CHALMERS | GÖTEBORG UNIVERSITY

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Workshop: Algebraic Methods in Functional Analysis

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Workshop

Algebraic Methods in Functional Analysis

List of Participants, Schedule and Abstracts of Talks

Mathematical Sciences,
Chalmers University of Technology and
Göteborg University

Workshop

Algebraic Methods in Functional Analysis

Mathematical Sciences,
Chalmers University of Technology and
Göteborg University
Gothenburg, SWEDEN
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Organizers:

- **Volodymyr Mazorchuk**, Department of Mathematics, Uppsala University;
- **Lyudmila Turowska**, Mathematical Sciences, Chalmers University of Technology and Göteborg University

List of Participants:

- **Eshaghi Gorji Majid**, Semnan University, Semnan, IRAN;
- **Iusenko Kostyantyn**, Institute of Mathematics, Kyiv, UKRAINE;
- **Juschenko Kate**, Chalmers University Technology and Göteborg University, SWEDEN;
- **Lattarulo Michele**, Universitá di Genova, ITALY;
- **Levene Rupert**, Queen's University, Belfast, UK;
- **Ludwig Jean**, Metz University, FRANCE;
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- **Nest Ryszard**, Copenhagen University, DENMARK;
- **Neshveyev Sergey**, Oslo University, NORWAY
- **Ortega Eduard**, Universitat Autónoma de Barcelona, SPAIN;
- **Irina Peterburgsky**, Suffolk University, Boston, US
- **Popovych Stanislav**, Chalmers University Technology and Göteborg University, SWEDEN;
- **Proskurin Daniel**, Kyiv Taras Shevchenko University, UKRAINE;
- **Rørdam Mikael**, University of Southern Denmark, Odense, DENMARK;
- **Savchuk Yurii**, Max Planck Institute für Mathematik in den Naturwissenschaften; GERMANY;
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- **Vinh Le Anh** Harvard University, US
- **Zhang Genkai**, Chalmers University Technology and Gteborg University, SWEDEN;

Schedule of Talks

Saturday, June 16-th, 2007.

10.00-10.50	Sergey Neshveev	Dirac operators on compact quantum groups
11.00-11.30	Coffee/Tea Break	
11.30-11.55	Stanislav Popovych	Matrix Ordered Operator Algebras
12.00-14.00	Lunch	
14.00-14.50	Mikael Rørdam	On the structure of $C(X)$-algebras
15.00-15.25	Ivan Todorov	Operator ranges and $C*$-algebras
15.30-16.00	Coffee/Tea Break	
16.00-16.30	Yurii Savchuk	Non-commutative analogues of 17th Hilbert problem.
16.30-17.30	Ryszard Nest	Existence and classification of deformations of gerbes.
19.00	Conference Dinner	Vivaldi Restaurang, Berzeliigatan 19

Sunday, June 17-th, 2007.

09.30-10.20	Martin Mathieu	The structure of Lie derivations
10.30-10.55	Majid Eshaghi Gorji	N-ideal amenability of Banach algebras
11.00-11.30	Coffee/Tea Break	
11.30-11.55	Kate Juschenko/ Lyudmila Turowska	Operator multipliers
12.00-12.50	Jean Ludwig	Simple modules of some group algebras

Abstracts

N-ideal amenability of Banach algebras

Majid Eshaghi Gorji
Semnan University, Iran

We introduce two notions of amenability for a Banach algebra $\mathcal{A}$. Let $n \in \mathbb{N}$ and let I be a closed two-sided ideal in $\mathcal{A}$, $\mathcal{A}$ is $n-I-$weakly amenable if the first cohomology group of $\mathcal{A}$ with coefficients in the n-th dual space $I^{(n)}$ is zero; i.e., $H^1(\mathcal{A}, I^{(n)}) = \{0\}$. Further, $\mathcal{A}$ is n-ideally amenable if $\mathcal{A}$ is $n-I-$weakly amenable for every closed two-sided ideal I in $\mathcal{A}$. We study the n-ideal amenability of some classes of Banach algebras. We show that $B(H)$ is n-ideally amenable for every $n \in \mathbb{N}$ and for every Hilbert space H. We show that every C^*-algebra is n-ideally amenable for $n = 2k + 1$. We study the n-ideal amenability of commutative Banach algebras."

