Thesis for the degree of Licentiate of Philosophy

Mathematical Models in Municipal Solid Waste Management

Michael K. Nganda

CHALMERS | GÖTEBORG UNIVERSITY

Department of Mathematical Sciences
Chalmers University of Technology and Göteborg University
Göteborg, Sweden

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Michael K. Nganda
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Department of Mathematical Sciences
Chalmers University of Technology and Göteborg University
SE-412 96 Göteborg, Sweden
Telephone: +46 (0)31 772 1000

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Abstract

Two mathematical models developed as tools for solid waste planners in decisions concerning the overall management of solid waste in a municipality are described. The models have respectively been formulated as integer and mixed integer linear programming problems. The choice between the two models from the practical point of view depends on the user and the technology used. One user may prefer to measure the transportation costs in terms of costs per trip made from the waste source, in which case the first model is more appropriate. In this case we replace the coefficients of the decision variables in the objective function with the total cost per trip from the waste collection point. At the same time, instead of measuring the amount of waste using the number of trucks used multiplied by their capacities, continuous variables can be introduced to measure directly the amount of waste that goes to the plants and landfills. The integer linear problem is then transformed into a mixed integer problem that gives better total cost estimates and more precise waste amount measurements, but measuring transportation costs in terms of costs per trip. For instance, at the moment the first model is more relevant to the Ugandan situation, where the technology to measure waste as it is carried away from the waste sources is not available. Another user may prefer to measure the transportation costs in terms of costs per unit mass of waste picked from the waste source, in which case the second model is more appropriate. The models allow to plan the optimal number of landfills and the treatment plants, and to determine the optimal quantities and type of waste that has to be sent to treatment plants, to landfills and to recycling. It is also possible to determine the number and the type of trucks, as well as the number and the type of replacement trucks and their depots. In either model there is one linear objective and linear constraints that cover waste flows among the sources-plants-landfills, capacity, site selection, environmental, and facility availability. The objective function in either model describes tipping fees, total investment and maintenance costs, costs for buying/hiring trucks, transportation costs as well as operational costs from the use of replacement trucks. The benefits from refused derived fuel, energy generation, compost, and recycling are also incorporated in the objective function. Validity and robustness tests conducted on the models, and applied on a hypothetical case study are promising; the models can be used in important tools for planners in municipal solid waste management in an urban environment. The models may as well be adapted for use in other areas of application like industrial warehouse location and product distributions among industry agents.

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1. MSW = municipal solid waste
2. RDF = refuse derived fuel
3. SWM = solid waste management
4. SOM = stabilized organic material

1 Introduction

1.1 Background to the study

The protection of the environment and natural resources is increasingly becoming very important through environmentally sustainable waste management programs. It is necessary to follow, on the part of waste managers, a sustainable approach to waste management and to integrate strategies that will produce the best practicable option. This is a very challenging task since it involves taking into account economic, technical, regulatory (normative), and environmental issues (Costi et al [18]). Waste management can become more complex if social and political considerations are also taken into account.

Municipal solid waste (MSW) management involves the collection of waste from its sources and the transportation of waste to processing plants where it can either be converted into fuel (refuse derived fuel), electrical energy, compost (stabilised organic material) or recycled for reuse. The unrecoverable waste can either be transported directly from the waste sources to landfills or from treatment plants to landfills. A careful planning is required in order to execute these activities in an optimal way. Municipal solid waste has several sources such as residential areas, commercial areas, institutional environments, construction and demolition areas, municipal services, etc. (Badran and El-Haggar [3]).

1.2 Statement of the Problem

Kampala, the capital city of Uganda, has a population of more than one million people, and it is estimated that more than one thousand tons of MSW is generated per day. About half of the waste generated is collected and disposed of at the only landfill at Kiteezi. Limited treatment is done at the landfill where some organic material is converted into compost; this is done in order to save the water streams near the landfill. A limited amount of waste is picked by individuals that sell it to some industries for reuse as raw material; this covers plastic bottles, tins and other metallic objects. A limited amount of organic material is also picked by individuals as animal feeds to cattle, pigs and dogs. Since less than half of the waste is collected and the waste is in the open, much of it litters the city whenever the wind blows and whenever it rains. This explains the incidence of the annual cholera outbreaks during the rainy seasons and the terrible stench from the city areas where the waste accumulated has decayed.

Until recently, the City Council of Kampala has been the sole body dealing with waste management in the entire city. Privatization has now taken place and Kampala City Council plays the overall supervisory role of making sure that the private companies follow the agreements made with it. The city is currently composed of six divisions and there may be more than one company in a single division. These companies are still in their infant stage and they currently collect and transport MSW to the single landfill at Kiteezi. Optimization of solid waste management based on operations research techniques has not yet been applied by any of the private companies. At the moment none of the companies treats waste and the decisions are basically based on intuition and experience. There is an urgent need to utilize scientific techniques as decision support tools in order to provide a healthy environment to all city dwellers and to optimally use the available resources in the day to day management of waste.

1.3 Objectives of the Study

The aim of this work is to present a detailed description of the mathematical models that can be used as tools for decision makers of a municipality in the day to day planning and management of integrated programs of solid waste collection, incineration, recycling, treatment, and disposal. The models can as well be used as design tools for the plants, landfills, and truck depots, in addition to the day to day planning of municipal solid waste. The main focus of the models, whose structures are described in this thesis, is to plan the MSW management, by defining the refuse flows that have to be sent to recycling or to different treatment plants or landfills, suggesting the number, the types, and the selection of plants or landfills that have to remain active at minimum total cost. Several treatment plants and facilities will be included within the desired MSW mathematical model: trucks for the transportation of waste, plants for recycling, production of refuse derived fuel (RDF), and treatment of organic material; incinerators with energy recovery; sanitary landfills; standby trucks and their depots.

The application of the models may take some time as it may involve testing the models, and the companies managing waste may need some time to implement the proposals in the models like setting up plants. The proposed models may as well undergo some modifications since almost all companies managing waste in the city have small financial budgets. What is likely to emerge with time are companies that only collect and transport waste, companies that only treat waste, and companies that only manage landfills. The models will however be a good starting point towards a sustainable municipal solid waste management and an understanding of integral waste management as a way of shaping the direction of municipal solid waste management in Ugandan cities or towns.

Figure 1 shows a compact representation of the key components of the models where the nodes stand for waste source locations (collection points), sanitary landfills, processing plants, and replacement trucks depot locations. The arrows from the truck depots node to the other three nodes indicate the flow of replacement trucks to those nodes. The arrows among waste sources, landfills, and processing plants nodes indicate the flow of waste among these nodes.

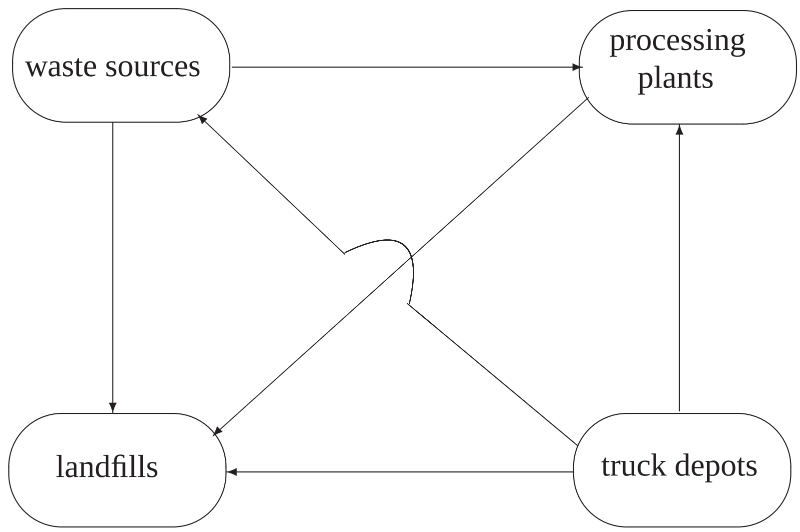

Figure 1: A compact representation of key components in a decision support mathematical model.

1.4 Justification

Waste management is very important for every country since it directly affects the health of her people and their environment. For example in Uganda cholera outbreaks are common in congested areas, especially during the rainy season. It is imperative that efficient municipal solid waste management methods are put in place. Municipal solid waste also serves as an ambit for disease vectors like rodents. Eutrophication, the increased presence of nutrients and its consequence has been one of the most serious lake water quality problems over the last decades. By allowing the rotting municipal waste to enter water channels to our rivers and lakes, we risk losing our water sources and fish because a fertile ground for water hyacinth and other water plants is generated. This too impairs the health and the economic power of the state - wetlands that are so important for a healthy environment are also affected by such waste. Noxious gases from rotting garbage also end up in the atmosphere and can be deadly to human, animal, and plant life. With population growth the land for waste disposal and agricultural production becomes scarce, and since it takes long to reclaim land that has once been used for waste disposal, it becomes crucial to put in place mechanisms for reducing waste to landfills.

A brief survey of the main approaches proposed in the literature for solid waste management (SWM) models, during the last two decades, is made in Section 2. Section 3 discusses the mathematical modelling of the municipal solid waste management problem. The model formulations are described in Sections 3.1 and 3.3 while the analyses of the models are outlined in Sections 3.2 and Section 3.4. The case study through which the validity and robustness of the models are tested is described in Section 4. The examples used to illustrate how the models can be solved are given in Section 3.5. The data used in the case study is presented in Section 4.1 while the results from the validity and robustness tests of the models are discussed in Sections 4.2 and 4.3. The programming language and the solver used are briefly described in Section 5. The conclusions and future developments are presented in Section 6.

2 Annotated Bibliography

The effective application of SWM mathematical models as tools for decision making by municipal solid waste planners, in developing countries, is still a big challenge. A considerable amount of research has been done in the last two decades on various aspects of SWM, and a number of economically based optimization models for waste streams allocation and collection vehicle routes, have been developed. Owing to an increasing awareness of environmental protection and conservation of natural resources, rising prices of raw materials, and energy conservation concerns, the current research in SWM is now guided by the aim of designing comprehensive models that take into account multi-disciplinary aspects involving economic, technical, regulatory, and environmental sustainability issues.

The solid waste models that have been developed in the last two decades have varied in goals and methodologies. Solid waste generation prediction, facility site selection, facility capacity expansion, facility operation, vehicle routing, system scheduling, waste flow and overall system operation, have been some of these goals (Badran and El-Haggar [3]).

Some of the techniques that have been used include linear programming, integer programming, mixed integer programming, non-linear programming, dynamic programming, goal programming, grey programming, fuzzy programming, quadratic programming, stochastic programming, two-stage programming, and interval-parameter programming, geographic information systems (Ghose et al [24] and Hasit Warner [27]).

The main objective of most of the models developed has been to minimize cost. Some models are dynamic, while others are static (Badran and EL-Haggar [3]). Morrissey and Browne [49] have classified municipal waste management models into three categories on the basis of decision making criteria: cost benefit analysis, life cycle analysis, and multi-criteria decision making.

A detailed description of the mathematical models that have most inspired the development of the models presented in Section 3 is made below, while the other relevant and interesting models are mentioned at the end of this section.

Costi et al [18] have proposed a mixed integer nonlinear programming decision support model to help decision makers of a municipality in the development of incineration, disposal, treatment, and recycling integrated programs. In that model several treatment plants and facilities have been considered: separators, plants for producing refuse derived fuel (RDF), incinerators with energy recovery, plants for treatment of organic material, and sanitary landfills. The main objective of that model is to plan the municipal solid waste (MSW) management, define the refuse flow that has to be sent to recycling or to different treatment or disposal landfills, and to determine the optimal number, the kinds, and the localization of the plants that are to be active. Some of the decision variables in the model are binary while others are continuous. The objective consists of all possible economic costs and subjected to technical, regulatory (normative), and environmental constraints. In particular, pollution and impacts induced by the overall solid waste management system, are considered through the formalization of constraints on incineration, emissions and on negative effects produced by disposal or other forms of treatments like RDF chemical composition. A case study, relevant to the municipality of Genova, Italy, has been presented.

Fiorucci et al [22] have presented a mixed integer nonlinear programming decision support model for assisting planners in decisions regarding the overall management of solid waste at a municipal level. By using that model, an optimal number of landfills and treatment plants, optimal quantities and the characteristics of refuse that have to be sent to treatment plants, to landfills and to recycling can be determined. Various classes of constraints are considered in the problem formulation, considering the regulations about the minimum requirements for recycling, incineration process requirements, sanitary landfill conservation, and mass balance. The objective function is composed of recycling, transportation and maintenance costs. The model has been tested on the municipality of Genova, Italy. Unlike Costi et al [18], Fiorucci et al [22], have not considered constraints associated with the environmental impact due to incineration, production of refuse derived fuel (RDF), or stabilized organic material (SOM).

Badran and El-Haggar [3] have proposed a mixed integer linear programming model for the optimal management of municipal solid waste at Port Said, Egypt. The idea is to choose a combination of collection stations from the possible locations in such a way as to minimize the daily transportation costs from the districts to the collection stations, from the collection stations to composting plants and landfills, and from the collection stations to landfills. The constraints of the single objective (i.e. total cost) are the capacity constraints for the collection stations, composting plants, and landfills. The model tests show positive results that can result in profit from the collection fees and the sales of sorted recyclable material.

Daskalopoulos et al [19] have presented a mixed integer linear programming model for the management of MSW streams, taking into account their rates and compositions, as well as their adverse environmental impacts. Using this model, they identify the optimal combination of technologies for handling, treatment and disposal of MSW in a better economical and more environmentally sustainable way. The single objective is composed of costs per tonne of waste treated at the recycling, composting, incinerating plants, and landfills. The constraints of the objective are capacity constraints for the plants and landfills. The model has been applied to the management of MSW in the UK. The findings have revealed that the current costs favour the landfill option of managing the MSW. It is however noted that the impact of a potential levy on waste land filled, can reduce the gap between the costs of land filling and the other alternative waste-treatment technologies.

Chang and Chang [6] have presented a non-linear programming model for municipal solid waste management based on the minimization of an overall cost considering energy and material recovery requirements. A set of continuous decision variables that express material flows to the various facilities are defined. Presorting facilities (separators) are part of the model. The objective function includes transportation, treatment, and fixed and operational costs, and takes into account possible benefits from the sale of electric energy and recyclable raw materials. The problem constraints cover mass balance, incinerator and landfill capacities, and minimum energy recovery constraints. The proposed model has been tested in the Taipei metropolitan region, Taiwan. The tests are encouraging.

Similarities and differences between the current work and the foregoing literature survey

Costi et al [18] have presented a comprehensive mixed integer nonlinear programming problem, whose planning horizon is a year. They give a detailed description of environmental constraints that cover RDF constraints, incineration constraints, and SOM constraints.

The nonlinearity of their model consists in the nature of the decision variables used. These decision variables are percentages (fractions) of waste that has to be sent to various plants and landfills in their model. The interaction between these percentages generates their products that appear in the objective function, in the regulatory (normative) constraints, in the technical and environmental constraints. Probably the choice of the variables is due to the desired goal, and consequently nonlinearity is inevitable. Transformation to a linear model may require change of variables.

In contrast to the work of Costi et al [18], we present two models; the first model is an integer linear programming problem. Some of the variables in this model measure the number of trucks (including replacement trucks) used per day. The amount of waste transported is then determined by multiplying the number of trucks of a given type used between any two nodes by the capacity of a single truck, and by the expected number of trips a single truck of that type makes per day between those nodes. The binary variables used in the model decide the existence of a plant of a given type, and a landfill of a given type or size.

The second model is a mixed integer linear programming problem where the continuous variables measure the amount of waste that flows between the nodes while the integer variables measure the number of trucks used per day. The binary variables, like in the first model, decide the existence of a plant or a landfill.

We have two models because of the realization that some users may want to measure transportation costs in terms of costs per trip from a waste collection point, in which case the first model is more appropriate. Others may prefer to measure the transportation costs in terms of costs per unit waste carried away from a waste collection point, in which case the second model is more appropriate. For instance, the first model is more desirable for the Ugandan situation since it is not yet possible to measure waste as it is transported from the waste collection points.

The planning horizon in both models is a day; decisions are to be taken on a day to day basis. This means a continuous monitoring and collection of data in order to make the required adjustments. This flexibility may be lost in a long period horizon model. In addition to the daily operational utility of the models, they can as well be used as design tools for the plants, landfills, and truck depots. Apart from the transportation costs, installation and operational costs for plants and landfills, and benefits from recycling, RDF sales, SOM sales, and energy sales, the objective function also includes truck purchase costs as well as costs due to the presence of replacement trucks depots.

Since our desire is not only that of locating plants and deciding waste flows to these plants and landfills, special attention has been given to deciding the number and the type of trucks that are used to transport a given type of waste from the waste collection points to the plants or landfills. Replacement trucks are also considered with the observation of possible breakdowns of the operational trucks. This is an aspect that is missing in the surveyed works, and since transportation costs play a big part in the daily operational costs, it is important that a waste management planner has a tool as a basis for the trucks deployed.

Although regulatory, technical, and environmental constraints are not comprehensively considered in our models as by Costi et al [18] and Fiorucci et al [22], it is our belief that they can be handled in detail without affecting the linearity of the models. The regulatory constraints give the minimum percentage of waste recycling; these percentages are proportions of the total waste generated. The technical constraints not only deal with plant capacities but also deal with the minimum amount of waste that has to be sent to the plants if these plants are to be economically beneficial. The environmental constraints are necessary to limit emissions during the combustion processes at incinerators, and to limit the presence of noxious substances in the RDF and in the SOM. Along the same line leachate and biogas production at landfills can be studied.

Unlike in the model of Costi et al [18], waste flows from RDF, recycling, and SOM plants to incinerators are not considered in our models. These are left out in order to first develop the key elements that deal with the determination of trucks used in the daily transportation exercise. These missing elements can however be incorporated by defining new sets of variables to cover the waste flows from the RDF plants, from the recycling plants, and from the SOM plants to the incinerators, since the aim is to recover as much waste as possible.

One similarity between our models and that of Costi et al [18] is that collection costs from waste sources to collection points are not part of the models. Other similarities are that our models are all static and deterministic (Murty [51]), and single objectives that minimize total costs are used. A major similarity is that the goal is to present integrated models that are comprehensive.

The model of Fiorucci et al [22] can be derived from that of Costi et al [18] by ignoring environmental constraints. Like in the model of Costi et al [18], the nonlinearity of their model consists in the nature of the decision variables used. These decision variables are percentages (fractions) of waste that has to be sent to various plants and landfills in their model. The interaction between these percentages generates their products that appear in the objective function, in the regulatory (normative) constraints, and in the technical constraints.

The model of Chang and Chang [6] minimizes overall cost (taking into account energy and material recovery) through the solution of a nonlinear programming problem. Unlike Costi et al [18], their model does not cater for regulatory and environmental constraints while technical constraints are not as extensively described as done by Costi et al [18]. We present linear models, and go at length in dealing with the waste transportation by determining the truck types and numbers as well as considering replacement trucks in our models. We also show environmental constraints can be included.

Badran and El-Haggar [3] present a mixed integer linear programming model whose objective covers collection costs from the districts to collection stations, transportation costs from collection stations to either composting plants or to landfills. Benefits from the sale of compost and recyclable material are incorporated into the objective function. Binary variables are used to decide the existence of collection stations. Incineration, recycling, and RDF production are not part of the model. Regulatory, technical, and environmental constraints are not covered in the model. Unlike Badran and El-Haggar [3], waste collection from the sources of generation is not considered in our models, and collection points are assumed to be known. Recycling, refused derived fuel, and energy generation are considered in our models. The determination of trucks as well as replacement trucks used everyday is not considered by Badran and El-Haggar [3]. Our models are linear like their model.

