda_{020}}\right) [degrees/hr]. \tag{20}$$

If we assume additionally that the field with the covariance structure in eq. 18 is separable, then $\lambda^i_{002} = \lambda^i_{200}(v_x^2 + v_y^2)$. That is, the matrices Λ^i defined in eq. 19 are singular which further suggests that the spatial field is actually drifting with velocity $\mathbf{v}_{max}$.

Remark 5 *The proposed covariance structure belongs to the family of covariances for a separable drifting field. More precisely,*

$$r(\mathbf{s}, t) = r_{sp}(x - v_x t, y - v_y t) \cdot r_t(t), \tag{21}$$

where $r_{sp}(\cdot)$ is the spatial covariance and $r_t(\cdot)$ is the temporal covariance. The velocity $\mathbf{v}_{max} = (v_x, v_y)$ is defined in eq. 20. This model, which is non-differentiable, can be thought of as a first-order auto-regression AR(1), on the spatial field with smooth noise. Some of the properties of this field are presented in section 6.2.

Remark 6 *Note that the two components of the field may move with different principal velocity and $\mathbf{v}_{max}$ is then used as a generic constant.*

4.2 Local spatial models along the NAr

The methodology for obtaining the mean and spatial covariance structure, described in section 4.1, is now applied to the remaining NAr. The area is splitted into smaller regions so that the requirements of stationarity and isotropy are satisfied. The results are presented in Table 1 and Figs 1 and 2.

region	geographic area	$\hat{\mu}$	$\hat{\sigma^2}$
R1	$[-12.8, -10] \times [48, 52]$	$1.0364 + 0.4294\cos(\phi t) + 0.0652\sin(\phi t)$	0.1667
R2	$[-15.5, -12.5] \times [48, 52]$	$1.0656 + 0.4926\cos(\phi t) - 0.0026\sin(\phi t)$	0.1555
R3	$[-18.5, -15.2] \times [48, 52]$	$1.1014 + 0.4532\cos(\phi t) + 0.0531\sin(\phi t)$	0.1457
R4	$[-21, -18] \times [48, 52]$	$1.1207 + 0.4230\cos(\phi t) + 0.0679\sin(\phi t)$	0.1491
R5	$[-23.5, -20] \times [48, 52]$	$1.1223 + 0.4005\cos(\phi t) + 0.1061\sin(\phi t)$	0.1391

R6	$[-28.5, -22.5] \times [48, 52]$	$1.1366 + 0.4184 \cos(\phi t) + 0.1231 \sin(\phi t)$	0.1313
R7	$[-33.8, -27.5] \times [48, 52]$	$1.1601 + 0.4517 \cos(\phi t) + 0.1298 \sin(\phi t)$	0.1287
R8	$[-37, -33] \times [48, 52]$	$1.1630 + 0.4316 \cos(\phi t) + 0.1546 \sin(\phi t)$	0.1361
R9	$[-40.2, -36] \times [48, 52]$	$1.1866 + 0.4138 \cos(\phi t) + 0.1284 \sin(\phi t)$	0.1362
R10	$[-42.5, -40] \times [46, 50]$	$1.1112 + 0.3938 \cos(\phi t) + 0.1229 \sin(\phi t)$	0.1471
R11	$[-45, -42.5] \times [46, 50]$	$1.0381 + 0.4027 \cos(\phi t) + 0.1222 \sin(\phi t)$	0.1260
R12	$[-46.4, -44] \times [46, 50]$	$0.9739 + 0.4357 \cos(\phi t) + 0.095 \sin(\phi t)$	0.1195
R13	$[-52, -46.4] \times [46, 48]$	$0.9032 + 0.4141 \cos(\phi t) + 0.0678 \sin(\phi t)$	0.1397
R14	$[-54, -52] \times [42, 46]$	$0.8674 + 0.3836 \cos(\phi t) + 0.0635 \sin(\phi t)$	0.1529
R15	$[-55.8, -54] \times [42, 46]$	$0.8632 + 0.3972 \cos(\phi t) + 0.1051 \sin(\phi t)$	0.1655
R16	$[-57.5, -55.6] \times [42, 46]$	$0.6763 + 0.4103 \cos(\phi t) + 0.1131 \sin(\phi t)$	0.1932
R17	$[-60, -57.3] \times [42, 46]$	$0.5354 + 0.3631 \cos(\phi t) + 0.1076 \sin(\phi t)$	0.2463

Table 1: Estimates of the mean value and variance for each one of the stationarity regions.

