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COMPUTATIONAL ALGORITHMS FOR OPTIMIZATION PROBLEMS FOR PERIODIC SYSTEMS

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Abstract. We give a survey of computational algorithms derived in a series of papers for solution of a synthesis problem for optimal stabilization of periodic systems. The proposed algorithms synthesize reliable control, with a given stability tolerance, for periodic systems optimized with respect to the quadratic performance criterion. A reliable controller guarantees the stability and is nearly a linear/quadratic regulator. We also consider the inverse problems for optimal synthesis corresponding to quadratic functionals. The algorithms designed to solve this problem are based on methods of linear matrix inequalities. The efficiency of the proposed algorithms are shown through implementing some examples.

Introduction

In this note we give an overview of computational algorithms for solution of a synthesis problem for optimal stabilization of periodic systems. Our study is based on stability of a discrete algebraic Riccati equation (ARE). The main feature of this study is to define an optimal regulator for linear discrete periodic control systems and minimize the corresponding quadratic functional under asymptotic stability condition of the closed-loop systems. To motivate this, we point out that in many applied problems all phase coordinates of an object are not available and therefore it is necessary to find controlling influence as a function, depending only on part of the phase vector coordinates: an optimal regulator with reduced dimension. The regulators with reduced dimension are recommended to stabilize the program trajectory in different mechanical systems. Reduced dimensions are especially attractive because of their lower computational costs. Moreover, excluding insignificant components of the phase vector minimizes the delay in the procedure of feedback chain.

In this context, to solve an optimization problem requires to find solution of an algebraic Lyapunov equation (ALE) and minimize the corresponding quadratic functional by feedback matrices. To this approach we rely on efficient computational methods. A variety of methods, for minimizing of the functionals of several variables, has already been developed and investigated. These approaches are based on finding the gradient of the functionals. One of these methods is the well-known *conjugated gradient method*.

Here we report on a project work dealing with developing new algorithms, for optimal control with reduced dimension in both continuous and discrete cases, where the results of optimization of all phase coordinates are used. We also develop methods for solution of reliable stabilization problem for the discrete periodic control systems. Finally we give algorithms for solution of periodic problems of optimal regulators based on the results of discrete inverse problems for stationary systems.

Key words and phrases. Mathematical Programming, Variational Analysis and Optimal Control, Modeling and Simulation, Periodic Riccati Equation, Inverse Problem, Reliable Stabilization
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1. Synthesis of optimal system stabilization for periodic discrete systems with respect to output variable

1.1. **Stabilization of periodic systems by static output feedback.** The objective of the control systems under consideration is given by

$$x(t+1) = A(t)\,x(t) + B(t)\,u(t) \quad t = 0,1,...,\ x(0) = x_0\ , y(t) = C(t)\,x(t)\,, \tag{1.1.1}$$

where $x(t)$ is a phase vector, $u(t)$ is a controlling influence vector, A, B are periodic matrices (with period τ). Let, the system (1.1.1) be optimized in the *quadratic quality criterion.*

$$J = \sum_{t=0}^{\infty} \left(x'(t)\,Q(t)\,x(t) + u'(t)\,R(t)\,u(t)\right), \tag{1.1.2}$$

where $Q(t) = Q'(t) \geq 0$, $R(t) = R'(t) \geq 0$ are periodic matrices with period τ, i.e. $Q(t+\tau) = Q(t)$**,** $R(t+\tau) = R(t)$**.** We need to determine the regulator (the periodic matrix $K(t)$ with period τ) such that:

$$u(t) = K(t)\,y(t)\,, \tag{1.1.3}$$

minimizes the criterion (1.1.2) on the class of stability of the closed-loop system (1.1.1), and (1.1.3), see [1]. We modify the equation (1.1.1) by substituting the matrix A(t) by $A_\mu(t)$:

$$A_\mu(t) = (1-\mu)A(t).$$

Using this substitute the relation (1.1.1) would become

$$x(t+1) = \bar{A}_\mu(t)\,x(t)\,, \qquad \text{with}$$
$$\bar{A}_\mu(t) = \left(A_\mu(t) + B(t)\,K(t)\,C(t)\right).$$

In the modified problem, initially, the parameter μ will be chosen so that

$$\left|\lambda\left((1-\mu)^{\tau} A(t-1)\,A(t-2)\,.....A(0)\right)\right| < 1.$$

For period $\tau = 2$ the gradient dependence for the functional (1.1.2) is given by

$$\frac{\partial J_\mu}{\partial K(0)} = 2\left(R(0)K(0)C(0) + B'(0)\left(\bar{Q}(1)\bar{A}_\mu(0) + \bar{A}_\mu(1)U\Psi_\mu\right)\right)\cdot P_\mu C'(0), \tag{1.1.4}$$

$$\frac{\partial J_\mu}{\partial K(1)} = 2\left(R(1)\,K(1)\,C(1)\,\bar{A}_\mu(0) + B'(0)\,U\Psi_\mu\right)\cdot P_\mu \bar{A}'_\mu(0)\,C'(1)\,, \tag{1.1.5}$$

$$U = \Psi_\mu U \Psi_\mu + T_\mu, \tag{1.1.6}$$

where

$$\Psi_\mu = \bar{A}_\mu(1)\,\bar{A}_\mu(0)\,,$$
$$T_\mu = \bar{Q}(0) + \bar{A}'_\mu(0)\,\bar{Q}(1)\,\bar{A}_\mu(0)\,,$$
$$\bar{Q}(t) = Q(t) + C'(t)K'(t)R(t)K(t)C(t), \qquad t = 0,1.$$

Thus, we have all necessary relations for numerically solving the optimization problem. Choosing an initial value for μ yields the choice of an initial value for the matrices $K(t)$. One may start, e.g., with $K(t) = 0$.

Below, we shall assume that the initial conditions x_0 are random vectors with zero mathematical expectation and covariance matrix

$$S = < x_0 x_0^{'} > .$$

We use the procedure of **fmini.m** MATLAB package for solving the optimization problem of the periodic system by output variable. To solve this problem we use the function

[E,K0,K1]=plqdc(A0,A1,B0,B1,C0,C1,Q0,Q1,R0,R1,S).

The problem is solved for $\tau = 2$, where $A0,\ A1, B0, B1, C0, C1, Q0, Q1, R0, R1$ denote the matrices $A(0)\,,\ A(1)\,, B(0)\,, B(1)\,, C(0)\,, C(1)\,, Q(0)\,, Q(1)\,, R(0)$ and $R(1)$, respectively. The outputs are:

$E-$ eigenvalues of the system.
$K0, K1-$ the coefficients of the optimal regulators corresponding to the matrices $K(0), K(1)$.
Below we give some examples of output displayed as the results of the program **plqdc**.