The model of Daskalopoulos et al [19] does not cover collection and transportation costs. Regulatory and technical constraints are not considered either. The costs in the objective function cater for the environmental considerations related to the emission of greenhouse gases. These costs are evaluated by costing all possible environmental damages that are associated with the waste management options, like potential crop yield reduction, forest damage, sea level rise, and damage to human health. Unlike our models where several aspects of waste management planning are considered, the model of Daskalopoulos [19] is restricted to waste treatment and environmental impact. Such a model can be useful to companies that solely deal with municipal solid waste treatment. It can also be expanded to cater for the missing elements.

ReVelle [53] presents a survey on the applications of operations research to a variety of environmental problem areas like water resource management, water quality management, solid waste operation and design, cost allocation for environmental facilities, and air quality management. He notes that despite four decades of such activities, challenging operational research problems still remain in all of those areas. The open problems described include the design of rationing strategies in a system of parallel reservoirs, hydro power production planning, simultaneous siting and efficiency determination of waste water treatment plants, design of the sequence of facilities in solid waste collection/disposal system, the achievement of equity as well as rationality in cost allocation, the planning of cost allocation when demands change over time, and the siting of air quality monitoring stations.

The other relevant solid waste management models are outlined below under the following seven distinct traits:

1) Linear models; 2) Nonlinear models; 3) Dynamic models; 4) Static models; 5) Stochastic models; 6) Deterministic models; 7) Multi-objective models; 8) Single objective models.

The models under trait one include Badran and El-Haggar [3], Daskalopoulos et al [19], Alidi [1], Amouzegar and Moshirvaziri [2], Bloemhof-Ruwaard et al [4], Caruso et al [5], Chang et al [7], Chang and Davila [9], Chang et al [11], Chang et al [12], Chang and Wang [13], Chang and Wang [14], Chang and Wang [15], Chang and Wang [17], Davila and Chang [20], Everett and Modak [21], Gottinger ([25], [26]), Huang et al [28], Huang et al [29], Huang et al [30], Huang et al [32], Huang et al [33], Huang et al [34], Huang et al [35], Huang et al [37], Huang et al [38], Huang et al [39], Hsin-Neng and Kuo-hua [40], Kühner and Harrington [42], Kulcar [43], Li and Huang [44], Li et al [45], Maqsood and Huang [46], Marks and Liebman [47], Nie et al [52], and Solano et al ([54], [55]).

Under trait two there is Costi et al [18], Fiorucci et al [22], Chang and Chang et al [6], Chang et al [8], Chang and Wang [16], Huang et al [31], Huang et al [36], Minciardi et al [48], and Wu et al [57].

In trait three there is Chang et al [11], Chang et al [12], Chang and Wang [13], Chang and Wang [15], Huang et al [32], Huang et al [34], and Kühner and Harrington [42].

Trait four comprises Costi et al [18], Fiorucci et al [22], Daskalopoulos et al [19], Alidi [1], Amouzegar and Moshirvaziri [2], Badran and El-Haggar [3], Bloemhof-Ruwaard et al [4], Caruso et al [5], Chang and Chang et al [6], Chang et al [7], Chang et al [8], Chang and Davila [9], Chang and Wang [14], Chang and Wang [16], Chang and Wang [17], Davila and Chang [20], Everett and Modak [21], Gottinger ([25], [26]), Huang et al [28], Huang et al [29], Huang et al [30], Huang et al [31], Huang et al [33], Huang et al [35], Huang et al [36], Huang et al [37], Huang et al [38], Huang et al [39], Hsin-Neng and Kuo-hua [40], Kühner and Harrington [42], Kulcar [43], Li and Huang [44], Li et al [45], Maqsood and Huang [46], Marks and Liebman [47], Minciardi et al [48], Nie et al [52], and Solano et al ([54], [55]), and Wu et al [57].

Trait five consists of Chang et al [7], Chang and Wang [16], Chang and Wang [17], Davila and Chang [20], Huang et al [28], Huang et al [29], Huang et al [30], Huang et al [31], Huang et al [32], Huang et al [33], Huang et al [34], Huang et al [35], Huang et al [36], Huang et al [37], Huang et al [38], Huang et al [39], Li and Huang [44], Li et al [45], Maqsood and Huang [46], Nie et al [52], and Wu et al [57].

Under trait six there is Costi et al [18], Fiorucci et al [22], Daskalopoulos et al [19], Alidi [1], Amouzegar and Moshirvaziri [2], Badran and El-Haggar [3], Bloemhof-Ruwaard et al [4], Caruso et al [5], Chang and Chang et al [6], Chang et al [8], Chang and Davila [9], Chang et al [11], Chang et al [12], Chang and Wang [13], Chang and Wang [14], Chang and Wang [15], Everett and Modak [21], Gottinger ([25], [26]), Hsin-Neng and Kuo-hua [40], Kühner and Harrington [42], Kulcar [43], Marks and Liebman [47], Minciardi et al [48], and Solano et al ([54], [55]).

Under trait seven there is Alidi [1], Caruso et al [5], Chang et al [7], Chang and Wang [14], Chang and Wang [17], and Minciardi et al [48].

Under trait eight there is Costi et al [18], Fiorucci et al [22], Daskalopoulos et al [19], Amouzegar and Moshirvaziri [2], Badran and El-Haggar [3], Bloemhof-Ruwaard et al [4], Chang and Chang et al [6], Chang et al [8], Chang and Davila [9], Chang et al [11], Chang et al [12], Chang and Wang [13], Chang and Wang [15], Chang and Wang [16], Davila and Chang [20], Everett and Modak [21], Gottinger ([25], [26]), Huang et al [28], Huang et al [29], Huang et al [30], Huang et al [31], ,Huang et al [32], Huang et al [33], Huang et al [34], Huang et al [35], Huang et al [36], Huang et al [37], Huang et al [38], Huang et al [39], Hsin-Neng and Kuo-hua [40], Kühner and Harrington [42], Kulcar [43], Li and Huang [44], Li et al [45], Maqsood and Huang [46], Marks and Liebman [47], Nie et al [52], Solano et al ([54], [55]), and Wu et al [57].

3 Models of the Problem

Building an exhaustive SWM management model is a very complex process as it is necessary to simultaneously consider conflicting objectives; such problems are usually characterized by an intrinsic uncertainty in estimates of costs and environmental impacts. A wide knowledge and a comprehensive analysis of all possible treatment processes of materials constituting the waste is required. The waste which is not recycled should be treated or disposed of at sanitary landfills. Since the aim is to minimize waste disposal and hence prolong the life span of landfills, an increment in recycling, refuse derived fuel (RDF) production, and energy generation may conflict. This is because these processes compete for waste with low humidity and high heating value like paper and plastic. Thus an optimal flow of waste to the plants is required; to achieve this it is necessary to express the humidity and heat values of each type of waste in the model (see Costi et al [18] and Fiorucci et al [22]). A detailed analysis will be considered in the future modifications of the model; processing plants are not yet part of waste management programs in Ugandan towns.

Furthermore, the benefits from waste recovery are measured in terms of income per unit (ton) of waste used in recycling, production of RDF, compost production, and energy. The environmental impact is dealt with by restricting the gaseous emissions from the plants as well the chemical composition of RDF and stabilized organic material (SOM); a detailed chemical characterization of these noxious materials (as done by Costi et al [18]) will be a point in the future modifications of the model. In general, municipal solid waste treatment covers paper, plastic, glass, metals, organic material, wood, inert material, scraps, and textile.

Figure 2 illustrates some of the key components in the decision support mathematical model. The variables x and y along the arcs give the waste flow amounts in terms of numbers of trucks, while the n variables give the numbers of replacement trucks. These decision variables have to be determined in the optimization process. Each of these variables is explained below in Section 3.1. The total daily waste production enters the sources where it is separated and then sent to the plants. Ideally, waste is separated at separators which are plants distinct from waste sources. In the proposed model, these sources can as well be assumed to be separators from where metals are taken to recycling, and organic material is taken for compost (SOM) production. Part of the waste with low humidity and high heating value is sent to incinerators for energy generation, or sent for RDF production, or disposed of in a sanitary landfill.

Recycling is considered for paper, glass, plastic, wood, organic material, and textiles. The fuel from RDF producing plants is sold while the scraps are sent to an incinerator or landfill. The generated energy is sold while the scraps are sent to a landfill. The SOM joins the market while the scraps are taken to an incinerator or landfill. Waste flows (scraps) from recycling, RDF producing plants, and SOM producing plants to incinerators will not be incorporated in the model.

With increased environmental concerns and shortage of land for landfills, waste disposal at landfills should be done for only unrecoverable waste; this can be achieved by restricting the maximum daily amount of waste flow to a landfill considering the amount of waste that saturates a landfill (in tons) and the minimum allowed time (in days) to saturate a landfill (Costi et al [18]). From this, landfill saturation constraints can be determined.

3.1 First Model Formulation

The model has been formulated as an integer linear programming problem (see Wolsey [56]). It has been presented as a decision making tool in the planning and management of integrated programs of solid waste collection, transportation, incineration, recycling, composting, and disposal.

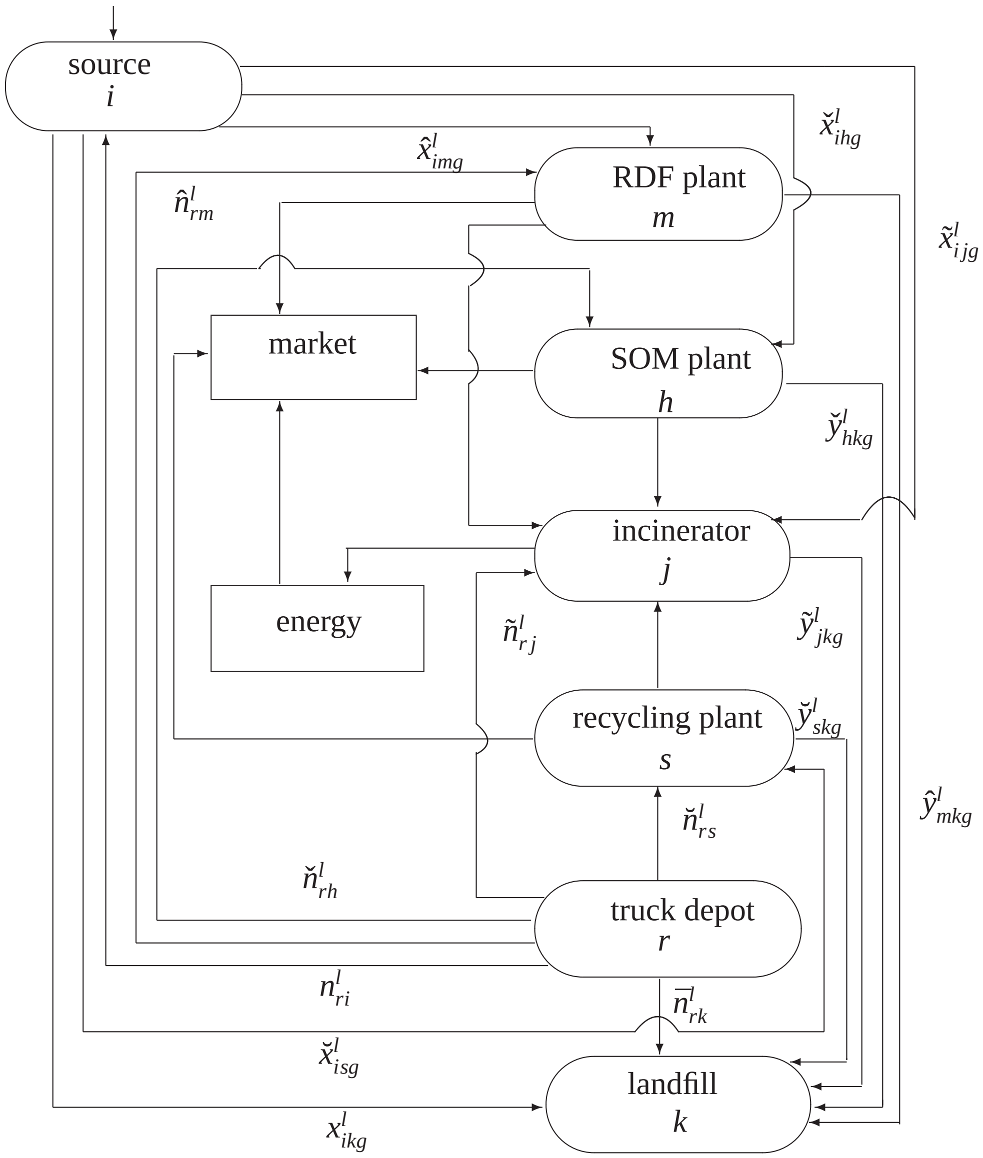

Figure 2: A detailed representation of key components in a decision support mathematical model.

The waste collection component has not been considered but several treatment plants and facilities have been included within the proposed model: trucks for the transportation of waste; replacement trucks and their depots; incinerators with energy recovery; sanitary landfills; plants for recycling, production of RDF, and treatment of organic material.

The objective function consists of total cost owing to investment and management costs, transportation costs, operational costs from the use of replacement trucks, benefits from energy generation, RDF production, compost, and recycling. The constraints include waste flow constraints due to the movement of waste among sources and plants and landfills as well as capacity, site selection, facility availability, environmental, and landfill saturation constraints. Constraints owing to the utilization of replacement trucks have also been included.

It is assumed that a waste manager in a municipality has a database of all the parameters on the computer well written in AMPL language as well as the model where all this data is supposed to be fed for a solution whenever required. There should also be an AMPL compatible solver like CPLEX on the computer. The advantage of having a database in AMPL is that it is easy to modify according to the changes in the parameters, and because modification of data does not require an expert in programming but one who can enter/change data in a proper way. The first use of the model determines all costs including investment costs; the subsequent uses, depending on whether significant changes have been observed in some of the key parameters, determine transport and operational costs, etc. The location of facilities will have been done in the first application of the model. Decisions are taken whenever required by considering the results. In the day to day application of the models, it may be more orderly and cheaper to hire replacement trucks instead of buying them.

The model has been built upon the following assumptions:

1. “Waste source” are located at the centres of waste generating areas.

2. Waste separation is done at the waste source locations (collection points). In other words, we can identify these sources with separators in this case. In practice sources and separators are distinct.

3. Waste handling operations proposed in the model are to be executed daily.

The ambiguity of the first assumption is that “radii” of waste generating areas are not specified; the point is that if the areas are almost “circular” and the “radii” are “small”, then the waste collection points at the centres are uniformly accessible from within the areas. The drawback is that the shapes and sizes of the waste areas can be very erratic so that the accessibility of the waste collection points at the centres may not be uniform from within the entire area; some of the waste may then not reach these collection points.

The advantage of the second assumption is that no money is then spent on establishing separators; it is however largely dependent on the cooperation of waste generators, the volume/weight and nature of waste generated. The most realistic option may be to have separators in the model, to which some of the waste is channelled for separation before transportation to recycling, SOM and RDF producing, and incinerating plants.

The third assumption is advantageous for daily heavy waste producing waste sources; the drawback is that waste sources that require weekly or monthly collections are not directly catered for in the model. Probably in the daily utilization of the model, some of the parameters (like costs) of such waste sources can be considered as “zeroes” until the days when they require collections.

Indices

$i = 1, 2, \ldots, I$: location of waste sources (collection points).

$j = 1, 2, \ldots, J$: location of incinerators.

$k = 1, 2, \ldots, K$: location of sanitary landfills.

$r = 1, 2, \ldots, R$: location of replacement trucks depots.

$m = 1, 2, \ldots, M$: location of refuse derived fuel (RDF) plants.

$h = 1, 2, \ldots, H$: location of composting (stabilized organic material, SOM) plants.

$s = 1, 2, \ldots, S$: location of recycling plants.

$l = 1, 2, \ldots, L$: truck type.

$g = 1, 2, \ldots, G$: waste type.

Variables

$\tilde{X}^l_{ijg}, \hat{X}^l_{img}, \check{X}^l_{ihg}, \breve{X}^l_{isg}, X^l_{ikg}$: respectively total number of trips made by trucks of type l used everyday to carry waste of type g from waste source i to an incinerator at j, an RDF plant at m, an SOM plant at h, a recycling plant at s, and a landfill at k.

$\tilde{x}^l_{ijg}, \hat{x}^l_{img}, \check{x}^l_{ihg}, \breve{x}^l_{isg}, x^l_{ikg}$: respectively number of trucks of type l used everyday to carry waste of type g from waste source i to an incinerator at j, an RDF plant at m, an SOM plant at h, a recycling plant at s, and a landfill at k.

$\tilde{Y}^l_{jkg}, \hat{Y}^l_{mkg}, \check{Y}^l_{hkg}, \breve{Y}^l_{skg}$: respectively total number of trips made by trucks of type l used everyday to carry waste of type g from an incinerator at j, an RDF plant at m, an SOM plant at h, and a recycling plant at s to a landfill at k.

$\tilde{y}^l_{jkg}, \hat{y}^l_{mkg}, \check{y}^l_{hkg}, \breve{y}^l_{skg}$: respectively number of trucks of type l used everyday to carry waste of type g from an incinerator at j, an RDF plant at m, an SOM plant at h, and a recycling plant at s to a landfill at k.

$\tilde{n}^l_{rj}, \hat{n}^l_{rm}, \check{n}^l_{rh}, \breve{n}^l_{rs}, \overline{n}^l_{rk}, n^l_{ri}$: respectively number of trucks of type l used everyday from a replacement trucks depot at r to an incinerator at j, an RDF plant at m, an SOM plant at h, a recycling plant at s, a landfill at k, and a waste source at i.

$\tilde{z}_j, \hat{z}_m, \check{z}_h, \breve{z}_s, z_k, \acute{z}_r$: 0-1 variables indicating respectively, the presence of an incinerator at j, an RDF plant at m, an SOM plant at h, a recycling plant at s, a landfill at k, and a replacement trucks depot at r.

$\tilde{w}_j, \hat{w}_m, \check{w}_h, \breve{w}_s, t_k$: amount of waste transported everyday respectively, to an incinerator at j, an RDF plant at m, an SOM plant at h, a recycling plant at s, and a sanitary landfill at k.

T_l : The number of trucks of type l used everyday.

T : The total number of trucks (excluding replacement trucks) used everyday.

$(RT)_l$: The number of replacement trucks of type l required everyday.

Input Data/Parameters

$\tilde{a}^l_{ij}$, $\hat{a}^l_{im}$, $\breve{a}^l_{ih}$, $\check{a}^l_{is}$, a^l_{ik}: expected number of trips a truck of type l can make respectively, per day between waste source at i and an incinerator at j, an RDF plant at m, an SOM plant at h, a recycling plant at s, and a landfill at k.

$\tilde{b}^l_{jk}$, $\hat{b}^l_{mk}$, $\check{b}^l_{hk}$, $\breve{b}^l_{sk}$: expected number of trips a truck of type l can make respectively, per day between an incinerator at j, an RDF plant at m, an SOM plant at h, a recycling plant at s, and a landfill at k.

α_l: capacity (in tonnes) of a truck of type l.

p_l: probability that a truck of type l breaks down in a day.

Ω_e: upper limit for noxious substance e; $e = 1, 2, \ldots, E$.