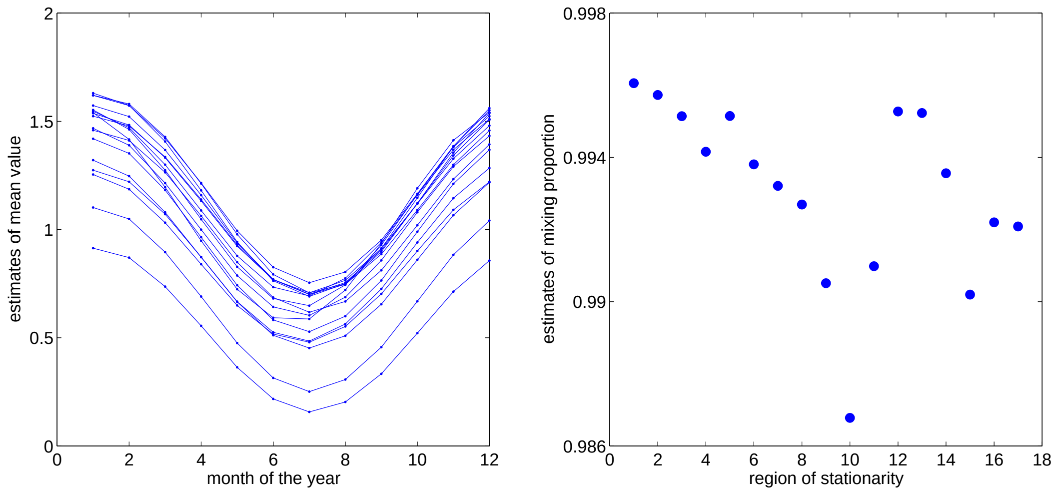

Figure 1: *Left:* Estimates of mean value $\mu(t)$ of $\log(H_s)$ for each stationarity region vs month of the year. *Right:* Estimates of the mixing proportion p for each stationarity region.

In Figs 1 and 2, the time dependence of the mean and the correlation parameters is illustrated. As expected the summer months have lower average H_s values than the winter months. Higher λ^i_{200} values correspond to faster decreasing correlation function. Hence, the space correlation during the summer is shorter than during the winter.

As can be seen in Table 1, the expected values of $\log(H_s)$ vary geographically. Area R9, a region in the middle of the Atlantic, has the highest monthly averages throughout the year, while area R17, the one closest to the coast of the North America, does not only have the lowest monthly averages but also the biggest variance. The covariance parameter estimates for region R17 as can be seen in Fig 1 *(Right)* and Fig. 2, differ substantially from those for the rest of the regions. It is our belief that this is due to the proximity of this region to the coast that makes it difficult to obtain the correct model using only satellite observations. Obviously this model needs further verification.