Example 1, cf [1].

A0 = [1.0201 0.2013 ; 0.2013 1.0201] ; B0 = [0.0201 ; 0.2013];

A1 =[1.0201 0.2013; 0.2013 1.0201]; B1 =[0.0201 ; 0.2013];

Q0 =[1 0; 0 1] ; Q1 =[1 0 ; 0 1]; R0 = 0; R1 =0;

S =[1 0 0 1]; C0 =[1 0] ; C1 =[0 1];

E =[0.6185 ; 0.4283] K0 = -6.9521; K1 = -3.8123.

Example 2, cf [1]. Computing the relation defining the gradient of the increasing object function. The optimization procedure **fmins.m** was used without calculating the gradient of the objective function. The example is performed with period 10.

A0 =[1.0201 0.2013;0.2013 1.0201]; B0 =[0.0201; 0.2013];

A1 =[1.0201 0.2013; 0.2013 1.0201];B1 =[0.0201 ; 0.2013];

m =0.2700; Q =[1 0; 0 1]; R =0; S =[1 0; 0 1];

C0 =[0 ; -1]; C1 =[-0.5878 ; -0.8090]; C2 =[-0.9511; -0.3090];

C3 =[-0.9511; 0.3090]; C4 =[-0.5878; 0.8090]; C5 =[-0.0000; 1.0000];

C6 =[0.5878 ; 0.8090]; C7 =[0.9511; 0.3090]; C8 =[0.9511; -0.3090]; C9 =[0.5878; -0.8090];

K0=-5.0055; K1=-8.8772; K2=-0.0211; K3=0.0010; K4=-0.0007; K5=-0.0182; K6=0.3013; K7=0.1191; K8=-0.0045; K9=-0.0339;

J = 8.6123; r =2.4275e-007.

1.2. **Algorithm for periodic discrete optimal system stabilization with respect to output variable.** Let the closed system "object + regulator" be described by the system of finite-difference equations

$$x(i+1) = (\Psi(i) + \Gamma(i) W(i)) x(i), x(0) = X_0, i = 0, 1, ... \tag{1.2.1}$$

If we aim to find a regulator over a part of the phase vector $x_1(i)$, then the equation of the regulator over all phase coordinates may have the form

$$u(i) = W(i) x(i) = [W_1(i) \quad W_2(i)] \begin{bmatrix} x_1(i) \\ x_2(i) \end{bmatrix}, \tag{1.2.2}$$

and the minimizing functional can be defined as

$$J = \sum_{i=0}^{\infty} x'(i) \left(Q(i) + W'(i) R(i) W(i) + \alpha \begin{bmatrix} 0 & 0 \\ 0 & E \end{bmatrix} W'(i) W(i) \begin{bmatrix} 0 & 0 \\ 0 & E \end{bmatrix} \right) x(i). \tag{1.2.3}$$

Here $x(i) = [x_1(i) \;\; x_2(i)]'$ is n-dimensional vector of the object phase coordinates, $x_1(i) - l$-dimensional vector, $u(i) - m$-dimensional vector of the controlling influence, X_0- the random value, $\langle X_0 \rangle = 0$ and $\langle X_0 X_0' \rangle = E$ is uniformly distributed over the unit sphere, where $\langle \; \rangle$ - is the symbol of mathematical expectation. The matrices $\Psi(i), \Gamma(i), Q(i), R(i)$ are periodic with period p, $R(i+p) = R(i) = R'(i) > 0$, $Q(i+p) = Q(i) = Q'(i) \geq 0$, and α is any positive number. Following the procedure in [2], $Q(0) \neq 0$, $Q(1) = ... = Q(p-1) = 0$. Under these conditions the solution of initial value problem (1.2.2), (1.2.3) is reduced to that of the stationary case. In the present work we consider the case when $Q(i) \neq 0.(i = \overline{0, p-1})$. For solution of this problem it is necessary to compute the gradient of the functional with respect to the feedback matrix. Here the gradient is calculated as in [3], and has an essential influence on the calculating process for finding the regulator. In [2] the gradient is taken over an scalar parameter, viz [4]: an approach which meets some difficulties with dimension raise, also also with increasing number of separation points. Therefore it is more secure, for calculating the gradient (1.2.3), to use the analytical formulas given in [3]. Here is the corresponding *algorithm*:

Step1. The inputs are $\Psi_i, \Gamma_i, Q_i, R_i$. The initial matrices W_i^0 are calculated as in [5,6]:

$$W_i^0 = -\left(R_i + \Gamma_i' S_{i+1} \Gamma_i\right)^{-1} \Gamma_i' S_{i+1} \Psi_{,i}$$

Step 2. Given a large α, the discrete ALE is solved by the method of [5].
Step 3. The gradient of the functional is calculated as in [3].
Step 4. The vector L_i^j is calculated using the recursive formula

$$L_i^j = \left(E - \sum_{r=1}^{j-1} \frac{L_i^r L_i^{r\prime}}{L_i^{r\prime} L_i^r}\right) \left.\frac{dSpS_0(W_i)}{dW_i}\right|_{W_i = W_i^j},$$

$$L_1 = \left.\frac{dSpS_0(W_i)}{dW(i)}\right|_{W_i = W_i^1}, j = 2, 3, ..., n, i = \overline{0, p-1}, p = 5,$$

where the matrices $\tilde{L}_i^j$ are reconstructed by

$$\tilde{L}^j(i) = \left[\begin{array}{l} L_{11}^j(i)\ L_{12}^j(i)\ ...\ L_{1n}^j(i) \\ .... \\ L_{m1}^j(i)\ L_{m2}^j(i)\ ...\ L_{mn}^j(i) \end{array}\right].$$

Step5. Feedback matrices W_i^{j+1} are defined by

$$W_i^{j+1} = W_i^j - \gamma^j L_i^j,$$

$$W_i^{\hat{t}+j+1} = W_i^{\hat{t}+j} + \beta^j \left(W_i^{\hat{t}+j} - W_i^{\hat{t}+1-j}\right), \quad i = \overline{0, p-1}, \quad j = 1, 2, ..., \hat{t},$$

where γ^j, β^j are calculated using "golden" section method.
Step 6. For any small and positive ε, the convergence condition

$$\left\|W_i^{2\hat{t}+1} - W_i^{2\hat{t}}\right\| \leq \varepsilon,$$

is checked-up. When this condition is not fulfilled, then we take $W_i^0 = W_i^{2\hat{t}+1}$ and return to step 2, else calculating procedure ends.