$\bar{e}^l_{rk}$, $\tilde{e}^l_{rj}$, $\hat{e}^l_{rm}$, $\check{e}^l_{rh}$, $\breve{e}^l_{rs}$, $\grave{e}^l_{ri}$: respectively the cost of moving a truck of type l from a replacement trucks depot at r to a landfill at k, an incinerator at j, an RDF plant at m, an SOM plant at h, a recycling plant at s, and a waste source at i.

$\tilde{c}^l_{ij}$, $\hat{c}^l_{im}$, $\check{c}^l_{ih}$, $\breve{c}^l_{is}$, c^l_{ik}: respectively transportation cost per unit of waste carried by a truck of type l from a waste source at i to an incinerator at j, an RDF plant at m, an SOM plant at h, a recycling plant at s, and a landfill at k.

$\tilde{d}^l_{jk}$, $\hat{d}^l_{mk}$, $\check{d}^l_{hk}$, $\breve{d}^l_{sk}$: respectively transportation cost per unit of waste carried by a truck of type l from an incinerator at j, an RDF plant at m, an SOM plant at h, and a recycling plant at s to a landfill at k.

$\tilde{c}_j$, $\hat{c}_m$, $\check{c}_h$, $\breve{c}_s$: revenue respectively, per unit of waste at an incinerator at j, an RDF plant at m, an SOM plant at h, and a recycling plant at s.

f_l: the cost of buying a new truck of type l, $l = 1 \ldots, L$.

d_i: amount of waste at source i.

$\tilde{\rho}_j$, $\hat{\rho}_m$, $\check{\rho}_h$, $\breve{\rho}_s$: fraction (%) of unrecovered waste respectively, at an incinerator at j, an RDF plant at m, an SOM plant at h, and a recycling plant at s that requires disposal to a landfill.

$\tilde{Q}_j$, $\hat{Q}_m$, $\check{Q}_h$, $\breve{Q}_s$, $\bar{Q}_k$, $\acute{Q}_r$: capacity per day respectively, for an incinerator at j, an RDF plant at m, an SOM plant at h, a recycling plant at s, a landfill at k, and a replacement trucks depot at r.

$\tilde{\delta}_j$, $\hat{\delta}_m$, $\check{\delta}_h$, $\breve{\delta}_s$, $\bar{\delta}_k$, $\acute{\delta}_r$: respectively fixed cost incurred in opening an incinerator at j, an RDF plant at m, an SOM plant at h, a recycling plant at s, a landfill at k, and a replacement trucks depot at r.

$\tilde{\gamma}_j$, $\hat{\gamma}_m$, $\check{\gamma}_h$, $\breve{\gamma}_s$, $\bar{\gamma}_k$: respectively variable cost incurred in handling a unit of waste at an incinerator at j, an RDF plant at m, an SOM plant at h, a recycling plant at s, and a landfill at k.

$\tilde{\mu}^e_j$, $\hat{\mu}^e_m$, $\check{\mu}^e_h$, $\breve{\mu}^e_s$, $\bar{\mu}^e_k$: respectively amount of noxious material e generated (per unit of waste) at an incinerator at j, an RDF plant at m, an SOM plant at h, a recycling plant at s, and a sanitary landfill at k; $e = 1, 2, \ldots, E$.

Objective Function

The objective function represents the overall daily waste management costs; the first component gives the investment and waste handling expenses as well as transportation costs, the second component gives expenses owing to the use of replacement trucks, and the third component the income from waste products like refuse derived fuel and energy.

The first component F_1 refers to the overall costs; the first part deals with the investment and management expenses while the second is concerned with the transportation costs. The two parts are separated by two sets of square brackets in (1). In this function we have the fixed cost parameters δ, and the variable cost parameters γ. The variables X and Y have been defined at the beginning of Section 3.1.

$$
\begin{aligned}
F_1(z,w,X,Y) \;=\;& [\sum_j(\tilde{\delta}_j\tilde{z}_j + \tilde{\gamma}_j\tilde{w}_j) + \sum_m(\hat{\delta}_m\hat{z}_m + \hat{\gamma}_m\hat{w}_m) \\
&+ \sum_h(\breve{\delta}_h\breve{z}_h + \breve{\gamma}_h\breve{w}_h) + \sum_s(\breve{\delta}_s\breve{z}_s + \breve{\gamma}_s\breve{w}_s) + \sum_k(\bar{\delta}_k z_k + \bar{\gamma}_k t_k)] \\
&+ [\sum_{glij}\tilde{c}^l_{ij}\alpha_l\tilde{X}^l_{ijg} + \sum_{glim}\hat{c}^l_{im}\alpha_l\hat{X}^l_{img} + \sum_{glih}\breve{c}^l_{ih}\alpha_l\breve{X}^l_{ihg} + \sum_{glis}\breve{c}^l_{is}\alpha_l\breve{X}^l_{isg} \\
&+ \sum_{glik}c^l_{ik}\alpha_l X^l_{ikg} + \sum_{gljk}\tilde{d}^l_{jk}\alpha_l\tilde{Y}^l_{jkg} + \sum_{glmk}\hat{d}^l_{mk}\alpha_l\hat{Y}^l_{mkg} + \sum_{glhk}\breve{d}^l_{hk}\alpha_l\breve{Y}^l_{hkg} \\
&+ \sum_{glsk}\breve{d}^l_{sk}\alpha_l\breve{Y}^l_{skg}]
\end{aligned}
\tag{1}
$$

Component F_2 concerns the total costs owing to the presence of replacement trucks (or standby trucks). Component F_3 gives the total cost for buying all trucks required in the daily management of waste. Component B gives the benefits at the plants owing to the production of electric energy, compost, refuse derived fuel, and recycled material.

$$
\begin{aligned}
F_2(n,z) \;=\;& \sum_{rkl}\vec{e}^l_{rk}\vec{n}^l_{rk} + \sum_{rjl}\tilde{e}^l_{rj}\tilde{n}^l_{rj} + \sum_{rml}\hat{e}^l_{rm}\hat{n}^l_{rm} + \sum_{rhl}\breve{e}^l_{rh}\breve{n}^l_{rh} \\
&+ \sum_{rsl}\breve{e}^l_{rs}\breve{n}^l_{rs} + \sum_{ril}\grave{e}^l_{ri}n^l_{ri} + \sum_r\acute{\delta}_r\acute{z}_r
\end{aligned}
\tag{2}
$$

$$
F_3(x,y,n) = \sum_l f_l(T_l + (RT)_l) \tag{3}
$$

$$
\begin{aligned}
B(w) \;=\;& \sum_j\tilde{c}_j(1-\tilde{\rho}_j)\tilde{w}_j + \sum_m\hat{c}_m(1-\hat{\rho}_m)\hat{w}_m + \sum_h\breve{c}_h(1-\breve{\rho}_h)\breve{w}_h \\
&+ \sum_s\breve{c}_s(1-\breve{\rho}_s)\breve{w}_s
\end{aligned}
\tag{4}
$$

So the objective function F, to be minimized, is

$$
F = F_1 + F_2 + F_3 - B \tag{5}
$$

Constraints

In general, the constraints include those linking waste flow among sources and plants and landfills, as well as capacity, site selection, facility availability, environmental, and landfill saturation constraints. The desire in constraint (6) is to clear all the waste generated at the source (collection point) i. So the total waste moved from each waste collection point i should at least be equal to the amount of waste found at that point.

$$\sum_{glj} \alpha_l \tilde{X}^l_{ijg} + \sum_{glm} \alpha_l \hat{X}^l_{img} + \sum_{glh} \alpha_l \check{X}^l_{ihg} + \sum_{glh} \alpha_l \check{X}^l_{isg} + \sum_{glk} \alpha_l X^l_{ikg} \geq d_i, \; i = 1, \ldots, I \quad (6)$$

In constraints (7)-(10), it is meant that the waste generated by a processing plant is disposed of in a landfill (or tip). Thus the amount of waste carried away from every plant to a landfill, should at least be equal to the amount of waste found at that plant. It is important to note that the weight of waste carried by a truck in a single trip (from a given source) is at most equal to its capacity, depending on the waste type and its amount; so the inequalities in (6) and (7)-(10) make sense.

$$\tilde{\rho}_j \tilde{w}_j \leq \sum_{gkl} \alpha_l \tilde{Y}^l_{jkg}, \qquad j = 1, \ldots, J \quad (7)$$

$$\hat{\rho}_m \hat{w}_m \leq \sum_{gkl} \alpha_l \hat{Y}^l_{mkg}, \qquad m = 1, \ldots, M \quad (8)$$

$$\check{\rho}_h \check{w}_h \leq \sum_{gkl} \alpha_l \check{Y}^l_{hkg}, \qquad h = 1, \ldots, H \quad (9)$$

$$\check{\rho}_s \check{w}_s \leq \sum_{gkl} \alpha_l \check{Y}^l_{skg}, \qquad s = 1, \ldots, S \quad (10)$$

Constraint (11) means that the amount of noxious material must not exceed National Environmental Management Authority or international levels, Ω_e.

$$\sum_j \tilde{\mu}^e_j (1 - \tilde{\rho}_j) \tilde{w}_j + \sum_m \hat{\mu}^e_m (1 - \hat{\rho}_m) \hat{w}_m + \sum_h \check{\mu}^e_h (1 - \check{\rho}_h) \check{w}_h$$
$$+ \sum_s \check{\mu}^e_s (1 - \check{\rho}_s) \check{w}_s + \sum_k \bar{\mu}^e_k t_k \leq \Omega_e, \; e = 1, \ldots, E \quad (11)$$

In constraints (12)-(15) the maximum capacities for the processing plants are accounted for. These constraints mean that the amount of waste taken to these plants should not exceed the plant capacities. In constraint (16) the same thing is done for sanitary landfills.

$$\tilde{w}_j \leq \tilde{Q}_j \tilde{z}_j, \qquad j = 1, \ldots, J \quad (12)$$
$$\hat{w}_m \leq \hat{Q}_m \hat{z}_m, \qquad m = 1, \ldots, M \quad (13)$$
$$\check{w}_h \leq \check{Q}_h \check{z}_h, \qquad h = 1, \ldots, H \quad (14)$$
$$\check{w}_s \leq \check{Q}_s \check{z}_s, \qquad s = 1, \ldots, S \quad (15)$$
$$t_k \leq \bar{Q}_k z_k, \qquad k = 1, 2, \ldots, K \quad (16)$$

Constraint (17), means that the total number of replacement trucks of type l cannot be less than the expected number of daily truck breakdowns of the type l. With constraint (18), we ensure that there is at least one depot for the replacement trucks. In constraint (19) we codify that the number of trucks in a depot cannot exceed its capacity. Constraint (20) means that the total number of replacement trucks is not too big compared to the total number of trucks used per day.

$$\sum_{rk} \vec{n}^l_{rk} + \sum_{rj} \tilde{n}^l_{rj} + \sum_{rm} \hat{n}^l_{rm} + \sum_{rh} \breve{n}^l_{rh} + \sum_{rs} \check{n}^l_{rs} + \sum_{ri} n^l_{ri} \geq p_l T_l, \quad l = 1, \ldots, L \tag{17}$$

$$\sum_{r=1}^{R} \acute{z}_r \geq 1 \tag{18}$$

$$\sum_{lk} \vec{n}^l_{rk} + \sum_{lj} \tilde{n}^l_{rj} + \sum_{lm} \hat{n}^l_{rm} + \sum_{lh} \breve{n}^l_{rh} + \sum_{ls} \check{n}^l_{rs} + \sum_{lri} n^l_{ri} \leq \acute{Q}_r \acute{z}_r, \quad r = 1, \ldots, R \tag{19}$$

$$\sum_{r} \acute{Q}_r \acute{z}_r \leq T \tag{20}$$

Constraints (21)-(29) mean that once the flow to either plant or sanitary landfill is positive, that plant or landfill must actually exist. The variables X and Y have been defined at the beginning of Section 3.1 and under definitions (51)-(59). We note here, as an example, that the expression $i, (j) = 1, \ldots, I, (J)$ means that i ranges from 1 up to I and j ranges from 1 up to J.

$$\alpha_l \tilde{X}^l_{ijg} \leq \tilde{Q}_j \tilde{z}_j, \quad l = 1, \ldots, L,\ i, (j) = 1, \ldots, I, (J),\ g = 1, \ldots, G \tag{21}$$

$$\alpha_l \hat{X}^l_{img} \leq \hat{Q}_m \hat{z}_m, \quad l = 1, \ldots, L,\ i, (m) = 1, \ldots, I, (M),\ g = 1, \ldots, G \tag{22}$$

$$\alpha_l \breve{X}^l_{ihg} \leq \breve{Q}_h \breve{z}_h, \quad l = 1, \ldots, L,\ i, (h) = 1, \ldots, I, (H),\ g = 1, \ldots, G \tag{23}$$

$$\alpha_l \check{X}^l_{isg} \leq \check{Q}_s \check{z}_s, \quad l = 1, \ldots, L,\ i, (s) = 1, \ldots, I, (S),\ g = 1, \ldots, G \tag{24}$$

$$\alpha_l X^l_{ikg} \leq \bar{Q}_k z_k, \quad l = 1, \ldots, L,\ i, (k) = 1, \ldots, I, (K),\ g = 1, \ldots, G \tag{25}$$

$$\alpha_l \tilde{Y}^l_{jkg} \leq \bar{Q}_k z_k, \quad l = 1, \ldots, L,\ j, (k) = 1, \ldots, J, (K),\ g = 1, \ldots, G \tag{26}$$

$$\alpha_l \hat{Y}^l_{mkg} \leq \bar{Q}_k z_k, \quad l = 1, \ldots, L,\ m, (k) = 1, \ldots, M, (K),\ g = 1, \ldots, G \tag{27}$$

$$\alpha_l \breve{Y}^l_{hkg} \leq \bar{Q}_k z_k, \quad l = 1, \ldots, L,\ h, (k) = 1, \ldots, H, (K),\ g = 1, \ldots, G \tag{28}$$

$$\alpha_l \check{Y}^l_{skg} \leq \bar{Q}_k z_k, \quad l = 1, \ldots, L,\ s, (k) = 1, \ldots, S, (K),\ g = 1, \ldots, G \tag{29}$$

Variable Conditions

The variables in constraints (30)-(38) are defined as non-negative integers. These give the number of trucks used between two nodes in the model per day, excluding replacement trucks.

$$\tilde{x}^l_{ijg}, \text{ integer } \geq 0, \quad i, (j) = 1, \ldots, I, (J),\ l = 1, \ldots, L,\ g = 1, \ldots, G \tag{30}$$

$$\hat{x}^l_{img},\ \text{integer} \ \geq \ 0, \qquad i,\ (m) \ = \ 1,\ldots,I,\ (M),\ l \ = \ 1,\ldots,L,\ g \ = \ 1,\ldots,\ G \tag{31}$$

$$\check{x}^l_{ihg},\ \text{integer} \ \geq \ 0, \qquad i,\ (h) \ = \ 1,\ldots,I,\ (H),\ l \ = \ 1,\ldots,L,\ g \ = \ 1,\ldots,\ G \tag{32}$$

$$\breve{x}^l_{isg},\ \text{integer} \ \geq \ 0, \qquad i,\ (s) \ = \ 1,\ldots,I,\ (S),\ l \ = \ 1,\ldots,L,\ g \ = \ 1,\ldots,\ G \tag{33}$$

$$x^l_{ikg},\ \text{integer} \ \geq \ 0, \qquad i,\ (k) \ = \ 1,\ldots,I,\ (K),\ l \ = \ 1,\ldots,L,\ g \ = \ 1,\ldots,\ G \tag{34}$$

$$\tilde{y}^l_{jkg},\ \text{integer} \ \geq \ 0, \qquad j,\ (k) \ = \ 1,\ldots,J,\ (K),\ l \ = \ 1,\ldots,L,\ g \ = \ 1,\ldots,\ G \tag{35}$$

$$\hat{y}^l_{mkg},\ \text{integer} \ \geq \ 0, \qquad m,\ (k) \ = \ 1,\ldots,M,\ (K),\ l \ = \ 1,\ldots,L,\ g \ = \ 1,\ldots,\ G \tag{36}$$

$$\check{y}^l_{hkg},\ \text{integer} \ \geq \ 0, \qquad h,\ (k) \ = \ 1,\ldots,H,\ (K),\ l \ = \ 1,\ldots,L,\ g \ = \ 1,\ldots,\ G \tag{37}$$

$$\breve{y}^l_{skg},\ \text{integer} \ \geq \ 0, \qquad s,\ (k) \ = \ 1,\ldots,S,\ (K),\ l \ = \ 1,\ldots,L,\ g \ = \ 1,\ldots,\ G \tag{38}$$

The variables in constraints (39)-(44) are defined as non-negative integers. These give the number of replacement trucks required everyday in the waste management program. We note that the breakdown of a truck can occur anywhere in the road network followed by the trucks. For purposes of locating the truck depots, it is assumed that these breakdowns occur at either a waste collection point or at a plant or at a landfill.

$$\vec{n}^l_{rk},\ \text{integer} \ \geq \ 0, \qquad r,\ (k) \ = \ 1,\ldots,R,\ (K),\ l \ = \ 1,\ldots,L \tag{39}$$

$$\tilde{n}^l_{rj},\ \text{integer} \ \geq \ 0, \qquad r,\ (j) \ = \ 1,\ldots,R,\ (J),\ l \ = \ 1,\ldots,L \tag{40}$$

$$\hat{n}^l_{rm},\ \text{integer} \ \geq \ 0, \qquad r,\ (m) \ = \ 1,\ldots,R,\ (M),\ l \ = \ 1,\ldots,L \tag{41}$$

$$\check{n}^l_{rh},\ \text{integer} \ \geq \ 0, \qquad r,\ (h) \ = \ 1,\ldots,R,\ (H),\ l \ = \ 1,\ldots,L \tag{42}$$

$$\breve{n}^l_{rs},\ \text{integer} \ \geq \ 0, \qquad r,\ (s) \ = \ 1,\ldots,R,\ (S),\ l \ = \ 1,\ldots,L \tag{43}$$

$$n^l_{ri},\ \text{integer} \ \geq \ 0, \qquad r,\ (i) \ = \ 1,\ldots,R,\ (I),\ l \ = \ 1,\ldots,L \tag{44}$$

The variables in (45)-(50) are defined as boolean. These are used to determine the existence of either a plant or a landfill.

$$\tilde{z}_j \ \in \ \{0,\ 1\}, \qquad j \ = \ 1,\ldots,J \tag{45}$$

$$\hat{z}_m \ \in \ \{0,\ 1\}, \qquad m \ = \ 1,\ldots,M \tag{46}$$

$$\check{z}_h \ \in \ \{0,\ 1\}, \qquad h \ = \ 1,\ldots,H \tag{47}$$

$$\breve{z}_s \ \in \ \{0,\ 1\}, \qquad s \ = \ 1,\ldots,S \tag{48}$$

$$z_k \ \in \ \{0,\ 1\}, \qquad k \ = \ 1,\ldots,K \tag{49}$$

$$\acute{z}_r \ \in \ \{0,\ 1\}, \qquad r \ = \ 1,\ldots,R \tag{50}$$

Definitions

In equations (51)-(59) the expected number of trips made per day by the trucks of type l from waste sources to plants, waste sources to landfills, and plants to landfills are given.