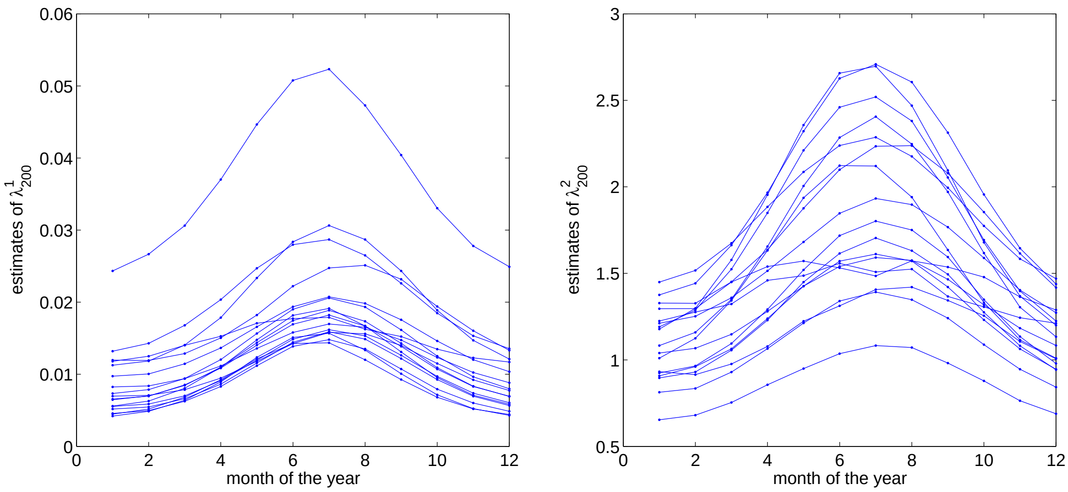

Figure 2: *Left:* Estimates of λ^1_{200} for each stationarity region vs month of the year. *Right:* Estimates of λ^2_{200} for each stationarity region vs month of the year.

4.3 Temporal model for H_s climate along the NAr

In this section we estimate the temporal variability of H_s along the NAr. For this, we use data from the US NODC Buoy 46005. An annual cycle plus trend was fitted to the data and the residuals were used to calculate the autocorrelation function. The correlation estimates for different time lags are presented in Table 2, which is taken from Anderson et al. 2001. Obviously there are certain limitations on the use of buoy data, since buoys are traditionally located close to coastal areas. Here, the specific data that is collected from the NE Pacific is used for illustration reasons only, although the estimates are very close to what we would expect for the North Atlantic. Use of hindcast data for verification of the method is a part of future research plans.

Before we continue some further simplifications / assumptions are required. Firstly, the correlation structure given in Table 2 is assumed to be valid also for the North Atlantic. This assumption, although it is more likely to be violated, was made since there is no other suitable data at the moment. Secondly, the storm systems are assumed to move from the coast of North America towards Europe along storm tracks that are parallel to the x-axis. This assumption is necessary since use of buoy or any temporal data allows estimation of the speed $||\mathbf{v}_{max}|| = \sqrt{v_x^2 + v_y^2}$ but provides with no information on the direction on which the storm systems are moving.

lag(hr)	1	2	3	4	5	6	7	8	10
Nb(10^3)	140	138	137	136	134	133	132	131	128
$\rho(t)$	0.9802	0.9710	0.9582	0.9425	0.9251	0.9066	0.8878	0.8688	0.8308

Table 2: Correlation of residual $\log(H_s)$ about an annual cycle and linear trend from all data, $1978 - 1999$. Nb indicates the number of pairs that were used in the estimation of the correlation parameters.

Under these assumptions, the temporal covariance given in eq. 18 is simplified to

$$r(t) = r(0,0,t) = \sigma^2[p\exp(-\frac{\lambda_{200}^1}{2\sigma^2}v_x^2t^2 - c|t|) + (1-p)\exp(-\frac{\lambda_{200}^2}{2\sigma^2}\tilde{v}_x^2t^2 - c|t|)]. \quad (22)$$

There are a number of methods that can be employed to fit the model to the data given in Table 2. The method used here is ordinary least squares; that is $r(t)$ is chosen so that it minimises

$$\sum_t (r(t) - \hat{r}(t))^2$$

where $\hat{r}(t)$ is the empirical correlation function.

Obviously, since the parameters p, σ^2 and λ_{200}^i, for $i = 1, 2$, vary both seasonally and geographically, so do $v_x, \tilde{v}_x$ and c. Application of the minimisation procedure provided with estimates for $v_x, \tilde{v}_x$ and c for each stationarity region and month of the year. The parameter c was found to be almost constant and hence was set to equal its mean value. The parameter $\tilde{v}_x$, the average velocity of the fast component of the field seems to be just noise. Hence for simplicity at this point it is also set to equal its average value. Finally, the estimates of the parameter v_x that can be seen in Fig. 3, vary both according to region and month.