For the processing of the program "art2.m" the initial data must be given as input, i.e. the values of the variables psi0, psi1,psi2, psi3, psi4, gam0, gam1,gam2, gam3, gam4, R0, R1,. R2, R3, R4, Q0, Q1, Q2, Q3, Q4 must be introduced to the program "art2.m". Then the outputs: $k0$, $k1$, $k2$, $k3$, $k4$ are the coefficients of the optimal regulator matrices corresponding to $W(0)$, $W(1)$, $W(2)$, $W(3)$, $W(4)$.

Examples 1.

psi0 = [0.920 -0.003 0 0;-0.003 1.00 0 0; 0.558 0.025 0.155 0.003;0.027 -0.030 0.003 0.000];

psi1 =[0.920 -0.003 0 0; -0.003 1.00 0 0;0,558 0.025 0.155 0.003; 0.027 -0.030 0.003 0.0001]

psi2 =[0.920 -0.003 0 0; -0.003 1.00 0 0; 0.558 0.025 0.155 0.003;0.027 -0.030 0.003 0.0001] psi3 =[0.920 -0.003 0 0;-0.003 1.003 0 0; 0.558 0.025 0.155 0.003; 0.027 -0.030 0.003 0.0001];

psi4 =[1.00 0 -1.00 0; 0 1.00 0 0.02; 0 0 -1.00 0; 0 0 0 0];

gam0 =[0.001 -0.000002;-0.000002 0.001;0.008 -0.00003;-0.00003 0.009];

gam1 =[0.001 -0.000002;-0.000002 0.001;0.008 -0.00003;-0.00003 0.009];

gam2 =[0.001 -0.000002;-0.000002 0.001;0.008 -0.00003;-0.00003 0.009];

gam3 =[0.001 -0.000002;-0.000002 0.001;0.008 -0.00003;-0.00003 0.009];

gam4 =[0 0 ;0 0 ;0 0; 0 0]; R0 =[0.0002 0; 0 0.0002]; R1 =[0.0002 0;0 0.0002];

R2 =[0.0002 0;0 0.0002]; R3 = [0.0002 0;0 0.0002]; R4 =[0.0002 0; 0 0.0002];

Q0 =[1000 0 0 0;0 100 0 0;0 0 0 0;0 0 0 0]; Q1 =[0 0 0 0;0 0 0 0;0 0 0 0;0 0 0 0]

Q2 =[0 0 0 0;0 0 0 0 ;0 0 0 0 ;0 0 0 0]; Q3 =[0 0 0 0;0 0 0 0;0 0 0 0; 0 0 0 0];

Q4 =[0 0 0 0;0 0 0 0;0 0 0 0; 0 0 0 0];

k0 =[0.200 9.69 0.0000006 0.000008;8.17 -1.19 -0.00002 -0.0000005];

k1 =[6.55 2.36 0.0000006 -0.000001; -5.32 -210.4 -0.000003 0.000002];

k2 =[2.11 2.32 0.000001 0.0000001; -7.25 -246.2 -0.000004 -0.000001];

k3 =[3.82 -9.64 -0.00001 -0.00001; 0.51 -356.2 0.0000002 0.00001];

k4 =[0 0 0 0;0 0 0 0].

2. Problem of reliable stabilization for periodic discrete systems

2.1. **Reliable stabilization of the periodic system with a stability tolerance.** Consider the periodic system

$$x\left(t+1\right)=A\left(t\right)x\left(t\right)+B\left(t\right)u\left(t\right),\qquad t=0,1,..., \tag{2.1.1}$$

where $x(t)$ is the phase vector, $u\left(t\right)$ is the control vector; A, B are periodic matrices (with period τ),i.e. $A\left(t+\tau\right)=A\left(t\right)$, $B\left(t+\tau\right)=B\left(t\right)$, $\forall t$. Assume, in addition to the set of matrices $A(0), A(1), ..., A(\tau-1), B(0), B(1), ..., B(\tau-1)$, we also compute the matrices $A\left(t\right)$ and $B\left(t\right)$ over one period, where we have given L sets as

$$B_i\left(0\right), B_i\left(1\right), ..., B_i\left(\tau-1\right),\ \ i=1, ..., L. \tag{2.1.2}$$

These are derived by replacing the entries of some columns in the matrices $B(0), B(1), ...,$ and $B(\tau-1)$ by zero. With the periodicity condition, the sets (2.1.2) and (2.1.1) define L-periodic systems

$$x\left(t+1\right)=A\left(t\right)x\left(t\right)+B_i\left(t\right)u\left(t\right)\ ,\ \ i=0,1,...L. \tag{2.1.3}$$

Let now the system (2.1.1) be optimized by minimizing the quadratic criterion

$$J=\sum_{t=0}^{\infty}\left(x'\left(t\right)P\left(t\right)x\left(t\right)+u'\left(t\right)R\left(t\right)u\left(t\right)\right), \tag{2.1.4}$$

where $P\left(t\right)=P'\left(t\right)\geq 0,\ \ R\left(t\right)=R'\left(t\right)\geq 0$ are periodic matrices with a period τ. The problem is to find a controller $K_o\left(t\right)$ with period τ

$$u(t)=K_o(t)x(t), \tag{2.1.5}$$

that minimizes the criterion (2.1.4) in [7] on the class of stability of the closed-loop systems (2.1.3) and (2.1.5). The regulator (2.1.5) would stabilize the system (2.1.1), if the eigenvalues of the matrix

$$\Phi = \left(A\left(\tau-1\right)+B\left(\tau-1\right)K\left(\tau-1\right)\right)....\left(A\left(0\right)+B\left(0\right)K\left(0\right)\right), \qquad (2.1.6)$$

lie inside the unit circle.

Definition 1. The regulator (2.1.5) reliably stabilizes the system (2.1.1), if it stabilize the (2.1.2) as well.

Definition 2. The regulator (2.1.5) stabilizes the system (2.1.1) with stability tolerance ρ, if it stabilizes the systems (2.1.1) and (2.1.3) with stability tolerance ρ for periodic system.