$$\tilde{X}^l_{ijg} \ = \ \tilde{a}^l_{ij}\tilde{x}^l_{ijg}, \qquad l \ = \ 1,\ldots,L,\ i,\ (j) \ = \ 1,\ldots,\ I,\ (J),\ g \ = \ 1,\ldots,\ G \tag{51}$$

$$\hat{X}^l_{img} \ = \ \hat{a}^l_{im}\hat{x}^l_{img}, \qquad l \ = \ 1,\ldots,L,\ i,\ (m) \ = \ 1,\ldots,\ I,\ (M),\ g \ = \ 1,\ldots,\ G \tag{52}$$

$$\check{X}^l_{ihg} = \check{a}^l_{ih}\check{x}^l_{ihg}, \quad l = 1,\ldots,L,\ i,\ (h) = 1,\ldots,\ I,\ (H),\ g = 1,\ldots,\ G \tag{53}$$

$$\breve{X}^l_{isg} = \breve{a}^l_{is}\breve{x}^l_{isg}, \quad l = 1,\ldots,L,\ i,\ (s) = 1,\ldots,\ I,\ (S),\ g = 1,\ldots,\ G \tag{54}$$

$$X^l_{ikg} = a^l_{ik}x^l_{ikg}, \quad l = 1,\ldots,L,\ i,\ (k) = 1,\ldots,\ I,\ (K),\ g = 1,\ldots,\ G \tag{55}$$

$$\tilde{Y}^l_{jkg} = \tilde{b}^l_{jk}\tilde{y}^l_{jkg}, \quad l = 1,\ldots,L,\ j,\ (k) = 1,\ldots,\ J,\ (K),\ g = 1,\ldots,\ G \tag{56}$$

$$\hat{Y}^l_{mkg} = \hat{b}^l_{mk}\hat{y}^l_{mkg}, \quad l = 1,\ldots,L,\ m,\ (k) = 1,\ldots,\ M,\ (K),\ g = 1,\ldots,\ G \tag{57}$$

$$\check{Y}^l_{hkg} = \check{b}^l_{hk}\check{y}^l_{hkg}, \quad l = 1,\ldots,L,\ h,\ (k) = 1,\ldots,\ H,\ (K),\ g = 1,\ldots,\ G \tag{58}$$

$$\breve{Y}^l_{skg} = \breve{b}^l_{sk}\breve{y}^l_{skg}, \quad l = 1,\ldots,L,\ s,\ (k) = 1,\ldots,\ S,\ (K),\ g = 1,\ldots,\ G \tag{59}$$

Definitions (60)-(63), also mentioned at the beginning of Section 3.1, indicate the amount of waste transported to processing plants while definition (64) gives the amount of waste from all waste sources to a landfill k. Similarly, definition (65) indicates the amount of waste disposed of in a sanitary landfill k everyday. Equation (66) gives the total amount of waste collected from all waste sources per day; this excludes waste generated by the plants. In equation (67) we give the total number of trucks of type l used per day in the model and, in definition (68) the total number of trucks required per day for the transportation, treatment, and disposal of waste is determined. In definition (69) the number of replacement trucks of type l in each depot is given, and in equation (70) the total number of replacement trucks in all depots is determined. It is assumed that the trucks are fully loaded as they leave the waste collection points.

$$\tilde{w}_j = \sum_{gli} \alpha_l \tilde{X}^l_{ijg}, \quad j = 1,\ldots,J \tag{60}$$

$$\hat{w}_m = \sum_{gli} \alpha_l \hat{X}^l_{img}, \quad m = 1,\ldots,M \tag{61}$$

$$\check{w}_h = \sum_{gli} \alpha_l \check{X}^l_{ihg}, \quad h = 1,\ldots,H \tag{62}$$

$$\breve{w}_s = \sum_{gli} \alpha_l \breve{X}^l_{isg}, \quad s = 1,\ldots,S \tag{63}$$

$$w_k = \sum_{gli} \alpha_l X^l_{ikg}, \quad k = 1,\ldots,K \tag{64}$$

$$t_k = w_k + \sum_{glj} \alpha_l \tilde{Y}^l_{jkg} + \sum_{glm} \alpha_l \hat{Y}^l_{mkg} + \sum_{glh} \alpha_l \check{Y}^l_{hkg} + \sum_{gls} \alpha_l \breve{Y}^l_{skg},\ k = 1,\ldots,K \tag{65}$$

$$W = \sum_{j} \tilde{w}_j + \sum_{m} \hat{w}_m + \sum_{h} \check{w}_h + \sum_{s} \breve{w}_s + \sum_{k} w_k \tag{66}$$

$$T_l = \sum_{gij} \tilde{x}^l_{ijg} + \sum_{gim} \hat{x}^l_{img} + \sum_{gih} \check{x}^l_{ihg} + \sum_{gis} \breve{x}^l_{isg} + \sum_{gik} x^l_{ikg} + \sum_{gjk} \tilde{y}^l_{jkg}$$
$$+ \sum_{gmk} \hat{y}^l_{mkg} + \sum_{ghk} \check{y}^l_{hkg} + \sum_{gsk} \breve{y}^l_{skg},\ l = 1,\ldots,L \tag{67}$$

$$T = \sum_{l} T_l \tag{68}$$

$$(RT)_l = \sum_{rk} \overline{n}^l_{rk} + \sum_{rj} \tilde{n}^l_{rj} + \sum_{rm} \hat{n}^l_{rm} + \sum_{rh} \check{n}^l_{rh} + \sum_{rs} \check{n}^l_{rs} + \sum_{ri} n^l_{ri}, \ l = 1, \ldots, L \tag{69}$$

$$RT = \sum_{l} (RT)_l \tag{70}$$

3.2 Analysis of the First Model

As mentioned at the beginning of Section 3.1, the model has been formulated as an integer linear programming problem (Wolsey [56]). The constraints include waste flow constraints for sources and plants and landfills, capacity, site selection, facility availability, environmental, and landfill saturation constraints. Truck flow constraints for replacement trucks from depots to landfills, waste sources, and processing plants have also been included.

We have a single objective function that covers the overall economic cost in the model. According to Costi et al [18], the definition of a decision model concerning the design of an urban solid waste management system would require the use of multi-objective decision concepts and techniques. Our model, like that of Costi et al [18], is particularly oriented to real-world applications; the multi-objective nature is taken into account by considering a single optimization objective comprising the overall economic cost, and transforming all other objectives (on pollution containment, impact minimization, etc) into constraints. Through these constraints it becomes easier to deal with regulations that specify bounds on the release of pollutants and other negative effects on the environment.

It is a deterministic model with integral decision variables; this was motivated by the desire of not only measuring waste quantities handled but also count the number of trucks of every type being used in the model. For instance in equation (67) we find the number of trucks of each type while in equation (68) we find the total number of trucks that operate daily in the model. Through equation (69) we determine the number of replacement trucks of each type that we may need daily while through equation (70) the total number of replacement trucks needed in the model per day is computed.

Since it is linear, it can be solved to optimality by several modelling/solver packages on the market like AMPL/CPLEX, LINGO/LINDO, GAMS/CPLEX, and MPL/CPLEX. The package, AMPL/CPLEX, we intend to use is briefly described in Section 5. The formulation of this model lies within the field of operations research that has been usefully applied to a wide variety of environmental problem areas (see ReVelle [53]).

The benefits from waste in the third component B of the objective function are measured in terms of economic gain per unit of waste. In actual terms it should be measured in terms of sales per litre of RDF produced, unit of SOM produced, unit of energy produced, unit item produced from recycling. To simplify the mathematics in the model, this precision was indirectly looked at in terms of economic gain per unit of waste. Another point is that we do not yet have processing plants in Uganda although it is under consideration. This explains why we do not have regulatory and technical constraints (see Costi et al [18] and Fiorucci et al [22]) in the model. We also have only one landfill. We have included economic gains from recycling in the benefits function; according to Costi et al [18] and Fiorucci et al [22], recycling in reality produces net cost. It is however encouraged because of environmental concerns and optimal use of limited resources.

The fractions ρ of unrecovered waste at the plants, and the amounts μ of noxious substances generated at the plants and landfills are assumed to be independent of the type of waste in both models. It is more realistic to consider dependence on the type of waste in order to formulate more precise environmental constraints, etc.

It is worth mentioning here that the current waste management trend in Uganda indicates that we are likely to have private companies that only collect waste, companies that only treat waste, and possibly companies that only manage sanitary landfills. The future waste models are likely to have these three scenarios of waste management where regulatory and technical constraints will be of major interest to companies running processing plants and landfills. Probably these companies will later merge to form integrated waste management programs. The proposed model is a good starting point upon which future variations can be built.

We shall also not go into a detailed description of environmental impacts as done by Costi et al [18], with specific attention paid to incineration emissions and RDF chemical composition. They consider pollutant content in the RDF, in the SOM, and incineration emissions. We measure pollutant content through unit waste handled at the plants; this may not be precise but a good illustration of how environmental impact can be considered. The consideration of the regulatory, technical, and a detailed description of environmental constraints may be done without affecting the linearity of the models. The biggest problem so far in Uganda with regard to waste pollution springs from the fact that much of the waste generated in Ugandan towns is not actually collected.

We have mentioned landfill saturation constraints (16) in the model in Section 3.1; the daily capacity $\overline{Q}_k$ imposed on the landfill k will be determined according to our desire of keeping that landfill active for a determined minimum number of years. The quality of the technology in place is very crucial here. That is to say, we shall determine the total amount of waste that saturates that landfill and divide it with the number of days that constitute the determined minimum number of years we want the landfill to remain active.

The environmental constraints (11) have been presented in a very elementary form in order to keep the mathematics simple; a detailed and precise description has been done by Costi et al [18]. A detailed description also requires a deep knowledge and analysis of all the processes involved. These constraints regulate the pollutant emissions at the plants as well as the toxic composition of the RDF and the SOM produced.

With constraint (18) we can, in theory, ensure that (it is presumed that these probabilities are known by the waste managers) there is at least one depot for replacement trucks. This may be ridiculous in practice in case of no breakdowns (especially if the trucks are new)! An alternative to buying replacement trucks may be hiring them in case of breakdowns. This may be more practical and can also keep the daily operational costs down. However, this constraint is not unreasonable since specialized trucks may be used in the management programs, and consequently not easily obtainable through hiring.

3.3 The Second Model of the Problem

In this section, a variant of the integer linear model described in Section 3.1 is presented with the hope of getting better total cost estimates and waste amount measurements. Continuous variables u's and v's have been introduced; they respectively measure the amount of waste collected everyday from waste sources to plants and from plants to landfills. A mixed integer linear program is thus obtained (see Wolsey [56]); the description of the new variables u and v now follows.

1. $\tilde{u}^l_{ijg}$, $\hat{u}^l_{img}$, $\breve{u}^l_{ihg}$, $\check{u}^l_{isg}$, u^l_{ikg}: respectively amount of waste (in tons) of type g collected everyday

by trucks of type l from a waste source i to an incinerator at j, an RDF plant at m, an SOM plant at h, a recycling plant at s, and a landfill at k.

2. $\tilde{v}^l_{jkg}$, $\hat{v}^l_{mkg}$, $\check{v}^l_{hkg}$, $\breve{v}^l_{skg}$: respectively amount of waste (in tons) of type g collected everyday by trucks of type l from an incinerator at j, an RDF plant at m, an SOM plant at h, and a recycling plant at s to a landfill at k.

The description of the rest of the variables and parameters in the model, remains the same as in Section 3.1.

Let it be noted that the emergence of the mixed integer linear model nowhere undermines the importance of the integer linear model; the choice between the two models from the practical point of view depends on the user and the technology used. One user may prefer to measure the transportation costs in terms of costs per trip made from the waste source, in which case the first model is more appropriate. In this case we replace the coefficients of the variables X and Y in the objective function with the total cost per trip from the waste collection point. At the same time, instead of measuring the amount of waste using the number of trucks used multiplied by their capacities, continuous variables can be introduced to measure directly the amount of waste that goes to the plants and landfills. The integer linear problem is then transformed into a mixed integer problem that gives better total cost estimates and more precise waste amount measurements. For instance, at the moment the first model is more relevant to the Ugandan situation, where the technology to measure waste as it is carried away from the waste sources is not available. Another user may prefer to measure the transportation costs in terms of costs per unit mass of waste picked from the waste source, in which case the second model is more appropriate.

Objective Function

The objective function, like in Section 3.1, represents the overall daily waste management costs; the first component gives the investment and waste handling expenses as well as transportation costs, the second component gives expenses owing to the use of replacement trucks, and the third component the income from waste products like refuse derived fuel and energy.

The first component F_1 refers to the overall costs; the first part deals with the investment and management expenses while the second is concerned with the transportation costs. The two parts are separated by two sets of square brackets in (71). In this function we have the fixed cost parameters δ, and the variable cost parameters γ. The variables u and v have been defined at the beginning of this section.

$$
\begin{aligned}
F_1(z,w,u,v) \;=\; & [\sum_j(\tilde{\delta}_j\tilde{z}_j + \tilde{\gamma}_j\tilde{w}_j) + \sum_m(\hat{\delta}_m\hat{z}_m + \hat{\gamma}_m\hat{w}_m) \\
& + \sum_h(\check{\delta}_h\check{z}_h + \check{\gamma}_h\check{w}_h) + \sum_s(\breve{\delta}_s\breve{z}_s + \breve{\gamma}_s\breve{w}_s) + \sum_k(\bar{\delta}_k z_k + \bar{\gamma}_k t_k)] \\
& + [\sum_{glij}\tilde{c}^l_{ij}\tilde{u}^l_{ijg} + \sum_{glim}\hat{c}^l_{im}\hat{u}^l_{img} + \sum_{glih}\check{c}^l_{ih}\check{u}^l_{ihg} + \sum_{glis}\breve{c}^l_{is}\breve{u}^l_{isg} + \sum_{glik}c^l_{ik}u^l_{ikg} \\
& + \sum_{gljk}\tilde{d}^l_{jk}\tilde{v}^l_{jkg} + \sum_{glmk}\hat{d}^l_{mk}\hat{v}^l_{mkg} + \sum_{glhk}\check{d}^l_{hk}\check{v}^l_{hkg} + \sum_{glsk}\breve{d}^l_{sk}\breve{v}^l_{skg}] \qquad (71)
\end{aligned}
$$

Component F_2 concerns the total costs owing to the presence of replacement trucks (or standby trucks). Component F_3 gives the total cost for buying all trucks required in the daily management of waste. Component B gives the benefits at the plants owing to the production of electric energy, compost, refuse derived fuel, and recycled material.

$$\begin{aligned} F_2(n,z) &= \sum_{rkl} \vec{e}^{l}_{rk}\vec{n}^{l}_{rk} + \sum_{rjl} \tilde{e}^{l}_{rj}\tilde{n}^{l}_{rj} + \sum_{rml} \hat{e}^{l}_{rm}\hat{n}^{l}_{rm} + \sum_{rhl} \check{e}^{l}_{rh}\check{n}^{l}_{rh} \\ &\quad + \sum_{rsl} \breve{e}^{l}_{rs}\breve{n}^{l}_{rs} + \sum_{ril} \grave{e}^{l}_{ri}n^{l}_{ri} + \sum_{r} \acute{\delta}_r\acute{z}_r \end{aligned} \tag{72}$$

$$F_3(x,y,n) = \sum_{l} f_l(T_l + (RT)_l) \tag{73}$$

$$\begin{aligned} B(w) &= \sum_{j} \tilde{c}_j(1 - \tilde{\rho}_j)\tilde{w}_j + \sum_{m} \hat{c}_m(1 - \hat{\rho}_m)\hat{w}_m + \sum_{h} \check{c}_h(1 - \check{\rho}_h)\check{w}_h \\ &\quad + \sum_{s} \breve{c}_s(1 - \breve{\rho}_s)\breve{w}_s \end{aligned} \tag{74}$$

So we then obtain the objective function F, to be minimized, defined as

$$F = F_1 + F_2 + F_3 - B \tag{75}$$

Constraints

In general, the constraints are the capacity, site selection, facility availability, environmental, and landfill saturation constraints. In constraint (76) we make sure that the total waste moved from each waste collection point i is at least be equal to the amount of waste found at that point.

$$\sum_{glj} \tilde{u}^{l}_{ijg} + \sum_{glm} \hat{u}^{l}_{img} + \sum_{glh} \check{u}^{l}_{ihg} + \sum_{glh} \breve{u}^{l}_{isg} + \sum_{glk} u^{l}_{ikg} \geq d_i,\ i = 1,\ldots,I \tag{76}$$

In constraints (77)-(80), we guarantee that the amount of waste carried away from every plant to a landfill, is at least be equal to the amount of waste found at that plant.

$$\tilde{\rho}_j\tilde{w}_j \leq \sum_{gkl} \tilde{v}^{l}_{jkg}, \qquad j = 1,\ldots,J \tag{77}$$

$$\hat{\rho}_m\hat{w}_m \leq \sum_{gkl} \hat{v}^{l}_{mkg}, \qquad m = 1,\ldots,M \tag{78}$$

$$\check{\rho}_h\check{w}_h \leq \sum_{gkl} \check{v}^{l}_{hkg}, \qquad h = 1,\ldots,H \tag{79}$$

$$\breve{\rho}_s\breve{w}_s \leq \sum_{gkl} \breve{v}^{l}_{skg}, \qquad s = 1,\ldots,S \tag{80}$$

In constraint (81) the amount of noxious material must not exceed National Environmental Management Authority or international levels, Ω_e.

$$\sum_j \tilde{\mu}_j^e(1 - \tilde{\rho}_j)\tilde{w}_j + \sum_m \hat{\mu}_m^e(1 - \hat{\rho}_m)\hat{w}_m + \sum_h \breve{\mu}_h^e(1 - \breve{\rho}_h)\breve{w}_h$$

$$+ \sum_s \breve{\mu}_s^e(1 - \breve{\rho}_s)\breve{w}_s + \sum_k \overline{\mu}_k^e t_k \leq \Omega_e, \; e = 1, \ldots, E \tag{81}$$

In constraints (82)-(85) the maximum capacities for the processing plants are determined. These constraints mean that the amount of waste taken to these plants should not exceed the plant capacities. In constraint (86) the same is done for sanitary landfills.

$$\tilde{w}_j \leq \tilde{Q}_j \tilde{z}_j, \quad j = 1, \ldots, J \tag{82}$$

$$\hat{w}_m \leq \hat{Q}_m \hat{z}_m, \quad m = 1, \ldots, M \tag{83}$$

$$\breve{w}_h \leq \breve{Q}_h \breve{z}_h, \quad h = 1, \ldots, H \tag{84}$$

$$\breve{w}_s \leq \breve{Q}_s \breve{z}_s, \quad s = 1, \ldots, S \tag{85}$$

$$t_k \leq \overline{Q}_k z_k, \quad k = 1, 2, \ldots, K \tag{86}$$

In constraint (87), the total number of replacement trucks of type l cannot be less than the expected number of daily truck breakdowns of the type l. With constraint (88), we ensure that there is at least one depot for the replacement trucks. In constraint (89) we codify that the number of trucks in a depot cannot exceed its capacity. Constraint (90) guarantees that the total number of replacement trucks is not too big compared to the total number of trucks used per day.

$$\sum_{rk} \overline{n}^l_{rk} + \sum_{rj} \tilde{n}^l_{rj} + \sum_{rm} \hat{n}^l_{rm} + \sum_{rh} \breve{n}^l_{rh} + \sum_{rs} \breve{n}^l_{rs} + \sum_{ri} n^l_{ri} \geq p_l T_l,$$

$$l = 1, \ldots, L \tag{87}$$

$$\sum_{r=1}^{R} \acute{z}_r \geq 1 \tag{88}$$

$$\sum_{lk} \overline{n}^l_{rk} + \sum_{lj} \tilde{n}^l_{rj} + \sum_{lm} \hat{n}^l_{rm} + \sum_{lh} \breve{n}^l_{rh} + \sum_{ls} \breve{n}^l_{rs} + \sum_{lri} n^l_{ri} \leq \acute{Q}_r \acute{z}_r,$$

$$r = 1, \ldots, R \tag{89}$$

$$\sum_r \acute{Q}_r \acute{z}_r \leq T \tag{90}$$

Constraints (91)-(99) mean that once the flow to either plant or sanitary landfill is positive, that plant or landfill must actually exist.