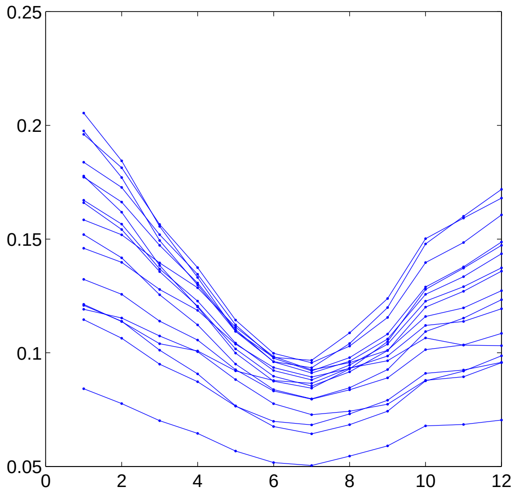

Figure 3: Estimates of v_x (units in degrees/hr) for each stationarity region vs month of the year.

To conclude, the field $\log(H_s(\mathbf{s}, t))$ although not stationary is locally stationary with variance that depends only on the region and mean value and correlation structure that varies monthly and geographically. Hence, a full regional probabilistic spatio-temporal model is attained but not a global one since the correlation between the different regions is still unknown although assumed to be non-negative, for the same reasons that applied for the local models and were explained in section 4.1. Next, we compute the fatigue damage accumulated by a vessel sailing between two harbours along the NAr and demonstrate how the prediction of fatigue life may be improved by also considering in the computations the variance of the fatigue damage.

4.4 Fatigue accumulated by a vessel sailing along the NAr

A merchant ship usually travels along the NAr with a speed around 10 m/s, that is 0.5046 (degrees/hr). We assume that the vessel travels with this speed independently of time and direction. The methodology presented in the previous sections, allows for variable speed although this case is not considered at the moment.

We start our analysis by providing a sketch on the computation of m_{jr} and s^2_{jr}. Assume that the vessel departs from the point $\mathbf{s}_0 = (x_0, y_0)$ at time t_0 and is travelling with constant velocity $\mathbf{V} = (V_1, V_2)$.

The significant wave height process encountered by the vessel is given by

$$(H_s)_{t_0}(t) = H_s(x(t), y(t), t), \quad t_0 \le t \le t_0 + T,$$

where $x(t) = x_0 + V_1(t - t_0), \quad y(t) = y_0 + V_2(t - t_0), \quad \mathbf{s}(t) = (x(t), y(t))$. Assuming the duration of the voyage to be too short so that the time variabililty of the mean $\mu(\mathbf{s}, t)$ can be neglected, the expected value is given by

$$\mu(t) = \mu(x_0 + V_1(t - t_0), y_0 + V_2(t - t_0), t_0), \quad t_0 \le t \le t_0 + T. \tag{23}$$

For the covariance function $r(t_1, t_2)$, we need to keep in mind that there are different regions of stationarity. Hence, for t_1, t_2, such that $\mathbf{s}(t_1)$ and $\mathbf{s}(t_2)$ are inside the same stationarity region,

$$r(t_1, t_2) = r(x(t_2) - x(t_1), y(t_2) - y(t_1), t_2 - t_1) = r(t), \tag{24}$$

with $t = t_2 - t_1$, otherwise $r(t_1, t_2) = 0$.