We propose the algorithm of synthesis for the reliable regulator with given (2.1.1) , tending to optimal regulator (in the sense of criteria (2.1.4)). Then the matrix $K_o\left(t\right)$ is defined by

$$K_o\left(t\right) = -\left[B'\left(t\right)S\left(t+1\right)B\left(t\right)+R\left(t\right)\right]^{-1}B\left(t\right)S\left(t+1\right)A\left(t\right), \qquad (2.1.7)$$

where the sequence of the periodic (with period τ) symmetric matrices $S(t)$ satisfies the following recurrent relation

$$\begin{aligned} S(t) = A(t)&\left[S(t+1)-S(t+1)B(t)\left(R(t)B'(t)S(t+1)B(t)\right)^{-1}S(t+1)\right]A(t)+ \\ &+P(t). \end{aligned} \qquad (2.1.8)$$

The solution of the equation (2.1.8) (i.e., to find the sequence of the matrices $S\left(1\right),...,S\left(\tau\right)$) is reduced to maximizing

$$Sp\left(S\left(1\right)+S\left(2\right)+....+S\left(\tau\right)\right), \qquad (2.1.9)$$

satisfying the matrix inequalities:

$$\begin{gathered} S\left(1\right)\geq 0,\ S\left(2\right)\geq 0,\ S\left(\tau\right)\geq 0, \\ \begin{bmatrix} A'(1)S(2)A(1)-S(1)+P(1) & A'(1)S(2)B(1) \\ B'(1)S(2)A(1) & R(1)+B'(1)S(2)B(1) \end{bmatrix}\geq 0, \\ \begin{bmatrix} A'(\tau)S(1)A(\tau)-S(\tau)+P(\tau) & A'(\tau)S(1)B(\tau) \\ B'(\tau)S(1)A(\tau) & R(\tau)+B'(\tau)S(1)B(\tau) \end{bmatrix}\geq 0. \end{gathered} \qquad (2.1.10)$$

Thus, after solving the problems (2.1.9), (2.1.10) the relation (2.1.7) defines the sequence of periodic matrices $K_o\left(t\right)$, realizing the optimal stabilization of system (2.1.1) with the criteria (2.1.4).

Examples.

Assume that the weights in (2.1.4) are unit matrices, i.e. $p=2$; $P=I$; $R=I$; to get the matrices $K_0\left(0\right)$ and $K_0\left(1\right)$ we use the MATLAB routine deqr.m. The matrices $B11$, $B12$ correspond to $B_1\left(0\right)$ and $B_1\left(1\right)$, respectively.
The parameter μ yields the stability margin $\frac{1}{\mu^2}$. Now we may determine ρ.
$K1$, $K2$ - are synthesis matrices of reliable regulators with given stability tolerance corresponding to matrices $K\left(0\right)$ and $K\left(1\right)$.
$e1$ and $e2$- characterize the closeness of the synthesis regulators $K\left(0\right), K\left(1\right)$ to the optimal regulators $K\left(0\right), K\left(1\right)$, and are given as relative errors:

$$e1 = \frac{\left\|K\left(0\right)-K_0\left(0\right)\right\|}{\left\|K_0\left(0\right)\right\|},$$

$$e2 = \frac{\left\|K\left(1\right)-K_0\left(1\right)\right\|}{\left\|K_0\left(1\right)\right\|}.$$

To solve this problem we use the following function algorithm:

[E,K1,K2,e1,e2]=intprf1(A1,A2,B1,B2,B11,B21,k1,k2,m).

Example 1, cf [7].

A1 =[1.4838 0.0002 0.0002 -0.0547 0.2022; 0 1 0.1747 0 0; 0 0 1 0 0; 0 0 0 1 0; 5.9480 0.0026 0.0026 -0.7115 1.4862];

A2 =[1.0003 0.0006 0.0006 0.0905 0.3256; 0 1 0.3253 0 0; 0 0 1 0 0; 0 0 0 1 0; 0.0018 0.0035 0.0035 0.5891 1.0018];

B1 =[-0.4066 0.3267 0.0167 -0.0023; 0 0 0 0.0147; 0 0 0 0.1687; 2.2261 2.5068 0 0.1551; -5.3757 3.8351 0.2055 -0.0516];

B2 =[-0.3299; 0; 0; 0; -1.9132];

B11 =[-0.4066 0 0.0167 -0.0023 ; 0 0 0 0.0147 ; 0 0 0 0.1687 ; 2.2261 0 0.0000 0.1551 ; -5.3757 0 0.2055 -0.0516];

B21 =[0 ; 0 ; 0 ; 0 ; 0];

k1 =[0.6038 0.0938 0.1182 -0.1490 0.1206; -0.4282 0.0966 0.1223 -0.0996 -0.1011; -17.7789 -0.1049 -0.1233 0.8524 -3.2036; 0.9377 -3.3184 -3.9446 -2.4637 0.1348];

k2 =[1.3405 0.0011 0.0007 0.1729 0.5390]; m = 0.6000;

E =[1.4336 -0.0133 + 0.0262i 0.0285 + 0.1816i; 0.0018 -0.0133 - 0.0262i 0.0285 - 0.1816i; 0.2480 + 0.3125i 0.0443 0.0214; 0.2480 - 0.3125i 0.5259 + 0.0873i 0.5441; 0.5506 0.5259 - 0.0873i 0.4132];

K1 =[1.1993 -0.0250 -0.1987 0.1401 0.2928 -0.0625 0.0137 0.0396 -0.0370 -0.0179 -19.1442 0.1721 0.8233 -0.0476 -3.5918 1.2939 -3.6439 -4.2895 -2.1456 0.2299];

K2 =[1.2286 -0.1228 -0.5609 0.7536 0.6537];

e1 = 0.1220 ; e2 = 0.5723.

2.2. **Algorithm for a reliable stabilization problem for the discrete periodic system using H2 optimization.** We describe the motion of an object by the following periodic finite-difference system, see [8].

$$x\,(i+1) = \Psi\,(i)\;\; x\,(i) + \Gamma\,(i)\;\; u\,(i)\,, \qquad x\,(0) = X_0\,, \quad i = 0,1,2,..., \tag{2.2.1}$$

with quadratic functional

$$J = \sum_{i=0}^{\infty}\left[x'\,(i)\,Q(i)\,x\,(i) + u\,(i)\,R(i)\,u\,(i)\right], \tag{2.2.2}$$

where $x\,(i)$ is n-dimensional vector of the phase coordinates, $u\,(i)$ is m-vector of the controlling influence, $\Psi\,(i)\,, \Gamma\,(i)$ are constant matrices with the period p ($p \in \mathrm{N}$).
We want to define a control

$$u(i) = K(i)x(i), \tag{2.2.3}$$

that minimizes (2.2.2) assuming that one of the components of the control action falls out:

$$Q(0) \neq 0;\; Q\,(1) = Q\,(2) = \cdots = Q\,(p-1) = 0. \tag{2.2.4}$$

For the corresponding discrete system we formulate a reliable stabilization problem as

$$y\,(i) = C\,(i)\,x\,(i) + D\,(i)\,u\,(i)\,,$$

where -$y\,(i)$ is an observed variable vector, $C\,(i)$ are $l\times n$ and $D\,(i)$ are $l\times m$ constant matrices satisfying

$$C\,(0)=C\,(1)=\cdots=C\,(p-1)=C\quad ;D\,(0)=D\,(1)=\cdots=D\,(p-1)=D.$$

Similarly

$$y(ip)=C_1x(ip)+D_1U(ip), \tag{2.2.5}$$

where $C_1\ =[C\,(0)\ \ ,\ C\,(1)\,,\,....C\,(p-1)]\ '$,

$$D_1=diag\ [D\,(0)\,,D\,(1)\,,\cdots,D\,(p-1)]\,.$$

C_1,D_1 are orthogonal matrices $C_1'\,D_1\ =\ 0,\ D_1'D_1\ >\ 0$ and $Q\,(i)\ =\ C^{'}\,(i)\ \ C\,(i);\ R\,(i)\ =\ D^{'}\,(i)\ \ D\,(i)$.