$$\tilde{u}^l_{ijg} \leq \tilde{Q}_j \tilde{z}_j, \quad l = 1, \ldots, L, \; i, \; (j) = 1, \ldots, \; I, \; (J), \; g = 1, \ldots, \; G \tag{91}$$

$$\hat{u}^l_{img} \leq \hat{Q}_m \hat{z}_m, \quad l = 1, \ldots, L, \; i, \; (m) = 1, \ldots, \; I, \; (M), \; g = 1, \ldots, G \tag{92}$$

$$\breve{u}^l_{ihg} \leq \breve{Q}_h \breve{z}_h, \quad l = 1, \ldots, L, \; i, \; (h) = 1, \ldots, \; I, \; (H), \; g = 1, \ldots, \; G \tag{93}$$

$$\check{u}^{l}_{isg} \leq \check{Q}_s \check{z}_s, \qquad l = 1,\ldots,L,\ i,\ (s) = 1,\ldots,\ I,\ (S),\ g = 1,\ldots,\ G \tag{94}$$

$$u^{l}_{ikg} \leq \bar{Q}_k z_k, \qquad l = 1,\ldots,L,\ i,\ (k) = 1,\ldots,\ I,\ (K),\ g = 1,\ldots,\ G \tag{95}$$

$$\tilde{v}^{l}_{jkg} \leq \bar{Q}_k z_k, \qquad l = 1,\ldots,L,\ j,\ (k) = 1,\ldots,\ J,\ (K),\ g = 1,\ldots,\ G \tag{96}$$

$$\hat{v}^{l}_{mkg} \leq \bar{Q}_k z_k, \qquad l = 1,\ldots,L,\ m,\ (k) = 1,\ldots,\ M,\ (K),\ g = 1,\ldots,\ G \tag{97}$$

$$\check{v}^{l}_{hkg} \leq \bar{Q}_k z_k, \qquad l = 1,\ldots,L,\ h,\ (k) = 1,\ldots,\ H,\ (K),\ g = 1,\ldots,\ G \tag{98}$$

$$\check{v}^{l}_{skg} \leq \bar{Q}_k z_k, \qquad l = 1,\ldots,L,\ s,\ (k) = 1,\ldots,\ S,\ (K),\ g = 1,\ldots,\ G \tag{99}$$

The constraints in (100)-(108) relate the amount of waste collected from any waste source per day with the number of trucks used to collect that waste. They mean that the waste carried by these trucks from the waste collection points cannot exceed the amount of waste they can carry when fully loaded.

$$\tilde{u}^{l}_{ijg} \leq \alpha_l \tilde{a}^{l}_{ij} \tilde{x}^{l}_{ijg}, \qquad l = 1,\ldots,L,\ i,\ (j) = 1,\ldots,\ I,\ (J),\ g = 1,\ldots,\ G \tag{100}$$

$$\hat{u}^{l}_{img} \leq \alpha_l \hat{a}^{l}_{im} \hat{x}^{l}_{img}, \qquad l = 1,\ldots,L,\ i,\ (m) = 1,\ldots,\ I,\ (M),\ g = 1,\ldots,\ G \tag{101}$$

$$\check{u}^{l}_{ihg} \leq \alpha_l \check{a}^{l}_{ih} \check{x}^{l}_{ihg}, \qquad l = 1,\ldots,L,\ i,\ (h) = 1,\ldots,\ I,\ (H),\ g = 1,\ldots,\ G \tag{102}$$

$$\check{u}^{l}_{isg} \leq \alpha_l \check{a}^{l}_{is} \check{x}^{l}_{isg}, \qquad l = 1,\ldots,L,\ i,\ (s) = 1,\ldots,\ I,\ (S),\ g = 1,\ldots,\ G \tag{103}$$

$$u^{l}_{ikg} \leq \alpha_l a^{l}_{ik} x^{l}_{ikg}, \qquad l = 1,\ldots,L,\ i,\ (k) = 1,\ldots,\ I,\ (K),\ g = 1,\ldots,\ G \tag{104}$$

$$\tilde{v}^{l}_{jkg} \leq \alpha_l \tilde{b}^{l}_{jk} \tilde{y}^{l}_{jkg}, \qquad l = 1,\ldots,L,\ j,\ (k) = 1,\ldots,\ J,\ (K),\ g = 1,\ldots,\ G \tag{105}$$

$$\hat{v}^{l}_{mkg} \leq \alpha_l \hat{b}^{l}_{mk} \hat{y}^{l}_{mkg}, \qquad l = 1,\ldots,L,\ m,\ (k) = 1,\ldots,\ M,\ (K),\ g = 1,\ldots,\ G \tag{106}$$

$$\check{v}^{l}_{hkg} \leq \alpha_l \check{b}^{l}_{hk} \check{y}^{l}_{hkg}, \qquad l = 1,\ldots,L,\ h,\ (k) = 1,\ldots,\ H,\ (K),\ g = 1,\ldots,\ G \tag{107}$$

$$\check{v}^{l}_{skg} \leq \alpha_l \check{b}^{l}_{sk} \check{y}^{l}_{skg}, \qquad l = 1,\ldots,L,\ s,\ (k) = 1,\ldots,\ S,\ (K),\ g = 1,\ldots,\ G \tag{108}$$

Constraints (109)-(117) can be referred to as waste flow fixing constraints. The reason is that when there are benefits at some node there is a tendency to move as much waste as possible to that node as long as there is space on the truck. In such a case, what is “carried” on the truck, that includes false waste, may go beyond the amount at a waste source; this is undesirable because the interest is in the precise amount of waste picked from the source.

$$\tilde{u}^{l}_{ijg} \leq d_i, \qquad l = 1,\ldots,L,\ i,\ (j) = 1,\ldots,\ I,\ (J),\ g = 1,\ldots,\ G \tag{109}$$

$$\hat{u}^{l}_{img} \leq d_i, \qquad l = 1,\ldots,L,\ i,\ (m) = 1,\ldots,\ I,\ (M),\ g = 1,\ldots,G \tag{110}$$

$$\check{u}^{l}_{ihg} \leq d_i, \qquad l = 1,\ldots,L,\ i,\ (h) = 1,\ldots,\ I,\ (H),\ g = 1,\ldots,\ G \tag{111}$$

$$\check{u}^{l}_{isg} \leq d_i, \qquad l = 1,\ldots,L,\ i,\ (s) = 1,\ldots,\ I,\ (S),\ g = 1,\ldots,\ G \tag{112}$$

$$u^{l}_{ikg} \leq d_i, \qquad l = 1,\ldots,L,\ i,\ (k) = 1,\ldots,\ I,\ (K),\ g = 1,\ldots,\ G \tag{113}$$

$$\tilde{v}^{l}_{jkg} \leq \rho_j w_j, \qquad l = 1,\ldots,L,\ j,\ (k) = 1,\ldots,\ J,\ (K),\ g = 1,\ldots,\ G \tag{114}$$

$$\hat{v}^l_{mkg} \leq \rho_m w_m, \quad l = 1,\ldots,L,\ m,\ (k) = 1,\ldots,\ M,\ (K),\ g = 1,\ldots,\ G \tag{115}$$

$$\breve{v}^l_{hkg} \leq \rho_h w_h, \quad l = 1,\ldots,L,\ h,\ (k) = 1,\ldots,\ H,\ (K),\ g = 1,\ldots,\ G \tag{116}$$

$$\check{v}^l_{skg} \leq \rho_s w_s, \quad l = 1,\ldots,L,\ s,\ (k) = 1,\ldots,\ S,\ (K),\ g = 1,\ldots,\ G \tag{117}$$

Variable Conditions

The variables in constraints (118)-(126) are defined as non-negative; these give the amount of waste that flows between various nodes.

$$\tilde{u}^l_{ijg} \geq 0, \quad i,\ (j) = 1,\ldots,I,\ (J),\ l = 1,\ldots,L,\ g = 1,\ldots,\ G \tag{118}$$

$$\hat{u}^l_{img} \geq 0, \quad i,\ (m) = 1,\ldots,I,\ (M),\ l = 1,\ldots,L,\ g = 1,\ldots,\ G \tag{119}$$

$$\breve{u}^l_{ihg} \geq 0, \quad i,\ (h) = 1,\ldots,I,\ (H),\ l = 1,\ldots,L,\ g = 1,\ldots,\ G \tag{120}$$

$$\check{u}^l_{isg} \geq 0, \quad i,\ (s) = 1,\ldots,I,\ (S),\ l = 1,\ldots,L,\ g = 1,\ldots,\ G \tag{121}$$

$$u^l_{ikg} \geq 0, \quad i,\ (k) = 1,\ldots,I,\ (K),\ l = 1,\ldots,L,\ g = 1,\ldots,\ G \tag{122}$$

$$\tilde{v}^l_{jkg} \geq 0, \quad j,\ (k) = 1,\ldots,J,\ (K),\ l = 1,\ldots,L,\ g = 1,\ldots,\ G \tag{123}$$

$$\hat{v}^l_{mkg} \geq 0, \quad m,\ (k) = 1,\ldots,M,\ (K),\ l = 1,\ldots,L,\ g = 1,\ldots,\ G \tag{124}$$

$$\breve{v}^l_{hkg} \geq 0, \quad h,\ (k) = 1,\ldots,H,\ (K),\ l = 1,\ldots,L,\ g = 1,\ldots,\ G \tag{125}$$

$$\check{v}^l_{skg} \geq 0, \quad s,\ (k) = 1,\ldots,S,\ (K),\ l = 1,\ldots,L,\ g = 1,\ldots,\ G \tag{126}$$

The variables in constraints (127)-(135) are defined as non-negative integers; these give the number of trucks used between any two nodes per day in the model, excluding replacement trucks.

$$\tilde{x}^l_{ijg},\ \text{integer} \geq 0, \quad i,\ (j) = 1,\ldots,I,\ (J),\ l = 1,\ldots,L,\ g = 1,\ldots,\ G \tag{127}$$

$$\hat{x}^l_{img},\ \text{integer} \geq 0, \quad i,\ (m) = 1,\ldots,I,\ (M),\ l = 1,\ldots,L,\ g = 1,\ldots,\ G \tag{128}$$

$$\breve{x}^l_{ihg},\ \text{integer} \geq 0, \quad i,\ (h) = 1,\ldots,I,\ (H),\ l = 1,\ldots,L,\ g = 1,\ldots,\ G \tag{129}$$

$$\check{x}^l_{isg},\ \text{integer} \geq 0, \quad i,\ (s) = 1,\ldots,I,\ (J),\ l = 1,\ldots,L,\ g = 1,\ldots,\ G \tag{130}$$

$$x^l_{ikg},\ \text{integer} \geq 0, \quad i,\ (k) = 1,\ldots,I,\ (K),\ l = 1,\ldots,L,\ g = 1,\ldots,\ G \tag{131}$$

$$\tilde{y}^l_{jkg},\ \text{integer} \geq 0, \quad j,\ (k) = 1,\ldots,J,\ (K),\ l = 1,\ldots,L,\ g = 1,\ldots,\ G \tag{132}$$

$$\hat{y}^l_{mkg},\ \text{integer} \geq 0, \quad m,\ (k) = 1,\ldots,M,\ (K),\ l = 1,\ldots,L,\ g = 1,\ldots,\ G \tag{133}$$

$$\breve{y}^l_{hkg},\ \text{integer} \geq 0, \quad h,\ (k) = 1,\ldots,H,\ (K),\ l = 1,\ldots,L,\ g = 1,\ldots,\ G \tag{134}$$

$$\check{y}^l_{skg},\ \text{integer} \geq 0, \quad s,\ (k) = 1,\ldots,S,\ (K),\ l = 1,\ldots,L,\ g = 1,\ldots,\ G \tag{135}$$

The variables in constraints (136)-(141) are defined as non-negative integers. These give the number of replacement trucks used in the daily waste management program. We note that the breakdown of a truck can occur anywhere in the road network followed by the trucks. For purposes of locating the truck depots, it is assumed that these breakdowns occur at either a waste collection point or at a plant or at a landfill.

$$\vec{n}^{l}_{rk}, \text{ integer } \geq 0, \quad r, (k) = 1, \ldots, R, (K), \; l = 1, \ldots, L \tag{136}$$

$$\tilde{n}^{l}_{rj}, \text{ integer } \geq 0, \quad r, (j) = 1, \ldots, R, (J), \; l = 1, \ldots, L \tag{137}$$

$$\hat{n}^{l}_{rm}, \text{ integer } \geq 0, \quad r, (m) = 1, \ldots, R, (M), \; l = 1, \ldots, L \tag{138}$$

$$\check{n}^{l}_{rh}, \text{ integer } \geq 0, \quad r, (h) = 1, \ldots, R, (H), \; l = 1, \ldots, L \tag{139}$$

$$\breve{n}^{l}_{rs}, \text{ integer } \geq 0, \quad r, (s) = 1, \ldots, R, (S), \; l = 1, \ldots, L \tag{140}$$

$$n^{l}_{ri}, \text{ integer } \geq 0, \quad r, (i) = 1, \ldots, R, (I), \; l = 1, \ldots, L \tag{141}$$

The variables in (142)-(147) are defined as boolean; they are used to decide the existence of a plant or a landfill.

$$\tilde{z}_j \in \{0, 1\}, \quad j = 1, \ldots, J \tag{142}$$

$$\hat{z}_m \in \{0, 1\}, \quad m = 1, \ldots, M \tag{143}$$

$$\check{z}_h \in \{0, 1\}, \quad h = 1, \ldots, H \tag{144}$$

$$\breve{z}_s \in \{0, 1\}, \quad s = 1, \ldots, S \tag{145}$$

$$z_k \in \{0, 1\}, \quad k = 1, \ldots, K \tag{146}$$

$$\acute{z}_r \in \{0, 1\}, \quad r = 1, \ldots, R \tag{147}$$

Definitions

Definitions (148)-(151), also mentioned at the beginning of Section 3.1, give the amount of waste transported to processing plants while definition (152) gives the amount of waste from all waste sources to a landfill k. Similarly, definition (153) indicates the amount of waste disposed of in a sanitary landfill k. Equation (154) gives the total amount of waste collected from all waste sources per day; this excludes waste generated by the plants. In equation (155) the total number of trucks of type l used per day is determined and, in definition (156) the total number of trucks required per day for the transportation, treatment, and disposal of waste is determined. In definition (157) the number of replacement trucks of type l in each depots is given, and in equation (158) the total number of replacement trucks in all depots is given. It is assumed that the trucks are fully loaded as they leave the waste collection points.

$$\tilde{w}_j = \sum_{gli} \tilde{u}^{l}_{ijg}, \quad j = 1, \ldots, J \tag{148}$$

$$\hat{w}_m = \sum_{gli} \hat{u}^{l}_{img}, \quad m = 1, \ldots, M \tag{149}$$

$$\check{w}_h = \sum_{gli} \check{u}^{l}_{ihg}, \quad h = 1, \ldots, H \tag{150}$$

$$\breve{w}_s = \sum_{gli} \breve{u}^{l}_{isg}, \quad s = 1, \ldots, S \tag{151}$$

$$w_k = \sum_{gli} u^{l}_{ikg}, \quad k = 1, \ldots, K \tag{152}$$

$$t_k = w_k + \sum_{glj} \tilde{v}^l_{jkg} + \sum_{glm} \hat{v}^l_{mkg} + \sum_{glh} \check{v}^l_{hkg} + \sum_{gls} \breve{v}^l_{skg}, \qquad k = 1, \ldots, K \tag{153}$$

$$W = \sum_j \tilde{w}_j + \sum_m \hat{w}_m + \sum_h \check{w}_h + \sum_s \breve{w}_s + \sum_k w_k \tag{154}$$

$$T_l = \sum_{gij} \tilde{x}^l_{ijg} + \sum_{gim} \hat{x}^l_{img} + \sum_{gih} \check{x}^l_{ihg} + \sum_{gis} \breve{x}^l_{isg} + \sum_{gik} x^l_{ikg} + \sum_{gjk} \tilde{y}^l_{jkg}$$

$$+ \sum_{gmk} \hat{y}^l_{mkg} + \sum_{ghk} \check{y}^l_{hkg} + \sum_{gsk} \breve{y}^l_{skg}, \; l = 1, \ldots, L \tag{155}$$

$$T = \sum_l T_l \tag{156}$$

$$(RT)_l = \sum_{rk} \vec{n}^l_{rk} + \sum_{rj} \tilde{n}^l_{rj} + \sum_{rm} \hat{n}^l_{rm} + \sum_{rh} \check{n}^l_{rh} + \sum_{rs} \breve{n}^l_{rs} + \sum_{ri} n^l_{ri},$$

$$l = 1, \ldots, L \tag{157}$$

$$RT = \sum_l (RT)_l \tag{158}$$

3.4 Analysis of the Second Model

The second model has been formulated as a mixed integer linear programming problem that is similar to the first model which is formulated as an integer linear problem; the major difference consists in the new variables introduced and which are continuous unlike in the first model where all variables are integral. In the second model the waste is measured differently using continuous variables and trucks are counted differently using integer variables. More exact values in total cost, waste amounts, and benefits are expected in the second model; the two models are, in general, expected to give the same number of active and replacement trucks. However, the performance of the first model can be enhanced if the transportation costs are measured by costing a trip made by a truck, instead of using a waste mass unit.

3.5 Examples illustrating how the Model Problems can be solved

Two examples are presented in order to facilitate appreciating and understanding the solution techniques to the two models, which have respectively been formulated as integer linear and mixed integer linear programming problems.

3.5.1 An Integer Linear Model Example

Let

1: denote a waste source (collection point).

2: denote an incinerator.

3: denote a replacement trucks depot.

4: denote a landfill.

Figure 3 illustrates a simple model, where the waste source, the incinerator, the landfill, the trucks depot are all known, and all trucks are of the same capacity.

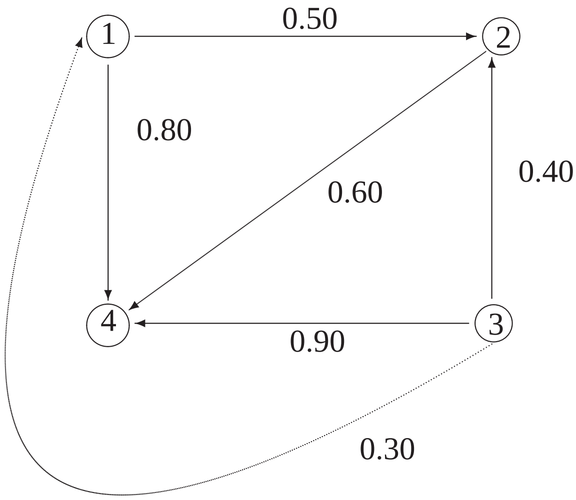

Figure 3: A simple model representation.

The figures along the arcs denote the costs which can be multiplied by 10,000 to get the actual value in Ug Shs (SEK 1 = Ug Shs 250).

Variables

$u_{12},\ u_{14}$: respectively represent the amount of waste (in tons) collected everyday by trucks of capacity 7.51 tons from a waste source at 1 to an incinerator at 2, and a landfill at 4.

v_{24} : represents the amount of waste (in tons) collected everyday by trucks of capacity 7.51 tons from an incinerator at 2 to a landfill at 4.