Each one of the crossings lasts around 4.5 days, which is short time compared to the time variability of the covariance parameters. Hence, during each crossing the parameters in the covariance function for each stationarity region depend only on the starting time of the trip, t_0. Hence,

$$r(t) = \sigma^2 \left(p e^{-\frac{\lambda^1_{200}}{2\sigma^2}[(V_1 - v_x)^2 + V_2^2]t^2 - c|t|} + (1-p) e^{-\frac{\lambda^2_{200}}{2\sigma^2}[(V_1 - \tilde{v}_x)^2 + V_2^2]t^2 - c|t|} \right).$$

Consequently, the expected value of the nominal damage accumulated during one passage can be computed by

$$E[D_X(T)] = \int_{t_0}^{t_0+T} e^{\alpha\mu(t) + \alpha^2 \frac{\sigma^2(t)}{2}} \, dt, \tag{25}$$

where $\sigma^2(t) = r(t,t)$, and the variance by

$$V[D_X(T)] = \int_{t_0}^{t_0+T} \int_{t_0}^{t_0+T} e^{\alpha\mu(t)+\alpha^2\frac{\sigma^2(t)}{2}} e^{\alpha\mu(s)+\alpha^2\frac{\sigma^2(s)}{2}} \cdot \left(e^{\alpha^2 r(s,t)} - 1\right) ds\, dt, \qquad (26)$$

with $\mu(t)$ and $r(s,t)$ defined in eqs. 23 and 24 respectively.

Remark 7 *It should be kept in mind that the direction affects only the covariance structure. The mean values are independent of the direction of the journey.*

The numerical values of $E[D(T)]$ and $\sqrt{V[D(T)]}$, defined in eqs 25 and 26, for two different sets of constants β and m_K are given in Tables 2 and 3. The constant velocity with which the vessel is travelling was set equal to $\mathbf{V} = (0.5046, 0)$ degrees/hr. We refer to the direction from Europe to the North America as *positive direction* and the opposite as *negative direction*.

month	$E[D_X(T)]$	$\sqrt{V[D_X(T)]}$, pos d.	$\sqrt{V[D(T)]}$, neg d.
January	0.0109	0.0062	0.0065
February	0.0088	0.0050	0.0053
March	0.0050	0.0028	0.0030
April	0.0024	0.0013	0.0014
May	0.0011	0.0006	0.0006
June	0.0006	0.0003	0.0004
July	0.0005	0.0003	0.0003
August	0.0006	0.0003	0.0004
September	0.0011	0.0006	0.0006
October	0.0025	0.0014	0.0014
November	0.0053	0.0030	0.0031
December	0.0091	0.0051	0.0054

Table 3: $\log(K) \in N(6.4523 \cdot 10^{-8}, 0.06)$

month	$E[D_X(T)]$	$\sqrt{V[D_X(T)]}$, pos d.	$\sqrt{V[D_X(T)]}$, neg d.
January	0.0104	0.0111	0.0119

February	0.0079	0.004	0.0090
March	0.0038	0.0040	0.0043
April	0.0015	0.0015	0.0016
May	0.0005	0.0005	0.0006
June	0.0003	0.0003	0.0003
July	0.0002	0.0002	0.0002
August	0.0003	0.0003	0.0003
September	0.0006	0.0006	0.0006
October	0.0015	0.0016	0.0017
November	0.0041	0.0044	0.0047
December	0.0082	0.0089	0.0094

Table 4: $\log(K) \in N(4.5625 \cdot 10^{-7}, 0.06)$

The failure time distribution given in eq. 17, is presented in Fig. 4 for two sets of constants, the same as before. The vessel crosses the middle Atlantic back and forth, four times in a month, twice on each direction. Here we present the fatigue accumulated after twenty five years of operation. As we can see, the history of the encountered sea states is a very important factor in the evaluation of the fatigue limits for a vessel. More importantly, we are now able not only to predict the fatigue damage but also to measure the risk involved.