Algorithm for the optimal design: Assume that $X_0\ =E$ and a component of control actions $u\,(i)$ is missing. Following [8] we construct $R_2=\{\bar{\Gamma}_l\ ;l=0,\bar{m}\ \}$. Label $\bar{\Gamma}_0=\bar{\Gamma}$ corresponding to the case when all components of control actions are present. $\bar{\Gamma}_i$ means that in all matrices $\Gamma\,(0)\,,\Gamma\,(1)\,,...,\Gamma\,(p-1)$ in (2.2.1) only the i-th column is zero. Inserting in (2.2.1) and (2.2.2),we get:

$$x_l\,((i+1)\,p)=\bar{\Psi}x_l\,(ip)+\bar{\Gamma}_lU\,(ip)\ ,\qquad l=0,1,...,m,$$

where

$$\bar{\Psi}=\Psi\,(p-1)\,\Psi\,(p-2)\,....\Psi\,(1)\,\Psi\,(0)\,,$$
$$\bar{\Gamma}_l=[\Psi\,(p-1)\,\Psi\,(p-2)\ldots\,\Psi\,(1)\,\Gamma_l\,(0)\,;\Psi\,(p-1)\,\Psi\,(p-2)\,...\Psi\,(2)\,\Gamma_l\,(1)\,;\ldots;\Gamma_l\,(p-1)]\,,$$
$$U\,(ip)=[u'\,(ip)\,,u'\,(ip+1)\,,\ldots,u'\,((i+1)\,p-1)]'\,,$$

$$H_l(z)=\left(C_1-D_1\bar{K}_l\right)\left[zE-\left(\bar{\Psi}+\bar{\Gamma}_l\bar{K}_l\right)\right]^{-1}X_0,l=0,1,...,m,$$

$$\bar{K}_l=\left[\begin{array}{l}K(0)\\ K(1)(\Psi(0)+\Gamma_l(0)K(0))\\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ K(p-1)\prod\limits_{i=1}^{p-1}(\Psi(p-1-i)+\Gamma_l(p-1-i)K(p-1-i))\end{array}\right]. \tag{2.2.6}$$

A reliable stabilization problem corresponds to choice of matrices
$K\,(0)\,,K\,(1)\,,...,K\,(p-1)$ such that the systems

$$x_l\,((i+1)\,p)=\left(\bar{\Psi}+\bar{\Gamma}_l\overline{K}\right)x_l\,(ip)\ ,\qquad l=0,1,\ldots,m,$$

becomes asymptotically stable for any$\bar{K}$ and its minimal value is given by

$$\|\mathrm{H}_l(z)\|=Sp(C_lL_lC_l'),$$

where L_l is a solution for the discrete Lyapunov equation

$$(\bar{\Psi}+\Gamma_l\bar{K})'L_l(\bar{\Psi}+\bar{\Gamma}_l\bar{K})-L_l+\bar{\Gamma}_l'\Gamma_l=0,\qquad l=0,1,...,m. \tag{2.2.7}$$

To use an algorithm as in [6], we introduce matrices depending on a parameter $\eta\in[0\ \ 1]$.

$$\bar{\Gamma}_l\,(\eta)=(1-\eta)\,\bar{\Gamma}_0+\eta\bar{\Gamma}_l\,,\ l=1,..,m. \tag{2.2.8}$$

To process the program '**danger.m** requires to declare the initial data, i.e., the values for psi0, psi1,psi2 gam0, gam1,gam2, Cc, D. Then the outputs are:
k0p,k1p,k2p- the synthesis matrices of the reliable regulators corresponding to the matrices K(0), K(1),K(2), and d,d1,d2- the eigenvalues of each system.

Example 1.

psi0 =[0.2113 0.0087 0.4524; 0.0824 0.8096 0.8075; 0.7599 0.8474 0.4832;];

psi1 =[0.2113 0.0087 0.4524; 0.0824 0.8096 0.8075; 0.7599 0.8474 0.4832];
psi2 =[0.2113 0.0087 0.4524; 0.0824 0.8096 0.8075; 0.7599 0.8474 0.4832];
gam0 =[0.6135 0.6538; 0.2749 0.4899; 0.8807 0.7741];
gam1 =[0.6135 0.6538; 0.2749 0.4899; 0.8807 0.7741]; gam2 =[0.6135 0.6538; 0.2749 0.4899; 0.8807 0.7741];
Q0 =[1 0 0; 0 1 0; 0 0 1]; Q1 =[0 0 0; 0 0 0; 0 0 0];
Q2 =[0 0 0 ;0 0 0; 0 0 0]; R0 =[1 0; 0 1];
R1 =[1 0; 0 1]; R2 =[1 0; 0 1];
Cc =[-0.577 0 0; 0 -0.577 0; 0 0 -0.577; 0 0 0; 0 0 0; -0.577 0 0;, 0 -0.577 0; 0 0 -0.577; 0 0 0; 0 0 0; -0.577 0 0; 0 -0.577 0; 0 0 -0.577; 0 0 0; 0 0 0];
D =[0 0 0 0 0 0; 0 0 0 0 0 0; 0 0 0 0 0 0; 1 0 0 0 0 0; 0 1 0 0 0 0; 0 0 0 0 0 0; 0 0 0 0 0 0; 0 0 0 0 0 0; 0 0 1 0 0 0; 0 0 0 0 1 0 0; 0 0 0 0 0 0; 0 0 0 0 0 0; 0 0 0 0 0 0; 0 0 0 0 1 0; 0 0 0 0 0 1];
k1p =[0.192 -0.823 -0.604; -0.3448 -0.803 -1.092]
k2p =[-0.315 -0.401 -0.200; -0.288 -0.503 -0.3541]
kop =[-0.523 -0.479 -0.864; -0.551 -0.351 -0.986]
d =[0.0285 - 0.0441i; 0.0285 + 0.0441i; -0.2223] ; d1 =[-0.1203 ; -0.0464; 0.2955] ;
d2 =[0.1482; -0.0413 ; 0.0058]
H1 = 3.1669 ; H2 = 3.3281; H3 = 3.1640.