$x_{12},\ x_{14}$: respectively represent the number of trucks of capacity 7.51 tons used everyday to carry waste from a waste source at 1 to an incinerator at 2, and to a landfill at 4.

y_{24} : number of trucks of capacity 7.51 tons used everyday to carry waste from an incinerator at 2 to a landfill at 4.

$n_{31},\ n_{32},\ n_{34}$: respectively represent the number of trucks of capacity 7.51 tons used everyday from a replacement trucks bank at 3 to a waste source at 1, an incinerator at 2, and a landfill at 4.

$w_2\ (=\ u_{12}),\ t_4$: respectively represent the amount of waste transported everyday to an incinerator at 2, and a landfill at 4.

Input data/Parameters

14, 6 : respectively are the expected number of trips (single trips) a truck of capacity 7.51 tons can make everyday from a waste source at 1 to an incinerator at 2, and a landfill at 4.

13 : is the expected number of trips a truck of capacity 7.51 tons can make everyday between and incinerator at 2, and a landfill at 4.

0.50, 0.80 : respectively are the transportation costs per ton of waste transported from a waste source at 1 to an incinerator at 2, and a landfill at 4.

0.60 : is the transportation per ton of waste moved from an incinerator at 2 to a landfill at 4.

0.30, 0.40, 0.90 : respectively are the costs of moving a replacement truck of capacity 7.51 tons from a replacement trucks depot at 3 to a waste source at 1, an incinerator at 2, and a landfill at 4.

3.78 : is the revenue per unit of waste from an incinerator at 2.

235 : is the amount of waste (in tons) at a waste source at 1.

0.30 : is the fraction (%) of unrecovered waste at an incinerator at 2.

1000.25, 3, 1850 : are the respective capacities for an incinerator at 2, a replacement trucks depot at 3, and a landfill at 4.

1.73, 0.93 : are the respective costs of handling a ton of waste at an incinerator at 2, and a landfill at 4.

5000 : cost of buying a new truck.

0.13 : probability that a truck breaks down in a day.

The Model

The first model is an integer program and we seek to minimize the total cost $F_1 + F_2 - B$, where

$$\begin{aligned} F_1 &= (0.50 \times 7.51 \times 14 \times x_{12} + 0.80 \times 7.51 \times 6 \times x_{14} + 0.60 \times 7.51 \times \\ &\quad 13 \times y_{24}) + (1.73 \times 7.51 \times 14 \times x_{12} + 0.93 \times t_4) && (159) \\ F_2 &= 5000 \times (T + RT) && (160) \\ B &= 2.646 \times 7.51 \times 14 \times x_{12} && (161) \end{aligned}$$

Constraints

$$\begin{aligned} 7.51 \times 14 \times x_{12} + 7.51 \times 6 \times x_{14} &\geq 235 && (162) \\ 0.3 \times 7.51 \times 14 \times x_{12} &\leq 7.51 \times 13 \times y_{24} && (163) \\ n_{31} + n_{32} + n_{34} &\geq 0.13 \times T && (164) \\ 7.51 \times 14 \times x_{12} &\leq 1000.25 && (165) \\ t_4 &\leq 1850 && (166) \\ n_{31} + n_{32} + n_{34} &\leq 3 \leq T && (167) \end{aligned}$$

Variable Conditions

$$\begin{aligned} x_{12}, x_{14}, y_{24} \text{ integer} &\geq 0 && (168) \\ n_{31}, n_{32}, n_{34} \text{ integer} &\geq 0 && (169) \end{aligned}$$

Definitions

$$t_4 = 7.51 \times 6 \times x_{14} + 7.51 \times 13 \times y_{24} \tag{170}$$
$$T = x_{12} + x_{14} + y_{24} \tag{171}$$
$$RT = n_{31} + n_{32} + n_{34} \tag{172}$$

The Solution

We begin by generating a feasible solution by carrying all the waste from node 1 to node 2, since there are benefits at node 2. From inequality (162), $x_{12} = \lceil 235/(7.51 \times 14) \rceil = 3$. Also from inequality (163), $y_{24} = \lceil (0.9 \times 14)/13 \rceil = 1$. So $T = 3 + 1 = 4$. Now $RT = n_{31} + n_{32} + n_{34}$; from inequalities (164) and (167), and definition (172), $RT = \lceil 0.52 \rceil = 1$. The feasible solution $x_{12} = 3$, $x_{14} = 0$, and $y_{24} = 1$ gives the total cost $F_1 + F_2 - B$ as

$$(0.50 \times 7.51 \times 14 \times 3 + 0 + 0.60 \times 7.51 \times 13 \times 1) + (1.73 \times 7.51 \times 14 \times 3+$$
$$0.93 \times 7.51 \times 13 \times 1) + 5000 \times (4 + 1) - 2.646 \times 7.51 \times 14 \times 3 = 25,018.15918.$$

Since there are benefits at node 2 it is possible that an optimal solution has been obtained; we check this claim by considering integral combinations of 4 (since $T = 4$). We note that the maximum number of trucks that can be used to move waste from node 1 to node 2 is 3, and no more than 1 truck can be used to carry the waste from node 2 to node 4.

(i) If two trucks are used to carry the waste from node 1 to node 2, and one truck is used to move the waste from node 2 to node 4, some waste will remain at node 1 and has to be moved to node 4. The waste moved to node 2 is $7.51 \times 14 \times 2 = 210.8$ tons leaving a balance of 24.72 tons at node 1 that must be carried to node 4 using $x_{14} = \lceil 24.72/(7.51 \times 6) \rceil = 1$ truck. $T = 4$ and $RT = \lceil 0.52 \rceil = 1$; thus the total cost $F_1 + F_2 - B$ is

$$(0.50 \times 7.51 \times 14 \times 2 + 0.80 \times 7.51 \times 6 \times 1 + 0.60 \times 13 \times 1) + (1.73 \times 7.51 \times 14 \times 2$$
$$+ 0.93 \times (7.51 \times 6 \times 1 + 7.51 \times 13 \times 1)) + 5000 \times (4+1) - 2.646 \times 7.51 \times 14 \times 2 = 25,089.07322.$$

(ii) If one truck is used to move the waste from node 1 to node 2, and one truck is used to carry the waste from node 2 to node 4, some waste remains at node 1 and has to be moved to node 4. With one truck from node 1 to node 2, the waste carried to node 2 is $7.51 \times 14 \times 1 = 105.14$ tons leaving a balance of 129.86 tons at node 1 which must be moved to node 4 using $x_{14} = \lceil 129.86/(7.51 \times 6) \rceil = 3$ trucks. $T = 5$ and $RT = \lceil 0.65 \rceil = 1$; thus the total cost $F_1 + F_2 - B$ is

$$(0.50 \times 7.51 \times 14 \times 1 + 0.80 \times 7.51 \times 6 \times 3 + 0.60 \times 13 \times 1) + (1.73 \times 7.51 \times 14 \times 1$$
$$+ 0.93 \times (7.51 \times 6 \times 3 + 7.51 \times 13 \times 1)) + 5000 \times (5+1) - 2.646 \times 7.51 \times 14 \times 1 =$$
$$30,339.49706.$$

(iii) If all the waste at node 1 is now transported to node 4, then from inequality (162), $x_{14} = \lceil 235/(7.51 \times 6) \rceil = 6$. So $T = 6$ and $RT = \lceil 0.78 \rceil = 1$. In this case the total cost $F_1 + F_2 - B$ is

$$0.80 \times 7.51 \times 6 \times 6 + 0.93 \times 7.51 \times 6 \times 6 + 5000 \times (6 + 1) = 35,467.7228.$$

Hence, the claimed optimal solution is indeed an optimal solution to the problem. Note that $T \leq 3$ is not possible because in that case waste will remain at node 1 which, is contrary to our desire. Also, $T \geq 7$ is obviously undesirable because some trucks will then be redundant. The optimal solution has been validated using AMPL/CPLEX, and a Pentium IV 2.66 GHz computer in less than two seconds.

Sensitivity Analysis

In order to demonstrate how sensitivity analysis can be conducted on the two models a parameter ρ, which measures the quality of the incinerator at node 2, has been chosen. A relationship between ρ and the total cost $F_1 + F_2 - B$ is studied over the interval [0.0, 0.6]. The first three of the total cost when ρ is respectively equal to $0.000, 0.025, 0.050$ are computed by hand using a calculator but the rest of the values over the interval are computed using the AMPL/CPLEX. The process and the findings are described below.

We begin by generating a feasible solution by carrying all the waste from node 1 to node 2, since there are benefits at node 2 (see Figure 4).

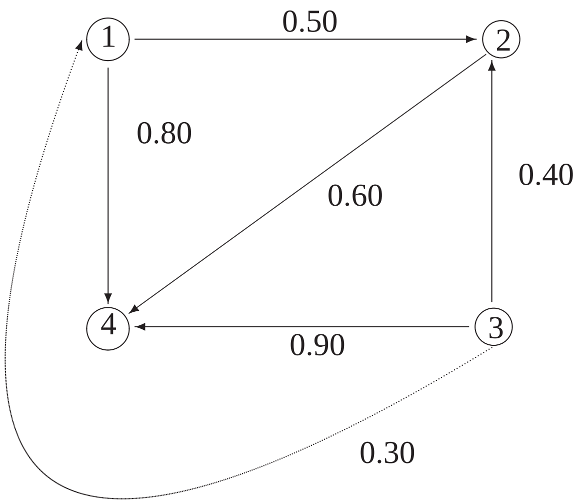

Figure 4: Second representation of the model.

For $\rho = 0.00$, equation (161) becomes

$$B = 3.78 \times 7.51 \times 14 \times x_{12} \tag{173}$$

and inequality (163) becomes

$$0 \times 7.51 \times 14 \times x_{12} \leq 7.51 \times 13 \times y_{24} \tag{174}$$

and $y_{24} = 0$. Thus, no waste will be moved from node 2 to node 4; also $x_{14} = 0$. From inequality (162), $x_{12} = \lceil 235/(7.51 \times 14) \rceil = 3$. So $T = 3$. Now $RT = n_{31} + n_{32} + n_{34}$; from inequalities (164) and (167), $RT = \lceil 0.39 \rceil = 1$. The feasible solution $x_{12} = 3, x_{14} = 0$, and $y_{24} = 0$ gives the total cost $F_1 + F_2 - B$ as

$$0.50 \times 7.51 \times 14 \times 3 + 1.73 \times 7.51 \times 14 \times 3 + 5000 \times (3 + 1) - 3.78 \times 7,51 \times 14 \times 3 \ = \ 19,511.099.$$

Since there benefits at node 2 it is likely that an optimal solution has been obtained; we check this claim by considering integral combinations of 3 (since $T = 3$). We note that the maximum number of trucks that can be used to move the waste from node 1 to node 2 is 3, and $y_{24} = 0$ since $\rho = 0$.

(i) If two trucks are used to carry the waste from node 1 to node 2, some waste will remain at node 1 and has to be moved to node 4. The waste moved to node 2 is $7.51 \times 14 \times 2 = 210.8$ tons leaving a balance of 24.72 tons that must be carried to node 4 using $x_{14} = \lceil 24.72/(7.51 \times 6) \rceil = 1$ truck. $T = 3$ and $RT = \lceil 0.39 \rceil = 1$; thus the total cost $F_1 + F_2 - B$ is

$$0.50 \times 7.51 \times 14 \times 2 + 0.80 \times 7.51 \times 6 \times 1 + 1.73 \times 7.51 \times 14 \times 2 + 0.93 \times 7.51 \times 6 \times 1 + 5000 \times (3 + 1) - 3.78 \times 7.51 \times 14 \times 2 = 19,752.0198.$$

(ii) If one truck is used to move the waste from node 1 to node 2 some waste remains at node 1 and has to be moved to node 4. With one truck from node 1 to node 2, the waste carried to node 2 is $7.51 \times 14 \times 1 = 105.14$ tons leaving a balance of 129.86 tons at node 1 which must be moved to node 4 using $x_{14} = \lceil 129.86/(7.51 \times 6) \rceil = 3$ trucks. $T = 4$ and $RT = \lceil 0.52 \rceil = 1$; thus the total cost $F_1 + F_2 - B$ is

$$0.50 \times 7.51 \times 14 \times 1 + 0.80 \times 7.51 \times 6 \times 3 + 1.73 \times 7.51 \times 14 \times 1 + 0.93 \times 7.51 \times 6 \times 3 + 5000 \times (4 + 1) - 3.78 \times 7.51 \times 14 \times 1 = 25,070.8944.$$

(iii) If all the waste at node 1 is now moved to node 4, then from inequality (162), $x_{14} = \lceil 235/(7.51 \times 6) \rceil = 6$. So $T = 6$ and $RT = \lceil 0.78 \rceil = 1$. In this case the total $F_1 + F_2 - B$ is

$$0.80 \times 7.51 \times 6 \times 6 + 0.93 \times 7.51 \times 6 \times 6 + 5000 \times (6 + 1) = 35,467.7228.$$

We note the $T \leq 2$ and $T = 5$ are not possible values since waste remains at node 1 or a truck is redundant. Hence, the claimed optimal solution is indeed an optimal solution to the problem.

We next try to find the optimal solution to the model if now $\rho = 0.025$. As in the previous case, we begin by generating a feasible solution by carrying all the waste from node 1 to node 2, since there are benefits at node 2 (see Figure 5).

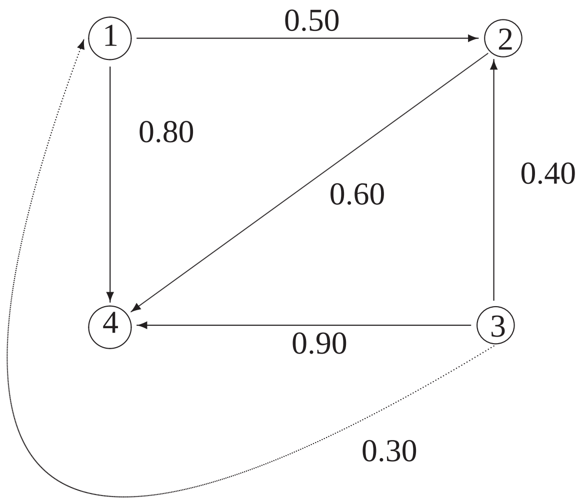

Figure 5: The third representation of the model.

Since $\rho = 0.025$, equation (161) becomes

$$B = 3.78 \times 0.975 \times 7.51 \times 14 \times x_{12} \tag{175}$$

and inequality (163) becomes

$$0.025 \times 7.51 \times 14 \times x_{12} \leq 7.51 \times 13 \times y_{24} \tag{176}$$

From inequality (162), $x_{12} = \lceil 235/(7.51 \times 14) \rceil = 3$. Also from inequality (176), $y_{24} = \lceil (0.075 \times 14)/13 \rceil = 1$. So $T = 3 + 1 = 4$. Now $RT = n_{31} + n_{32} + n_{34}$; from inequalities (164) and (167), $RT = \lceil 0.52 \rceil = 1$. The feasible solution $x_{12} = 3, x_{14} = 0$, and $y_{24} = 1$ gives the total cost $F_1 + F_2 - B$ as

$$0.50 \times 7.51 \times 14 \times 3 + 0 + 0.60 \times 7.51 \times 13 \times 1 + 1.73 \times 7.51 \times 14 \times 3 \times 0.93 \times 7.51 \times 13 \times 1 +$$
$$5000 \times (4 + 1) - 3.78 \times 0.975 \times 7,51 \times 14 \times 3 = 24,690.28009.$$

Since there are benefits at node 2 it is possible that an optimal solution has been obtained; we check this claim by considering integral combinations of 4 (since $T = 4$). We note that the maximum number of trucks that can be used to move the waste from node 1 to node 2 is 3, and no more than one truck can be used to carry waste from node 2 to node 4.

(i) If two trucks are used to carry the waste from node 1 to node 2 and one one truck is used to move the waste from node 2 to node 4, some waste will remain at node 1 and has to be moved to node 4. The waste moved to node 2 is $7.51 \times 14 \times 2 = 210.8$ tons leaving a balance of 24.72 tons that must be carried to node 4 using $x_{14} = \lceil 24.72/(7.51 \times 6) \rceil = 1$ truck. $T = 4$ and $RT = \lceil 0.52 \rceil = 1$; thus the total cost $F_1 + F_2 - B$ is

$$0.50 \times 7.51 \times 14 \times 2 + 0.80 \times 7.51 \times 6 \times 1 + 0.60 \times 7.51 \times 13 \times 1 + 1.73 \times 7.51 \times 14 \times 2+$$
$$0.93 \times (7.51 \times 6 \times 1 + 7.51 \times 13 \times 1) + 5000 \times (4 + 1) - 3.78 \times 0.975 \times 7.51 \times 14 \times 2 =$$
$$24,921.26516.$$

(ii) If one truck is used to move the waste from node 1 to node 2, and one truck is used to carry the waste from node 2 to node 4, some waste remains at node 1 and has to be moved to node 4. With one truck from node 1 to node 2, the waste carried to node 2 is $7.51 \times 14 \times 1 = 105.14$ tons leaving a balance of 129.86 tons at node 1 which must be moved to node 4 using $x_{14} = \lceil 129.86/(7.51 \times 6) \rceil = 3$ trucks. $T = 5$ and $RT = \lceil 0.65 \rceil = 1$; thus the total cost $F_1 + F_2 - B$ is

$$0.50 \times 7.51 \times 14 \times 1 + 0.80 \times 7.51 \times 6 \times 3 + 0.60 \times 7.51 \times 13 \times 1 + 1.73 \times 7.51 \times 14 \times 1+$$
$$0.93 \times (7.51 \times 6 \times 3 + 7.51 \times 13 \times 1) + 5000 \times (5 + 1) - 3.78 \times 0.975 \times 7.51 \times 14 \times 1 =$$
$$30,230.20403.$$

(iii) If all the waste at node 1 is now moved to node 4, then from inequality (162), $x_{14} = \lceil 235/(7.51 \times 6) \rceil = 6$. So $T = 6$ and $RT = \lceil 0.78 \rceil = 1$. In this case the total $F_1 + F_2 - B$ is

$$0.80 \times 7.51 \times 6 \times 6 + 0.93 \times 7.51 \times 6 \times 6 + 5000 \times (6 + 1) = 35,467.7228.$$

So $T = 4$ or 5 or 6. We note that $T \leq 3$ and $T \geq 7$ are not possible values since waste remains at node 1 or some trucks will be redundant. Hence, the claimed optimal solution is indeed an optimal solution to the problem.

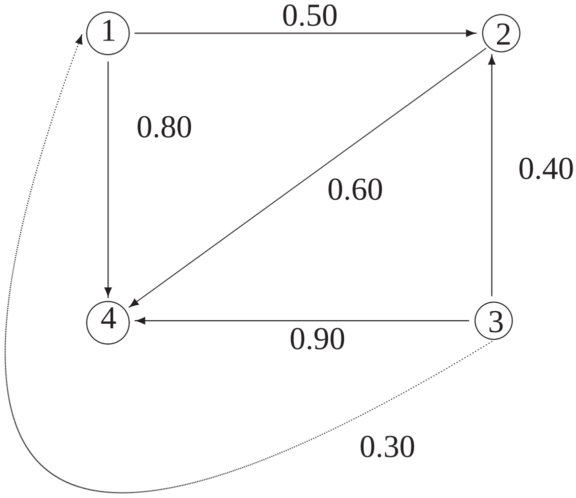

Figure 6: The fourth representation of the model.