Simple modules of some group algebras

Jean Ludwig
Metz University, France

We determine the simple modules of the L^1-algebras of Heisenberg's and Boidol's group and of $Sl(2, \mathbb{R})$. We show that up to equivalence these modules are the finite rank submodules of the L^p principal series (and of the discrete series representations in the case of $SL_2(\mathbb{R})$). The same result holds for every exponential Lie group.

We derive necessary and sufficient conditions for an ambiskew polynomial ring to have a Hopf algebra structure of a certain type. This construction generalizes many known Hopf algebras, for example $U(sl_2)$, $U_q(sl_2)$ and the enveloping algebra of the 3-dimensional Heisenberg Lie algebra. In a torsion-free case we describe the finite-dimensional simple modules, in particular their dimensions and prove a Clebsch-Gordan decomposition theorem for the tensor product of two simple modules. We construct a Casimir type operator and prove that any finite-dimensional weight module is semisimple.

The structure of Lie derivations

Martin Mathieu
Queen's University, Belfast, UK

The structure of Lie derivations on a C^*-algebra has a surprisingly simple pattern. Yet, to establish this result, which was completed in joint work with Armando Villena (Granada), a substantial amount of theory had to be developed. In fact, this was one of the main motivations to investigate local multipliers of C^*-algebras in our monograph with Pere Ara (Barcelona). The approach is essentially algebraic with a few analytic tweaks; these, however, turned out to be fairly tricky at times. In our talk we plan to discuss this work in some detail with the overall theme of the workshop in mind.

Dirac operators on compact quantum groups

Sergey Neshveev
Oslo University, Norway

For the q-deformation G_q, $0 < q < 1$, of any simply connected simple compact Lie group G we construct an equivariant spectral triple which is an isospectral deformation of that defined by the Dirac operator D on G. The construction depends on the choice of a twist, which can be thought of as a 2-cochain on the dual discrete quantum group $\hat{G}$. It turns out, the key properties of our quantum Dirac operators depend not on the twist but on the associator, that is, the corresponding coboundary on $\hat{G}$. What allows us to say something nontrivial about the quantum Dirac operators, is that by results of Drinfeld and Kazhdan-Lusztig we can always find a twist such that the corresponding associator is determined by the monodromy of a system of partial differential equations. (Joint work with Lars Tuset.)

Existence and classification of deformations of gerbes.

Ryszard Nest
Copenhagen University, Denmark

In this talk we will study deformation quantization of gerbes. After basic definitions we will interpret deformations of a stack as Maurer-Cartan elements of a differential graded Lie algebra and classify deformations of a given gerbe in terms of Maurer-Cartan elements of the DGLA of Hochschild cochains twisted by the cohomology class of the gerbe. In particular we will get a classification of all deformations of a given gerbe on a symplectic manifold.

Operators on Spaces of Abstract Valued Functions and Their Norms. Some Applications.

Irina Peterburgsky
Suffolk University, Boston, US

We proved that under certain conditions norms of linear operators over corresponding classes of scalar valued and Hilbert or Banach space valued functions coincide. Various applications of this general fact were found.

In particular, extremal problems for norms of linear operators over spaces of analytic functions in several variables were studied. Using our technique, we generalized E. Landau coefficient problem for a case of Hilbert or Banach codomain space.