3. Inverse problem of optimization of periodic system.

3.1. Time continuous inverse problems for synthesis of optimal output variable regulators.

Assume that the movement of a system is described by

$$\dot{x} = Ax + Bu, \quad y = Cx, \quad x(0) = x_0. \tag{3.1.1}$$

Here, x, y, u are phase, observable and control vectors respectively. The matrices A, B and C are time independent. Assuming the law of feedback circuit

$$u = -Fy, \tag{3.1.2}$$

the problem is reduced to find a solution to (3.1.1) (by varying the constant matrix F in (3.1.2)) that optimizes the quality criterion:

$$J =< \int_0^\infty \left(x'Qx + u'Ru\right) dt >, \quad Q = Q' \geq 0, \quad R = R' > 0. \tag{3.1.3}$$

Assume that the initial condition of the system (3.1.1) is a random vector with the following properties:

$$< x_0 >= 0, \quad \text{and} \quad < x_0 x_0' >= S. \tag{3.1.4}$$

We formulate the problem of optimizing the system (3.1.1) as

$$\min_F \{J = Sp\,[SW]\}. \tag{3.1.5}$$

Here S is defined by (3.1.4), and the matrix W satisfies the Lyapunov equation

$$\bar{A}'W + W\bar{A} + Q + C'F'RFC = 0, \quad \bar{A} = A + BFC. \tag{3.1.6}$$

The gradient G of functional (3.1.3) with respect to matrix $F$is given by

$$G = 2\Omega TC', \quad \Omega = RFC + B'W, \tag{3.1.7}$$

$$\bar{A}T + TA' + S = 0. \tag{3.1.8}$$

Setting the gradient to zero (optimality condition), we obtain:

$$F = -R^{-1}B'WTC'\left(CTC'\right)^{-1}. \tag{3.1.9}$$

To formulate different settings of inverse LQ-problems, one should consider possible cases when the gradient G vanishes.

1. $\Omega \neq 0$, $TC' \neq 0$, $G = 0$.
2. $\Omega = 0$, $TC' \neq 0$.
3. $\Omega \neq 0$, $TC' = 0$.

Assume that $\bar{A} = A + BFC$ is Hurwitz matrix. We get two settings cases.

1a. Assume that A, B, C , F, and S defined by (3.1.4) are given, see [9]. The problem is to establish the existence and then to find matrices $Q \geq 0$ and $R > 0$ such that the gradient G vanishes. This problem is solved by the algorithm

$$[Q, R] \ = \ natan1a\,(A, B, C, F, S)\,.$$

Example 1, cf [9].

A =[0 1; -1 0]; B =[0; 1]; F = -0.8165;
Q =[0.4262 0.0314; 0.0314 0.3825]; R = 1; KK = -0.8165;
N6 = 1.2238e-015.

Example 2, cf [9].

A =[-0.0366 0.0271 0.0188 -0.4555; 0.0482 -1.0100 0.0024 -4.0208; 0.1002 0.3681 -0.7070 1.4200 0 0 1.0000 0]

B =[0.4422 0.1761; 3.5446 -7.5922; -5.5200 4.4900 0 0]

F =[-1.6278; 6.5100]; C = [0 1 ;0 0]

Q =[0.1737 0.4647 -0.1136 0.0405; 0.4647 100.3649 -2.0264 9.1228; -0.1136 -2.0264 0.6249 -0.0911; 0.0405 9.1228 -0.0911 1.3315]

R =[0.9535 0.1052; 0.1052 0.7623]; KK=[-1.6278; 6.5100]; N6 =[5.1006e-015].

1b. In addition to A, B, C and F, we also assume that the matrices $Q \geq 0$ and $R > 0$ are given. Then the problem is to establish the existence and then find the matrices S and T such that the gradient G vanishes. The algorithm is:

[S,T]=natan1b(A,B,C,F,Q,R).

Example 1, cf [8].

A =[0 1;-1 0]; B =[0; 1]; C =[0 1]; R =1; Q =[1 0; 0 0];
F =-1; T =[0.0015 0.0005; 0.0005 0.0010]; S =[-0.0010 0.0009; 0.0009 0.0030].

2. Given A, B, C and F the problem is to establish the existence and then to find the matrices $Q \geq 0$ and $R > 0$ and KK such that the gradient G becomes zero. The function algorithm for this case is given by

[Q,R,KK]=natan2(A,B,C,F).

Example 1, [8].

A =[0 1; -1 0]; B =[0; 1]; F =[-1.6278; 6.5100]; C = [0 1 ; 0 0]

R = 1; Q =[0.0000 0.0000; 0.0000 0.6667]; W =[0.8165 0.0000; 0.0000 0.8165];

d = [0.4082 + 0.9129i 0; 0 0.4082 - 0.9129i]; KK = 0.8165.

Example 2, [9].

A =[-0.0366 0.0271 0.0188 -0.4555; 0.0482 -1.0100 0.0024 -4.0208; 0.1002 0.3681 -0.7070 1.4200 0 0 1.0000 0]

B =[0.4422 0.1761; 3.5446 -7.5922; -5.5200 4.4900; 0 0]; F =[-1.6278; 6.5100]; C =[0 1 0 0];

W =[0.0034 -0.0006 -0.0009 -0.0009; -0.0006 0.0124 0.0026 0.0014 -0.0009 0.0026 0.0038 0.0020 -0.0009 0.0014 0.0020 0.0063];

Q =1.0e-007 *[0.0005 -0.0017 -0.0004 0.0001; -0.0017 0.6556 0.0032 0.0489 -0.0004 0.0032 0.0015 -0.0000 0.0001 0.0489 -0.0000 0.0048];

R = [0.9412 0.2353; 0.2353 0.0588]; KK =[1.6885;-6.7527].

3. Assuming that the matrices A, B, C, F, $Q \geq 0$ and $R > 0$ are known. The problem is to establish the existence and then find the matrices $S \geq 0$ and N such that the gradient G vanishes. To solve this problem we use the function algorithm:

[S,N]=natan3(A,B,C,F,Q,R).

Example 1.

A =[0 1; -1 0]; B =[0; 1]; C = [0 1]; Q =[1 0; 0 0]; R = 1;

S = [0 -1; -1 0]; N = [-1; 0].

3.2. **Inverse problems for stationary discrete time systems.** Analogous reasoning may be applied to the case of a stationary system with discrete time

$$x\,(i+1) = Ax\,(i) + Bu\,(i) \quad , \quad i = 0,1,2,... \quad y(i) = Cx(i), \quad x\,(0) = x_0. \tag{3.2.1}$$

Assuming the law of the feedback circuit in the form

$$u\,(i) = -Fy\,(i)\,, \tag{3.2.2}$$

one needs to find a matrix F stabilizing the system (3.2.1), i.e. a matrix such that all eigenvalues of $(A + BFC)$ are inside the unit circle and in addition the functional

$$J = \sum_{i=0}^{\infty} x'\,(i)\,Qx\,(i) + u'\,(i)\,Ru\,(i)\,, \tag{3.2.3}$$

is minimal. Here Q and R are constant matrices as in [8].