We further try to find the optimal solution to the model if now $\rho = 0.05$. As in the previous cases, we begin by generating a feasible solution by carrying all the waste from node 1 to node 2, since there are benefits at node 2 (see Figure 6).

Since $\rho = 0.05$, equation (161) becomes

$$B = 3.78 \times 0.95 \times 7.51 \times 14 \times x_{12} \tag{177}$$

and inequality (163) becomes

$$0.05 \times 7.51 \times 14 \times x_{12} \leq 7.51 \times 13 \times y_{24} \tag{178}$$

From inequality (162), $x_{12} = \lceil 235/(7.51 \times 14) \rceil = 3$. Also from inequality (178), $y_{24} = \lceil (0.15 \times 14)/13 \rceil = 1$. So $T = 3 + 1 = 4$. Now $RT = n_{31} + n_{32} + n_{34}$; from inequalities (164) and (167), $RT = \lceil 0.52 \rceil = 1$. The feasible solution $x_{12} = 3, x_{14} = 0$, and $y_{24} = 1$ gives the total cost $F_1 + F_2 - B$ as

$$0.50 \times 7.51 \times 14 \times 3 + 0 + 0.60 \times 7.51 \times 13 \times 1 + 1.73 \times 7.51 \times 14 \times 3 \times 0.93 \times 7.51 \times 13 \times 1 + 5000 \times (4 + 1) - 3.78 \times 0.975 \times 7,51 \times 14 \times 3 = 24,720.08728.$$

As there are benefits at node 2 it is possible that an optimal solution has been attained; we check this claim by considering integral combinations of 4 (since $T = 4$). We once more note that the maximum number of trucks that can be used to move the waste from node 1 to node 2 is 3, and no more than one truck can be used to carry the waste from node 2 to node 4.

(i) If two trucks are used to carry the waste from node 1 to node 2 and one one truck is used to move the waste from node 2 to node 4, some waste will remain at node 1 and has to be moved to node 4. The waste moved to node 2 is $7.51 \times 14 \times 2 = 210.8$ tons leaving a balance of 24.72 tons that must be carried to node 4 using $x_{14} = \lceil 24.72/(7.51 \times 6) \rceil = 1$ truck. $T = 4$ and $RT = \lceil 0.52 \rceil = 1$; thus the total cost $F_1 + F_2 - B$ is

$$0.50 \times 7.51 \times 14 \times 2 + 0.80 \times 7.51 \times 6 \times 1 + 0.60 \times 7.51 \times 13 \times 1 + 1.73 \times 7.51 \times 14 \times 2 + 0.93 \times (7.51 \times 6 \times 1 + 7.51 \times 13 \times 1) + 5000 \times (4 + 1) - 3.78 \times 0.95 \times 7.51 \times 14 \times 2 = 24,941.13662.$$

(ii) If one truck is used to move the waste from node 1 to node 2, and one truck is used to carry the waste from node 2 to node 4, some waste remains at node 1 and has to be moved to node 4. With one truck from node 1 to node 2, the waste carried to node 2 is $7.51 \times 14 \times 1 = 105.14$ tons leaving a balance of 129.86 tons at node 1 which must be moved to node 4 using $x_{14} = \lceil 129.86/(7.51 \times 6) \rceil = 3$ trucks. $T = 5$ and $RT = \lceil 0.65 \rceil = 1$; thus the total cost $F_1 + F_2 - B$ is

$$0.50 \times 7.51 \times 14 \times 1 + 0.80 \times 7.51 \times 6 \times 3 + 0.60 \times 7.51 \times 13 \times 1 + 1.73 \times 7.51 \times 14 \times 1 + 0.93 \times (7.51 \times 6 \times 3 + 7.51 \times 13 \times 1) + 5000 \times (5 + 1) - 3.78 \times 0.95 \times 7.51 \times 14 \times 1 = 25,240.13976.$$

(iii) If all the waste at node 1 is now moved to node 4, then from inequality (162), $x_{14} = \lceil 235/(7.51 \times 6) \rceil = 6$. So $T = 6$ and $RT = \lceil 0.78 \rceil = 1$. In this case the total $F_1 + F_2 - B$ is

$$0.80 \times 7.51 \times 6 \times 6 + 0.93 \times 7.51 \times 6 \times 6 + 5000 \times (6 + 1) = 35,467.7228.$$

So $T = 4$ or 5 or 6. We note that $T \leq 3$ and $T \geq 7$ are not possible values since waste remains at node 1 or some trucks will be redundant. Hence, the claimed optimal solution is indeed an optimal solution to the problem. The remaining values of the optimal solution to the model as ρ varies over the interval [0.0, 0.6] have been computed using the AMPL/CPLEX, and they are displayed in Table 1. The values found by hand and calculator also agree with those computed using the programs. A graphical illustration of the behaviour of the total cost function as ρ varies over the interval [0.0, 0.6] is given in Figure 7. It is observed that the total cost falls with lower values of ρ; this is because the lower ρ is the more efficient the plant is, and consequently the more benefits will be obtained.

Table 1: Total costs and fractions of unrecovered waste for the incinerator at node 2.

ρ	$obj(F)$	$T(tot)$	$RT(tot)$	B	$W(tot)$
0.000	19511.09900	3	1	1192.28760	315.42
0.025	24690.28010	4	1	1162.48041	315.42
0.050	24720.08730	4	1	1132.67322	315.42
0.100	24779.70170	4	1	1073.05884	315.42
0.150	24839.31600	4	1	1013.44446	315.42
0.200	24898.93040	4	1	953.83008	315.42
0.250	24958.54480	4	1	894.21570	315.42
0.300	25018.15920	4	1	834.60132	315.42
0.400	25219.33710	4	1	476.91504	255.34
0.500	30405.99060	5	1	596.14380	315.42
0.600	30458.72580	5	1	158.97168	240.32
0.700	30498.46870	5	1	119.22876	240.32

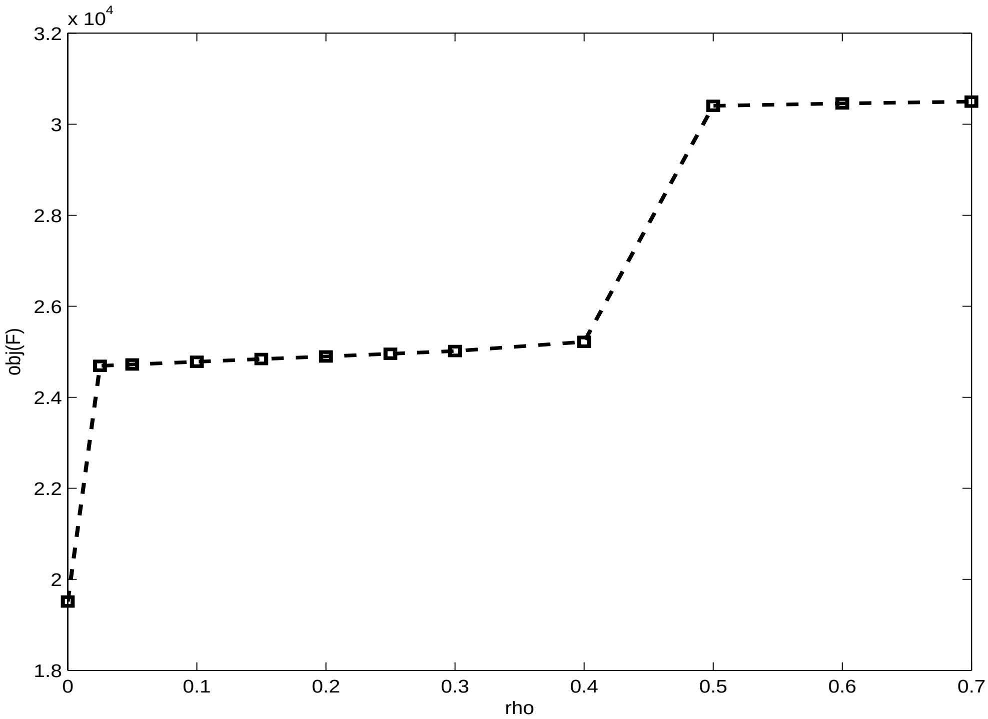

Figure 7: Total costs plotted against the fractions of unrecovered waste

3.5.2 A Mixed Integer Linear Model Example

The Model

The second model is a mixed integer linear program and we seek to minimize the total cost $F_1 + F_2 - B$,
where

$$F_1 = (0.50u_{12} + 0.80u_{14} + 0.60v_{24}) + (1.73u_{12} + 0.93t_4) \quad (179)$$
$$F_2 = 5000 \times (T + RT) \quad (180)$$
$$B = 2.646u_{12} \quad (181)$$

Constraints

$$u_{12} + u_{14} \geq 235 \quad (182)$$
$$0.3u_{12} \leq v_{24} \quad (183)$$
$$n_{31} + n_{32} + n_{34} \geq 0.13T \quad (184)$$
$$u_{12} \leq 1000.25 \quad (185)$$
$$t_4 \leq 1850 \quad (186)$$
$$n_{31} + n_{32} + n_{34} \leq 3 \leq T \quad (187)$$
$$u_{12} \leq 105.14x_{12},\ u_{14} \leq 45.06x_{14},\ v_{24} \leq 97.63y_{24} \quad (188)$$
$$u_{12} \leq 235,\ u_{14} \leq 235,\ v_{24} \leq 70.5 \quad (189)$$

Variable Conditions

$$u_{12},\ u_{14},\ v_{24} \geq 0 \tag{190}$$
$$x_{12},\ x_{14},\ y_{24} \text{ integer} \geq 0 \tag{191}$$
$$n_{31},\ n_{32},\ n_{34} \text{ integer} \geq 0 \tag{192}$$

Definitions

$$t_4 = u_{14} + v_{24} \tag{193}$$
$$T = x_{12} + x_{14} + y_{24} \tag{194}$$
$$RT = n_{31} + n_{32} + n_{34} \tag{195}$$

The Solution

Let us first suppose that all the waste of 235 tons is carried from node 1 to node 2 (see Figure 8). We determine the number of trucks required to carry this waste from node 1 to node 2 and then from node 2 to node 4, using inequalities (188).

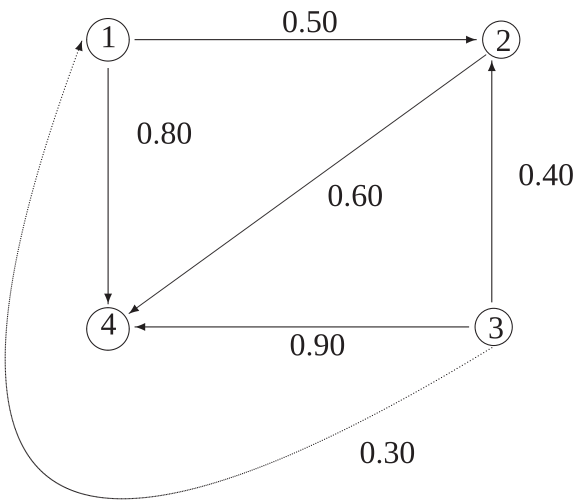

Figure 8: The fourth representation of the model.

We note that $u_{12} = 235, u_{14} = 0, v_{24} = 0.3 \times 235 = 70.5$. Therefore $x_{12} = \lceil 235/105.14 \rceil = 3$ and $y_{24} = \lceil 70.5/97.63 \rceil = 1$. So the the total number of trucks required to transport the waste of 235 from node 1 to node 2, and then the waste of 0.3×235 tons from node 2 to node 4, using equation (194), is $T = 4$.

To obtain the required number of replacement trucks we use inequalities (184) and (187), and obtain

$$0.52 \leq n_{31} + n_{32} + n_{34} \leq 4$$

Since, from equation (195), $RT = n_{31} + n_{32} + n_{34}$, it is reasonable to take $RT = \lceil 0.52 \rceil = 1$.

It is important to note that although an upper bound T on $n_{31} + n_{32} + n_{34}$ is reasonable, it is not the only one; the upper bound can be less or greater than T.

Since the waste at node 1 is transported to the incinerator at node 2, the transportation cost of 0.50 per ton and a handling fee of 1.73 per ton are incurred. There are benefits of 3.78 per ton; so the cost is

$$0.50 \times 235 + 1.73 \times 235 - 235 \times 0.7 \times 3.78 = -97.76.$$

A proportion, 0.3 of the waste taken to the incinerator has to be moved to the landfill at 4 at a cost of 0.60 per ton in addition to the handling charges at 4 of 0.93 per ton. The cost is therefore

$$0.3 \times 235 \times 0.60 + 0.3 \times 235 \times 0.93 = 107.865.$$

The cost for buying the trucks is $5000 \times (4+1) = 25,000$. Thus the total cost is

$$25,000 - 97.76 + 107.865 = 25,010.105.$$

Since there are benefits at node 2, the optimal cost is possibly 25,010.105; we check this claim by considering integral combinations of 4 (since $T = 4$). We note that the maximum number of trucks that can be used to move the waste from node 1 to node 2 is 3, and no more than 1 truck can be used to carry the waste from node 2 to node 4.

(i) If two trucks are used to carry the waste from node 1 to node 2, and one truck is used to move the waste from node 2 to node 4, some waste will remain at node 1 and has to be moved to node 4. The waste moved to node 2 is $7.51 \times 14 \times 2 = 210.8$ tons leaving a balance of 24.72 tons at node 1 that must be carried to node 4 using $x_{14} = \lceil 24.72/(7.51 \times 6) \rceil = 1$ truck. $T = 4$ and $RT = \lceil 0.52 \rceil = 1$; thus the total cost $F_1 + F_2 - B$ is

$$0.50 \times 210.8 + 0.80 \times 24.72 + 1.73 \times 210.8 - 0.7 \times 210.8 \times 3.78 + (0.3 \times 210.8 \times 0.6$$
$$+ 5000 \times (4+1) = 25,028.6172.$$

(ii) If one truck is used to move the waste from node 1 to node 2, and one truck is used to carry the waste from node 2 to node 4, some waste remains at node 1 and has to be moved to node 4. With one truck from node 1 to node 2, the waste carried to node 2 is $7.51 \times 14 \times 1 = 105.14$ tons leaving a balance of 129.86 tons at node 1 which must be moved to node 4 using $x_{14} = \lceil 129.86/(7.51 \times 6) \rceil = 3$ trucks. $T = 5$ and $RT = \lceil 0.65 \rceil = 1$; thus the total cost $F_1 + F_2 - B$ is

$$(0.50 \times 105.14 + 0.80 \times 129.86 - 0.7 \times 105.14 \times 3.78) + 0.3 \times 105.14 \times 0.60 + 0.3 \times$$
$$105.14 \times 0.93 + 5000 \times (5+1) = 29,926.51682.$$

(iii) If all the waste at node 1 is now transported to node 4, then from inequalities (188), $x_{14} = \lceil 235/(7.51 \times 6) \rceil = 6$. So $T = 6$ and $RT = \lceil 0.78 \rceil = 1$. In this case the total cost $F_1 + F_2 - B$ is

$$0.80 \times 235 + 0.93 \times 235 + 5000 \times (6+1) = 35,406.55.$$

Hence, the claimed optimal solution is indeed an optimal solution to the problem. As noted at the end of example1, $T \leq 3$ is not possible because in this case waste will remain at node 1 which, is contrary to the desired goal. Also, $T \geq 7$ is obviously undesirable because some trucks will then be redundant. The optimal solution has been validated using AMPL/CPLEX, and a Pentium IV 2.66 GHz computer in less than two seconds.

Sensitivity Analysis

The optimal solutions for the mixed integer model as ρ varies over the interval [0.0, 0.6] can be found in the same way as for the integer model. The reasoning is also similar to that used to find the optimal solution for the mixed integer model just above; so only the optimal solutions are presented for the mixed integer model. The hand computed optimal solutions when ρ is respectively equal to $0.0, 0.025$, and 0.05 are stated just below. The remaining values found using AMPL/CPLEX are presented in Table 2. A graphical illustration of the behaviour of the total cost function as ρ varies over the interval [0.0, 0.6] is given in Figure 9. It is observed that the total cost falls with lower values of ρ; this is because the lower ρ is the more efficient the incinerator at node 2 is, and consequently the more benefits will be obtained. The performance of both models can as well be compared from this figure. In general, the values of the objective functions are close to each other except when $\rho = 0.5$; the big difference in values here is due to the fact that the integer model uses one more truck that costs more money to buy (see Tables 1 and 2).

The extra truck is "used" in transporting extra "waste" from node 2 to node 4 (see Table 1). When $\rho = 5$, there is increased "waste" from node 1 to node 2 which results in increased benefits from node 2; this should not happen since the quality of the incinerator is getting worse. However this strange behaviour consists in the fact that the waste carried is determined by the trucks used, and since there may be some benefit (even if the incinerator is getting worse) some pseudo waste is likely to be transported on partially full trucks in order to keep the total transportation costs lower. Such strange isolated cases may not be many but they are likely to happen, and it is not easy to put control constraints, because of the assumption that the trucks leave the waste collection points when they are fully loaded. The best control may be to relax these assumptions by having continuous variables to measure the amount of waste carried; in this case it may be easier to control what is being transported as observed in the mixed integer linear model.

(i) We note that when $\rho = 0.000$ the equation (181) changes to

$$B = 3.78 \times u_{12} \tag{196}$$

and inequality (183) becomes

$$0 \times u_{12} \leq v_{24} \tag{197}$$

The optimal solution is found to be $u_{12} = 235, u_{14} = 0, v_{24} = 0, x_{12} = 3, x_{14} = 0, y_{24} = 0$, and the total cost is $19,635.75$ with $T = 3$ and $RT = 1$.

(ii) If instead $\rho = 0.025$, the equation (181) changes to

$$B = 3.78 \times 0.975 \times u_{12} \tag{198}$$

and inequality (183) becomes

$$0.025 \times u_{12} \leq v_{24} \tag{199}$$

The optimal solution is found to be $u_{12} = 235, u_{14} = 0, v_{24} = 5.875, x_{12} = 3, x_{14} = 0, y_{24} = 1$, and the total cost is $24,666.94625$ with $T = 4$ and $RT = 1$.

(iii) If now $\rho = 0.05$, the equation (181) changes to

$$B = 3.78 \times 0.95 \times u_{12} \tag{200}$$

and inequality (183) becomes

$$0.05 \times u_{12} \leq v_{24} \tag{201}$$

The optimal solution is found to be $u_{12} = 235, u_{14} = 0, v_{24} = 11.75, x_{12} = 3, x_{14} = 0, y_{24} = 1$, and the total cost is $24,698.1425$ with $T = 4$ and $RT = 1$.

Table 2: Total costs and fractions of unrecovered waste for the incinerator at node 2.