Matrix Ordered Operator Algebras

Ekaterina Juschenko, Stanislav Popovych
Chalmers University of Technology and Göteborg University

We present a characterization of C^*-representability of an arbitrary $*$-algebra in terms of algebraically admissible cones. It is analogues to Choi and Effros characterization of abstract operator systems. Then we discuss a question when for a given $*$-algebra $\mathcal{A}$ a sequence of cones $C_n \in M_n(\mathcal{A})$ can be realized as cones of positive operators in a faithful $*$-representation of $\mathcal{A}$ on a Hilbert space. As an application of the above results we present a characterization of operator algebras which are completely boundedly isomorphic to C^*-algebras. Some connections with Kadison's Similarity problem will be discussed.

References

[1] M.D. Choi, E.G. Effros, Injectivity and operator spaces. *J. Functional Analysis* 24 (1977), no. 2, 156–209.

[2] E. Juschenko, S. Popovych, Matrix Ordered Operator Algebras., *Chalmers & Göteborg University math. preprint* 2007:9.

[3] S. Popovych, On O^*-representability and C^*-representability of $*$-algebras. *Chalmers & Göteborg University math. preprint* 2006:35.

On the structure of $C(X)$-algebras

Mikael Rørdam
University of Southern Denmark, Odense

$C(X)$-algebras form a special class of non-simple C-algebras that extends the class of continuous field C-algebras. A $C(X)$-algebra is "assembled" over a compact Hausdorff space X from fibre algebras (one for each point in the space X). First, we shall spend some time giving the appropriate definitions and looking at examples. Then we shall address a number of recent results that describe when given properties (eg. stability, tensorially absorbing certain C-algebras, being properly infinite) of the fibre algebra

pass to the $C(X)$-algebra itself. In many cases one has nice results when the space X has finite dimension, and "counterexamples" when the space X has infinite dimension.

Non-commutative analogues of 17th Hilbert problem

Yurii Savchuk,
Max Planck Institute für Mathematik in den Naturwissenschaften, GERMANY

A self-adjoint element c of a $*$-algebra A is called positive if $\pi(c)$ is positive operator for all "good" $*$-representations of A. For some $*$-algebras we prove the Positivstellensatz: for every positive element c there exist elements $x \neq 0, x_1, \dots, x_n$ such that $x^*cx = x_1^*x_1 + \cdots + x_n^*x_n$. The Positivstellensatz for algebra of polynomials $\mathbb{C}[t_1, \dots, t_k]$ is the Artin's solution to 17th. Hilbert problem.

Operator ranges and C*-algebras

Ivan G. Todorov
Queen's University, Belfast, UK

The collection of all ranges of operators on a Hilbert space or, more generally, in a given von Neumann algebra, is a lattice with respect to intersection and (non-closed) linear span. This property does not hold for arbitrary C*-algebras of operators. Moreover, given a C*-algebra $\mathcal{A}$ and a faithful representation π of $\mathcal{A}$, whether or not $\pi(\mathcal{A})$ possesses this property depends on π. Say that π possesses property L if the operator ranges from $\pi(\mathcal{A})$ form a lattice under the above operations.

The talk will be concerned with the study of the above property. In particular, property L will be related to the so called "directed set property" of a C*-algebra $\mathcal{A}$, namely the property that the set of all finite dimensional C*-subalgebras of $\mathcal{A}$ be directed by inclusion. A theorem on the structure of the collection of all representations of $\mathcal{A}$ possessing L will be discussed.

The talk will be based on a joint work with M. Anoussis (Samos) and A. Katavolos (Athens)

Operator multipliers

Kate Juschenko/Lyudmila Turowska
Chalmers University of Technology and Göteborg University, Sweden

Operator multipliers were recently introduced by Kissin ad Shulman as a non-commutative version of Schur multipliers. They are elements of the minimal tensor product of two C^*-algebras satisfying certain boundedness conditions.

In this talk we will descuss certain universal operator multipliers. We establish a non-commutative version of the characterisations by Grothendieck and Peller which shows that universal operator multipliers can be obtained as certain weak limit of elements of the algebraic tensor product of the corresponding C^*-algebras.

The talk will be based on a joint work with Ivan Todorov (Belfast).