In what follows, we assume that the matrices A, B, C and F are known. In addition, we assume $\bar{A} = A + BFC$ is a Hurwitz matrix. We begin with the first case. Here, we have two possible settings: We formulate the problem of optimizing system (3.2.1) -(3.2.3) as

$$\min_F \{J = Sp\,(SL)\}\,, \tag{3.2.4}$$

here, the matrix S is defined by

$$< x_o\, x'_o >= S, \tag{3.2.5}$$

and the matrix L is defined by the solution of the discrete Lyapunov equation:

$$\bar{A}'L\bar{A} + Q + C'F'RFC = L\,, \quad \bar{A} = A - BFC. \tag{3.2.6}$$

The gradient of functional (3.2.3) with respect to the matrix of the feedback circuit is given by the relation:

$$\frac{\partial J}{\partial F} = 2\left[B'L\bar{A} + RFC\right]P_xC'. \tag{3.2.7}$$

Let $\Omega = B'L\bar{A} + RFC$ then

$$\frac{\partial J}{\partial F} = 2\Omega P_xC', \tag{3.2.8}$$

where P_x is the solution to the Lyapunov equation

$$\bar{A}P_x\bar{A}' + S = P_x. \tag{3.2.9}$$

Setting the gradient to zero, the optimal solution gives the following expression for F:

$$F = \left(R + B'LB\right)^{-1} B'LAP_xC'\left(CP_xC'\right)^{-1}. \tag{3.2.10}$$

As in the continuous case, we consider all cases when the gradient (3.2.8) vanishes:

1. $\Omega \neq 0,\ \ P_xC' \neq 0,\ \ \frac{\partial J}{\partial F} = 0.$
2. $\Omega = 0,\ \ P_xC' \neq 0.$
3. $\Omega \neq 0,\ \ P_xC' = 0.$

Below, we formulate various statements of the inverse problem:

1a. Assume that the matrices A, B, C and F, are given and the matrix S defined by (3.2.5) as in [8]. The problem is then to establish the existence and then to find matrices $Q \geq 0$ and $R > 0$ such that the gradient G vanishes, cf [8]. This problem is solved using:

[Q,R,KK,N6]=adisk1a(A,B,C,F).

The data matrices A, B, C, F are appearing in (3.2.1) and (3.2.2). The result of solution Q, R correspond to the matrices appeared in (3.2.3).

Example 1.

A =[2.0000 1.0000 0; 0 -0.1000 1.0000; 0 0 3.0000];

B =[1 0; 0 0; 0 1]; C =[1 0 0; 0 0 1];

F =[-1.9000 -0.1300; -0.0008 -2.9000];R =[0.9998 -0.0093; -0.0093 0.6398]; Q =[5.9552 1.1396 -0.2945; 1.1396 4.6075 -0.0748; -0.2945 -0.0748 5.5623];

KK =[-1.9000 -0.1300; -0.0008 -2.9000]; N6 =4.2197e-016.

1b. Assume that the matrices A, B, C, F, $Q \geq 0$ and $R > 0$ are known. The problem is to establish the existence and then to find the matrix S satisfying (3.2.5) such that the gradient of functional (3.2.3) equals zero. The solution is obtained using the function

[S,G]=adisk1b(A,B,C,F,Q,R).

Example 2.

A =[2.0000 1.0000 0; 0 -0.1000 1.0000; 0 0 3.0000]; B =[1 0; 0 0; 0 1];

C =[1 0 0; 0 0 1]; F = [1.9000 0.1300; 0.0008 2.9000]; Q =[10 0 0; 0 10 0; 0 0 10];

R = [1 0; 0 1]; S =[4.5524 -1.1051 0.1442; -1.1051 0.4476 -0.0348; 0.1442 -0.0348 0.0047];

G = [0.0000 0.0001; -0.0001 0.0021].

2. Given the matrices A, B, C and F. The problem is to establish the existence and then find matrices $Q \geq 0$, $R > 0$ and KK such that the gradient G vanishes. The function below is used to solve this problem:

[Q,R,KK]=adisk2(A,B,C,F).

The results of the program adisk2 are displayed as in the following example

A =[2.0000 1.0000 0; 0 -0.1000 1.0000; 0 0 3.0000]; B =[1 0; 0 0; 0 1];

C =[1 0 0; 0 0 1]; F = [-1.9000 -0.1300; -0.0008 -2.9000]; KK =[1.9309 0.1786; 0.0007 2.9000];

G =[-0.0001 0.0019 0.0002; -0.0009 -0.0000 -0.0000]; N6 = 2.0035;

Q =[0.0019 -0.0330 0.0593; -0.0330 12.6696 1.2193 0.0593 1.2193 8.4182]; R= [0.0005 -0.0119; -0.0119 0.9999]

KK =[1.9309 0.1786; 0.0007 2.9000].

3. The matrices A, B, C, F, $Q \geq 0$ and $R > 0$ are assumed to be known. The problem is to establish the existence and then to find $S \geq 0$, and P_x such that the gradient G vanishes. This is performed through the function

[S,P_x]=adisk3(A,B,C,F,Q,R).

Example3

A =[2.0000 1.0000 0; 0 -0.1000 1.0000; 0 0 3.0000]
B =[1 0; 0 0; 0 1]
C =[1 0 0; 0 0 1] F =[1.9000 0.1300; 0.0008 2.9000]

Q =[10 0 0 ; 0 10 0; 0 0 10]; R =[1 0; 0 1]

Px =[0 0 0 ; 0 1 0; 0 0 0] ; S =[-1.0000 0.1000 0 ;0.1000 0.9900 0; 0 0 0].