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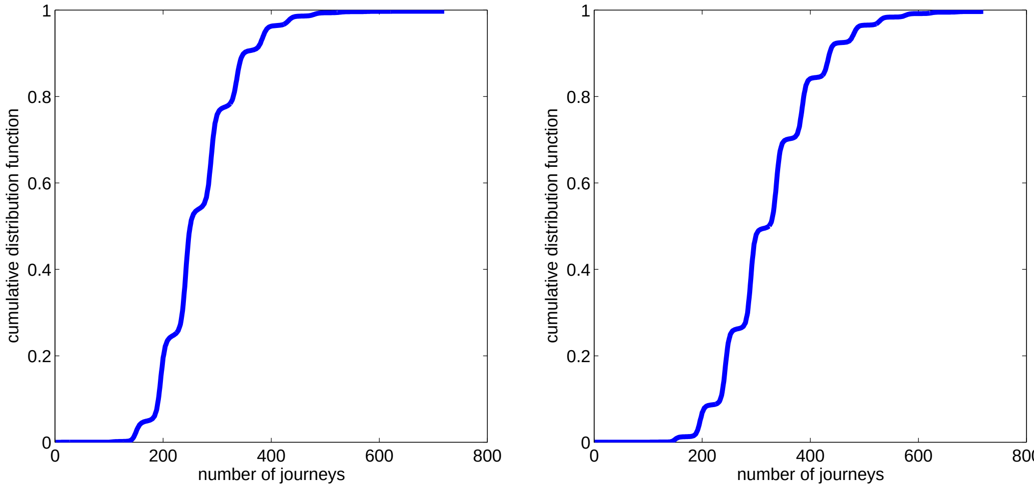

Figure 4: *Left:* Fatigue life distribution for a vessel with $\beta = 4$ and $\log(K) \in N(6.4523 \cdot 10^{-8}, 0.06)$. *Right:* Fatigue life distribution for a vessel with $\log(K) \in N(4.5625 \cdot 10^{-7}, 0.06)$.

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6 Appendix

6.1 Proofs of formulas

In this section, we derive the first two moments for the variability of the load as expressed through the integral $\int_0^T H_s(t)^\alpha \, dt$.

We start by reminding that if X is a normally distributed random variable, that is if $X \in N(\mu, \sigma^2)$, then its moment generating function (m.g.f) is given by

$$E[e^{tX}] = e^{t\mu + \frac{t^2\sigma^2}{2}}.$$

Now, let us consider a Gaussian random process $X(t)$ with mean function $\mu(t)$ and covariance function $r(t,s)$. Obviously $\sigma^2(t) = r(t,t)$. Then taking $X(t) = \log(H_s(t))$, the first moment of $\int_0^T H_s(t)^\alpha \, dt$ is derived as follows

$$E[\int_0^T H_s(t)^\alpha \, dt] = \int_0^T E[H_s(t)^\alpha] \, dt = \int_0^T E[e^{\alpha X(t)}] \, dt = \int_0^T e^{\alpha\mu(t) + \frac{1}{2}\alpha^2\sigma^2(t)} \, dt. \tag{27}$$

For the second moment we have

$$\begin{aligned} E[\left(\int_0^T H_s(t)^\alpha \, dt\right)^2] &= E[\int_0^T \int_0^T e^{\alpha(X(t)+X(s))} \, dt \, ds] \\ &= \int_0^T \int_0^T E[e^{\alpha(X(t)+X(s))}] \, dt \, ds \\ &= \int_0^T \int_0^T e^{\alpha(\mu(t)+\mu(s)) + \frac{\alpha^2}{2}(\sigma^2(t)+\sigma^2(s)) + \alpha^2 r(t,s)} \, dt \, ds. \end{aligned} \tag{28}$$

Combining (27) and (28) we obtain for the variance of the integral $\int_0^T H_s(t)^\alpha \, dt$

$$V[\int_0^T H_s(t)^\alpha \, dt] = \int_0^T \int_0^T e^{\alpha(\mu(t)+\mu(s)) + \frac{\alpha^2}{2}(\sigma^2(t)+\sigma^2(s))} \left(e^{\alpha^2 r(t,s)} - 1\right) \, dt \, ds. \tag{29}$$

6.2 Drifting AR(1) fields

Consider a three dimensional Gaussian random field described by

$$W(x,y,idt) = \rho W(x - v_x dt, y - v_y dt, (i-1)dt) + \sqrt{1-\rho^2}\epsilon_i(x,y), \tag{30}$$

for $i \geq 1$ and $|\rho| < 1$ and

$$W(x,y,0) := W(x,y) = \epsilon_0(x,y), \text{for } i = 0,$$

with $\epsilon_i(x, y)$ independent, stationary and identically distributed two dimensional random fields with covariance structure $r_{sp}(x, y)$. This is a discrete time random field and just for convenience in notation we assume the time points to be equidistant. That is $t_i = i\,dt$ for some time unit dt.