3.3. **Inverse problem of optimization of periodic system.** The main purpose of this problem is to describe the motion of an object by the periodic system of the finite difference equations:

$$x\left(i+1\right)=A\left(i\right)x\left(i\right)+B\left(i\right)u\left(i\right),\ \ i=0,1,2,...,\ \ x\left(0\right)=x_0. \tag{3.3.1}$$

Assume that $x\left(0\right)\neq 0$, find a suitable control matrices $K(i)$ in regulator equation

$$u\left(i\right)=K\left(i\right)x\left(i\right), \tag{3.3.2}$$

that minimize the quadratic performance index

$$I=\sum_{i=0}^{\infty} x'\left(i\right)Q\left(i\right)x\left(i\right)+u'\left(i\right)R\left(i\right)u\left(i\right), \tag{3.3.3}$$

on the class of asymptotically stable system (3.3.1)-(3.3.2) $\left(\lim_{i\to\infty} x\left(i\right)=0\right)$. Here $x\left(i\right), u\left(i\right)$ are vectors of phase coordinate and controlling influence, respectively, $A(i), B(i), Q(i)=Q'(i)\geq 0, R(i)=R'(i)\geq 0$ are periodic matrices with period p, i.e. $A\left(i+p\right)=A\left(i\right)$, etc. cf. [10].
It is known, that the optimal equation (3.3.2) have the control matrix:

$$K\left(i\right)=-\left[B'\left(i\right)S\left(i+1\right)B\left(i\right)+R\left(i\right)\right]^{-1}B\left(i\right)S\left(i+1\right)A\left(i\right), \tag{3.3.4}$$

where $S(i)$ satisfies the recurrent relation.

$$S(i) = A'(i)\left[S(i+1) - S(i+1)B(i)\left(B(i) + B'(i)S(i+1)B(i)\right)^{-1}B'(i)S(i+1)\right]\times \\ \times A(i) + Q(i), \tag{3.3.5}$$

$$S\left(i+p\right) = S\left(i\right). \tag{3.3.6}$$

Algorithm for solving inverse problem: Given the periodic (with period p) sequence matrices $A\left(i\right), B\left(i\right), K\left(i\right)$, we need to define the matrices $Q(i), (R(i))$, and $S(i)$ stabilizing (3.3.5),(3.3.6) viz the control matrix $K\left(i\right)$ given by (3.3.4). Here we used the method of LMI for solving the inverse problem.
The initial data $A1,\ A2,\ B1,\ B2$ correspond to the matrices $A\left(0\right),\ A\left(1\right),\ B\left(0\right),\ B\left(1\right).$
$k1,\ k2$ correspond to the matrices $K_0(0)$ and $K_0\left(1\right)$.
The output $Q1, Q2, R1, R2$ correspond to the matrices $Q\left(0\right), Q\left(1\right), R\left(0\right), R\left(1\right)$.
The problem is solved employing the algorithm function

[Q1,Q2,R1,R2]=invpdf(A1,A2,B1,B2,K1,K2).

Examples on performance of the program invpdf:

A1 =[1.483 0.0002 0.0002 -0.0546 0.2022; 0 1.000 0.1746 0 0; 0 0 1.000 0 0; 0.0000 0.0000 0.0000 1.000 0.0000; 5.9479 0.0025 0.002 -0.711 1.486]

A2 =[1.0002 0.00057 0.0005 0.0905 0.3256; 0 1.000 0.325 0 0; 0 0 1.000 0 0; 0 0 0 1.000 0; 0.0017 0.0035 0.0035 0.58909 1.001];

B1 =[-0.4066 0.3266 0.0167 -0.00228 ; 0 0 0 0.0147 ; 0 0 0 0.1686 ; 2.2260 2.506 0.0000 0.155; -5.3757 3.835 0.205 -0.0515]

B2 =[-0.32987; 0; 0; 0; -1.9131]

K1 =[0.6037 0.0937 0.1182 -0.1490 0.1205 ; -0.4281 0.0965 0.1223 -0.0995 -0.1010 ; -17.7788 -0.1049 -0.1233 0.8523 -3.2036 ; 0.9376 -3.3183 -3.9446 -2.46367 0.1348]

K2 =[1.34054 0.0010 0.0007 0.1728 0.53896]

Q1 =[12.3826 -1.2191 -1.3656 1.0742 -2.1500; -1.2191 -0.5593 -5.6275 -2.5678 0.0220; -1.3656 -5.6275 15.2250 4.6141 -0.0021; 1.0742 -2.5678 4.6141 14.1180 0.0764; -2.1500 0.0220 -0.0021 0.0764 0.3192];

Q2 =[17.5962 1.5840 1.9366 -2.9796 4.0085; 1.5840 1.5862 4.9198 1.3909 -0.0667; 1.9366 4.9198 -11.8186 -4.6242 -0.0283; -2.9796 1.3909 -4.6242 -11.7295 -0.1665; 4.0085 -0.0667 -0.0283 -0.1665 1.5158];

R1 =[11.2647 11.2088 -0.0249 -0.0057; 11.2088 12.3187 -0.0137 -0.0086; -0.0249 -0.0137 0.0008 -0.0000; -0.0057 -0.0086 -0.0000 0.0001];

R2 =1.

Command References

In this part we summarize the algorithms with their corresponding examples cited in the text.

art2.m Algorithm to the synthesis of periodic discrete optimal system stabilization with respect to output variable. See examples on section 1.2.

adisk1a.m Algorithm for inverse problems with discrete time in the stationary system. See examples on section 3.2.

[Q,R,KK,N6]=adisk1a (A,B,C,F)

adisk1b.m Algorithm for inverse problems with discrete time of a stationary system. See examples on section 3.2.

[S,G]=adisk1b (A,B,C,F,Q,R)

adisk2.m Algorithm for inverse problems with discrete time for the stationary system. See examples on section 3.2.

[Q,R,KK]=adisk2 (A,B,C,F)

danger.m The calculation algorithm to the solution of the reliable stabilization problem for the discrete periodic system uses H_2 optimization . See example on section 2.2.

intprf1.m Algorithm for problem of reliable stabilization of a system with a given margin stability tolerance. See examples on section 2.1.

[E,K1,K2,e1,e2]=intprf1(A1,A2,B1,B2,B11,B21,k1,k2,m).

Invpdf.m The optimization procedure invpdf was used without calculation of gradient of the functionals. See examples on section 2.1

[Q1,Q2,R1,R2]=invpdf(A1,A2,B1,B2,K1,K2).

natan1a.m Algorithm for inverse problems for stationary continuous time system. See example on section 3.1.

[Q,R]=natan1a (A,B,C,F,S)

natan1b.m Algorithm for inverse problems for stationary continuous time system. See example on section 3.1.

[S,T]=natan1b (A,B,C,F,Q,R)

natan2.m Algorithm for inverse problems for stationary continuous time system: See examples on section 3.1.

[Q,R,KK]=natan2 (A,B,C,F)

plqdc.m Algorithm for the problem of stabilization periodic systems by static output feedback. See example on section 1.1.

The purpose of this documentation is to give a survey on MATLAB codes generated checking the results of papers, published by the financial support of INTAS project Ref. Nr: 04-77-6902.

Due to the license of the used packages, all codes will be electronically available through a link from the coordinator's web-site: "http://www.math.chalmers.se/˜ mohammad"

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