Since the field $W(x, y)$ is stationary so are the fields defined in eq. 30. Their covariance is given by:

$$r(x, y, t) = r_{sp}(x - v_x t, y - v_y t) r_t(t),$$

with $\rho = r_t(dt)$.

The field in eq. 30 can be rewritten in a recursive form as:

$$W(x, y, idt) = \rho^i W(x - iv_x dt, y - iv_y dt) + \sum_{j=1}^{i} \sqrt{1-\rho^2} \rho^{i-j} \epsilon_j(x - v_x(i-j)dt, y - v_y(i-j)dt). \tag{31}$$

The fields defined by eq. 30 or equivalently by eq. 31 are particularly attractive since they exhibit certain properties. For example although they are three dimensional fields in order to simulate them is enough to simulate a sequence of independent two dimensional random fields with covariance structure $r_{sp}(x, y)$ on appropriate grids and use either eqs 30 or 31.

6.3 Damage accumulated by a vessel - A simplified approach

Let $X(t)$ be a variable load, for example the stress a vessel experiences at some critical location. In this subsection we provide the formula for the damage accumulated by a vessel due to the variable load process $X(t)$.

It is customary in ocean engineering to model the sea surface as a zero mean Gaussian random field and the vessel as a linear structure. Therefore the random load $X(t)$ can be thought for simplicity, as a Gaussian process. Under the additional assumption that the process is stationary or locally stationary, the damage accumulated during the time period $[0, T]$ depends only on the variance of the load, $\sigma^2_{X(t)}$, and of its derivative, $\sigma^2_{\dot{X}(t)}$.

Although, a realistic computation of $\sigma^2_X(t)$ and $\sigma^2_{\dot{X}}(t)$ requires use of a dedicated software, a simplified model could be considered assuming that the standard deviation of the load process $X(t)$ is proportional to the encountered sea elevation measured at a fixed point on the vessel. This assumption is not uncommon in the literature, see

Lindemann 1986: *the loading effect may be assumed proportional to the wave height.* Setting the proportionality constant to be equal to one, the load $X(t)$ can be further assumed to be the encountered sea elevation in which case $\sigma_X(t) = \frac{H_s(t)}{4}$ and $\sigma_{\dot{X}}(t) = 2\pi\frac{H_s(t)}{4T_z^e(t)}$. T_z^e is the average zero down-crossing period of the encountered sea surface elevation at time t and depends on the velocity of the vessel as well as higher spectral moments of the sea surface field (an integral with no closed form needs to be evaluated). Hence, in order to preserve the transparency and simplicity of the presentation we consider the average zero down-crossing period of the sea elevation instead of the average zero down-crossing period of the encountered sea elevation, i.e. we assume that $T_z^e = T_z$. Labeyrie 1990 has found, using data coming from radar distance measurements made from the QP structure in the Frigg Field, that for a given H_s a quadratic relation could be fitted to the conditional expectation of T_z to obtain

$$T_z \approx \sqrt{aH_s + b}, \tag{32}$$

with $a = 6.17 \cdot 10^{-7}$ and $b = 1.62 \cdot 10^{-6}$ and a relative error less than 1% for the Frigg Field data. Clearly these parameters may be location dependent and careful investigation is required, this being a preliminary analysis though, eq. 32 is adopted for the whole NAr with $b = 0$ for simplicity. Thus, the standard deviation of the derivative may be written as $\sigma_{\dot{X}}(t) = 225\sqrt{2} \cdot 2\pi\sqrt{H_s(t)}$ (units in hours).