ρ	$obj(F)$	$T(tot)$	$RT(tot)$	B	$W(tot)$
0.000	19635.7500	3	1	888.30000	235
0.025	24666.9463	4	1	866.09250	235
0.050	24698.1425	4	1	843.88500	235
0.100	24760.5350	4	1	799.47000	235
0.150	24822.9275	4	1	755.05500	235
0.200	24885.3200	4	1	710.64000	235
0.250	24947.7125	4	1	666.22500	235
0.300	25010.1050	4	1	621.81000	235
0.400	25134.8900	4	1	532.98000	235
0.500	25284.5125	4	1	369.04140	235
0.600	30384.4600	5	1	355.32000	235
0.700	30450.1713	5	1	113.19588	235

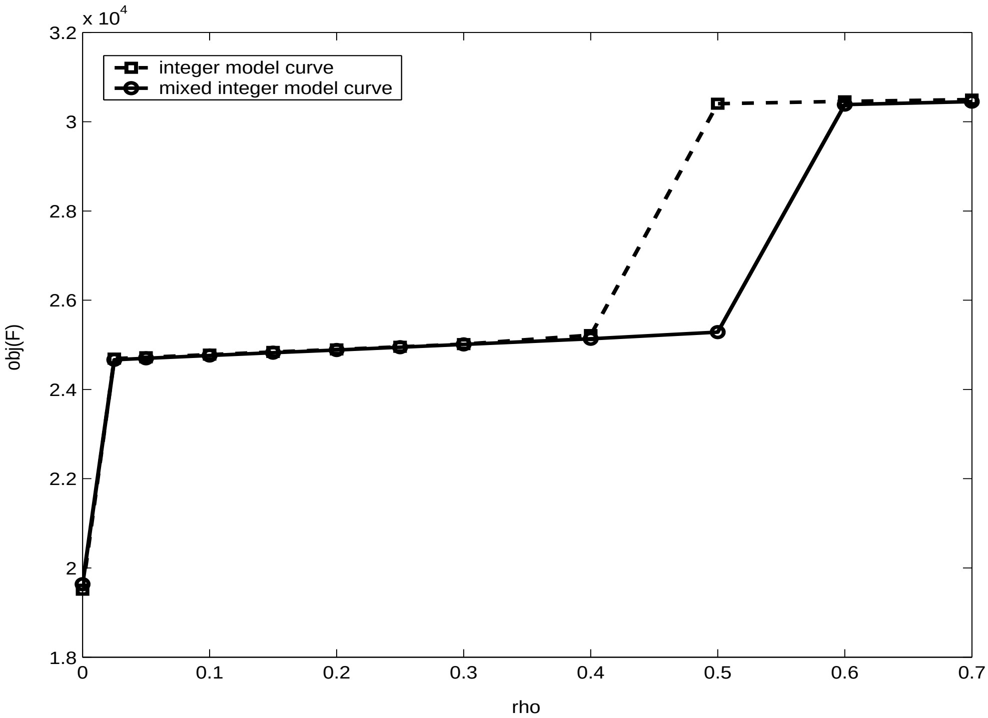

Figure 9: Total costs plotted against the fractions of unrecovered waste

4 Case Study

In order to illustrate how the models described in the above section may be useful in practice to waste managers, a (hypothetical) case study has been chosen and the data of the parameters is subjective. A hypothetical case study has been used because it has been difficult to get the actual data in time. The data of the parameters is chosen according to the situation in Kampala, the capital city of Uganda. At the moment none of the waste management systems in the City has a treatment plant; consequently the driving parameters are the transportation costs, waste amounts at the sources, and possibly the waste handling fees at the landfills. We have decided to include incinerator parameters because of the growing awareness in utilizing the waste to generate energy. We have not included waste handling fees at landfills because we think they are in the same category as transportation costs. In this hypothetical case study there are five waste source locations, three sanitary landfill locations, three replacement trucks depots, and three incinerators all of the same type. All trucks involved are all of the same type.

The names of landfills, waste source locations, replacement truck depots, and incinerators have been taken from regions or subdivisions of Kampala City. It is important to note that care has been taken to choose transport cost parameter data that is as close to the reality in Kampala as possible. The most difficult data to imagine has been that pertaining to incinerator parameters for instance, the fixed costs of opening incinerators, and the waste handling charges. Landfill data has also been hard to imagine because landfill charges do not directly go to waste managing systems.

At the moment it is the World Bank that is paying (per ton of waste deposited) the single landfill managing company with an aim of helping this company to determine with time a reasonable fee that can be paid by waste managing companies that transport waste to that landfill. At the same time, the other waste managing systems have also in time to determine a reasonable fee that can be paid by waste generators whose waste they handle. More than five hundred tons of waste are deposited everyday at the only landfill at Kiteezi; it is estimated that this is less than half of the total amount of waste accumulated daily in the entire city.

4.1 Data used to test the Models

The data for the parameters is given in Tables 3-18, that are given below. This data has also been used to study the validity and robustness of the models described in Section 3. Tables 19-22 and Figures 10-13 give the findings from the sensitivity analysis tests conducted on total expenditure against the amount of waste at the sources, transportation costs, the fractions of waste that remains at the incinerators, and the benefits from the incinerators. In Table 3 the locations for waste sources, sanitary landfills, replacement trucks depots, and incinerators are given. Table 4 gives waste source

Table 3: Node types and their locations

node type	locations				
waste sources (i)	Kamwokya	Kireka	Ntinda	Nalya	Kiwatule
landfill (k)	Kiteezi	Namugongo	Najeera		
truck depot (r)	Kyambogo	Nakasero	Lugogo		
incinerator (j)	Kawempe	Kiwatule	Kasubi		

locations as well as the waste amounts (in tons) at these locations. Table 5 gives the incinerator

Table 4: Waste amounts at waste sources.

waste source location (i)	waste amount, d_i (in tons)
Kamwokya	250.0
Kireka	350.0
Ntinda	400.0
Nalya	245.7
Kiwatule	268.5

locations j with their respective capacities $\tilde{Q}_j$ in tons, costs for treating a ton of waste $\tilde{\gamma}_j$, fixed costs in opening these incinerators $\tilde{\delta}_j$, and revenues per ton of waste at these incinerators $\tilde{c}_j$. Table

Table 5: Capacities, costs of opening and waste handling, revenues from incinerators.

j	$\tilde{Q}_j$	$\tilde{\delta}_j$	$\tilde{\gamma}_j$	$\tilde{c}_j$
Kawempe	1200.00	975	1.57	1.67
Kiwatule	1500.25	1079	1.73	6.78
Kasubi	1300.50	1354	1.96	1.89

6 gives the landfill locations k as well as landfill capacities $\overline{Q}_k$, fixed costs $\overline{\delta}_k$ incurred in opening these landfills or tips, and costs $\overline{\gamma}_k$ in handling a ton of waste at these landfills. Table 7 gives the replacement trucks depots locations r as well as fixed costs $\acute{\delta}_r$ in opening them, and their capacities $\acute{Q}_r$ in terms of the number of trucks that can be kept in them. For simplicity we only have one type

of trucks. In Table 8 truck capacity in tons α_l, the probability p_l of a truck breaking down in a day, and the cost f_l of buying a new truck are given. Table 9 gives the incinerator locations j, and the proportions of waste ρ_j that cannot be recovered by these incinerators. The waste that remains has to be disposed of at landfills. In Table 10 waste source locations i and sanitary landfill locations k are given. Transportations costs per trip c^l_{ik} made by a truck from waste sources to landfills have also been included. In Table 11 waste source locations i and landfill locations k are given. The expected number of trips a^l_{ik} a truck can make per day between a waste source i and a landfill k are also given. Table 12 gives waste source locations i and incinerator locations j. Transportation

Table 6: Landfill locations, landfill capacities, costs of opening landfills, and unit waste handling charges.

k	$\overline{Q}_k$	$\overline{\delta}_k$	$\overline{\gamma}_k$
Kiteezi	1850	1500	0.93
Namugongo	2500	1470	1.06
Najeera	3750	1575	1.10

Table 7: Replacement trucks depot locations as well as their capacities and fixed costs in opening them.

r	$\acute{\delta}_r$	$\acute{Q}_r$
Kyambogo	545.7	5
Nakasero	590.3	7
Lugogo	587.5	9

Table 8: Truck capacity, breakdown probability, and truck cost.

α_l	p_l	f_l
7.51	0.13	5500

Table 9: Incinerator locations and waste proportions that remain at these incinerators.

j	ρ_j
Kawempe	0.3
Kiwatule	0.3
Kasubi	0.3

Table 10: Transportation costs between waste sources and landfills.

$k \backslash i$	Kamwokya	Kireka	Ntinda	Nalya	Kiwatule
Kiteezi	0.70	0.70	0.80	0.80	0.80
Namugongo	0.70	0.50	0.60	0.40	0.50
Najeera	0.60	0.70	0.80	0.60	0.60

costs c^l_{ij} per ton of waste per day from a waste source to an incinerator are given as well. Table 13 gives waste source locations i and incinerator locations j. It also gives the expected number of trips $\tilde{a}_{ij}$ a truck can make per day between a waste source i and an incinerator j. Table 14 gives landfill locations k and incinerator locations j. Transportation costs c^l_{jk} per ton of waste from incinerators to landfills are also given. In Table 15 incinerator j and landfill k locations are provided. The expected number of trips $\tilde{b}^l_{jk}$ a truck can make everyday between incinerator j and landfill k are also given. Table 16 gives replacement trucks depots r and landfill k locations. Transportation costs

Table 11: Expected number of trips a truck makes per day between a waste sources and a landfills.

$k\backslash i$	Kamwokya	Kireka	Ntinda	Nalya	Kiwatule
Kiteezi	8	9	6	5	5
Namugongo	9	14	12	15	14
Najeera	12	8	7	13	12

Table 12: Transportation costs between waste sources and incinerators.

$j\backslash i$	Kamwokya	Kireka	Ntinda	Nalya	Kiwatule
Kawempe	0.60	0.80	0.60	0.60	0.60
Kiwatule	0.50	0.70	0.50	0.50	0.40
Kasubi	0.70	0.90	0.70	0.70	0.70

Table 13: Expected number of trips a truck can make a day between a waste source and an incinerator.

$j\backslash i$	Kamwokya	Kireka	Ntinda	Nalya	Kiwatule
Kawempe	13	8	12	12	11
Kiwatule	14	10	14	14	17
Kasubi	10	4	4	8	8

Table 14: Transportation costs between incinerators and landfills.

$j\backslash k$	Kiteezi	Namugongo	Najeera
Kawempe	0.40	0.60	0.70
Kiwatule	0.60	0.70	0.60
Kasubi	0.60	0.80	0.70

Table 15: Expected number of trips a truck can make a day between an incinerator and a landfill.

$j\backslash k$	Kiteezi	Namugongo	Najeera
Kawempe	17	12	10
Kiwatule	13	10	13
Kasubi	11	7	6

c^l_{rk} from depots to landfills are also given. In Table 17 replacement trucks depots r and incinerator

Table 16: Transportation costs between replacement trucks depots and landfills.

$r\backslash k$	Kiteezi	Namugongo	Najeera
Kyambogo	0.90	0.70	0.70
Nakasero	0.80	0.70	0.80
Lugogo	0.90	0.70	0.90

j locations are provided. Transportation costs $\overline{c}^l_{rj}$ from a depot r to an incinerator j are also given. Table 18 provides transportation costs of moving replacement trucks between their depots r and

Table 17: Transportation costs between replacement trucks depots and incinerators.

$r\backslash j$	Kawempe	Kiwatule	Kasubi
Kyambogo	0.70	0.40	0.80
Nakasero	0.60	0.60	0.70
Lugogo	0.70	0.50	0.80

waste sources i.

Table 18: Transportation costs between replacement trucks depots and waste sources.

$r\backslash i$	Kamwokya	Kireka	Ntinda	Nalya	Kiwatule
Kyambogo	0.50	0.40	0.30	0.30	0.20
Nakasero	0.50	0.70	0.60	0.70	0.60
Lugogo	0.30	0.50	0.60	0.60	0.60

4.2 Solution, Validity and Robustness of the First Model

The solution to the model has been obtained using a Pentium IV 2.66 GHz computer in less than two seconds. All data from the previous section has been used in the validity test; only some of it has been used in the robustness tests. The reason is that in the robustness tests the attention has been more on the sensitivity to changes in some of the most important parameters as far as the situation of Kampala is concerned. Also, some of the data is in the same category like the transportation costs between waste sources and incinerators, and the transportation costs between incinerators and landfills, etc. In particular the following parameters and functions have been studied; revenue from an incinerator at Kiwatule $rev(kiw)$, total costs $obj(F)$, total truck number used per day $T(tot)$, total replacement trucks number used per day $RT(tot)$, waste amount a source $waste(amt)$ in Ntinda, total waste amount collected from all waste sources per day $W(tot)$, fractions ρ of waste that remains per day at incinerators, and transportation costs from a waste source at Kamwokya and a landfill (tip) at Najeera.

The changes in the parameters $rev(kiw)$, $waste(amt)$, ρ, and the transportation cost between Kamwokya and Najeera have been studied against the total cost $obj(F)$. The findings are given in

Tables (19)-(22), and graphically depicted in Figures (10)-(13). The graphs in these figures have been plotted using matlab 6.5.

The validity of the model has been tested by doubling the values of the parameters except for the values of waste fractions that remain at incinerators and the daily truck breakdown probability. This has not affected the values of the decision variables; it has also indicated that the model has been well formulated.

The robustness of the model described in Section 3.1 has been studied by looking at the changes in some of the key parameters; revenues at an incinerator at Kiwatule, waste source amounts at Ntinda, transportation costs between a waste source at Kamwokya and a landfill at Najeera, changes in the fractions of wastes that is unrecovered at an incinerator at Kiwatule. The changes in these parameters have specifically been observed with respect to total costs; changes in other data like total waste amount from all waste sources every day $W(tot)$ have also been reflected as shown in the tables below. The changes in these parameters have been significant and they have greatly affected the total cost; small changes have induced small changes while big changes have caused big changes in the total cost.

Part of the solution from the case study is shown in the fifth row from the top of Table 19. Since our decision variables measure truck numbers, the sharpness of this solution can be studied by looking at the total amount of waste collected per day from all sources; the actual total waste amount from all waste sources is 1514.2 tons. From Table 19 it is given as 1659.71 tons when $rev(kiw)$ is 6.78, giving a deviation of 145.51 tons from the actual value. In general, the biggest deviation has been observed to be 243.14 tons in Table 22 when the total waste from all sources is given as 1757.34 tons. This inflation in waste amounts is also observed in the total cost $obj(F)$ and the benefits B.

The observed inflation consists in the assumption that the trucks leave the waste collection points when they are fully loaded. Some of these trucks may actually be partially full; so there is an overestimation if the truck number is simply multiplied by the truck capacity to obtain the amount of waste collected. This problem has been overcome by using the modified model described in Section 3.3. The results from using that model are given in Tables 23-26, and Figures 14-17.

Table 19 represents the relationship between the total cost $obj(F)$ and the income per ton of waste $rev(kis)$ from the incinerator at Kiwatule. As the revenue in the interval [4.78, 9.98] increases, the total cost $obj(F)$ decreases. The change in the revenue affects also the benefits B; there are no benefits if the value of $rev(kiw)$ is at most equal to 6.28. The relationship between $obj(F)$ and $rev(kiw)$ is shown in Figure 10; this figure shows that as the revenue increases, the total cost reduces. The total cost does not change when the benefits are at most 6.28 per ton of waste; in this case no waste is taken for incineration because it is not economically profitable.

Table 20 shows a variation in the total cost $obj(F)$ with the variation in the waste amount $waste(amt)$ at a source in Ntinda over the interval [300, 500]. The total cost increases with an increase in waste amount. The variation in $waste(amt)$ also affects $T(TOT)$ but $RT(TOT)$ remains stable. However, small changes in the values of $waste(amt)$ cause small changes in the values of F and $T(TOT)$ over this interval. The relationship between F and $waste(amt)$ is depicted in Figure 11; it is clear from the graph that the values of $obj(F)$ are increasing with the values of $waste(amt)$.

Table 21 shows a variation in the values of $obj(F)$ with respect to changes in the transportation cost $c(soti)$ between a waste source at Kamwokya and a landfill at Najeera over the interval [0.10, 21.00]. It is clear from the table that the higher the transportation costs the higher the total costs. There are slight changes in the values of $T(TOT)$ and B. The values of $RT(TOT)$ do not change while the values of $W(tot)$ change slightly. The relationship between F and $c(soti)$ is given in Figure 12; it is evident from the graph that when eventually the cost to Najeera landfill is very high

Table 19: Total costs and benefits from Kiwatule.

$rev(kiw)$	$obj(F)$	$T(tot)$	$RT(tot)$	B	$W(tot)$
4.78	121803.010	18	3	0000.00000	1667.22
5.28	121803.010	18	3	0000.00000	1667.22
5.78	121803.010	18	3	0000.00000	1667.22
6.28	121803.010	18	3	0000.00000	1667.22
6.78	121705.704	18	3	1496.98332	1659.71
7.28	121595.307	18	3	1607.38032	1659.71
7.78	121484.910	18	3	1717.77732	1659.71
8.28	121374.513	18	3	1828.17432	1659.71
8.78	121264.116	18	3	1938.57132	1659.71
9.28	121153.719	18	3	2048.96832	1659.71
9.98	120999.163	18	3	2203.52412	1659.71

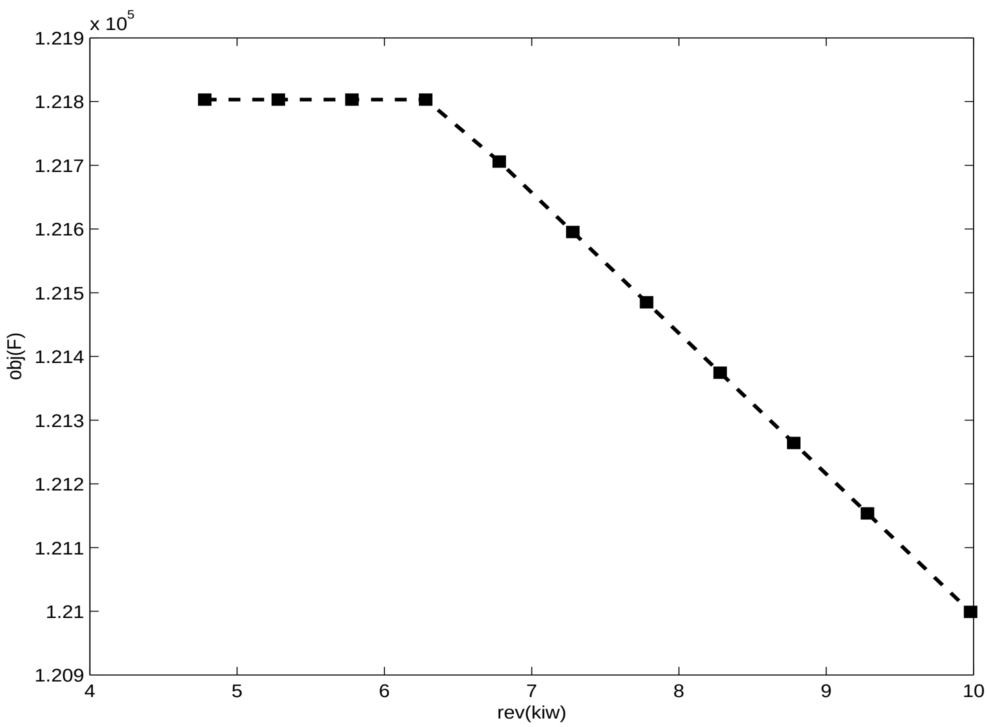

Figure 10: Total cost plotted against benefits from incineration

Table 20: Total costs and waste amounts at Ntinda.

$waste(amt)$	$obj(F)$	$T(tot)$	$RT(tot)$	B	$W(tot)$
300	116018.254	17	3	1496.98332	1547.060
320	116153.411	17	3	0000.00000	1577.100
340	116203.127	17	3	0000.00000	1614.650
360	116203.127	17	3	0000.00000	1614.650
380	121667.853	18	3	1496.98332	1637.180
400	121705.704	18	3	1496.98332	1659.710
420	121852.726	18	3	0000.00000	1704.770
440	121852.726	18	3	0000.00000	1704.770
460	127136.186	19	3	1496.98332	1697.260
480	127317.452	19	3	1496.98332	1727.300
500	127452.609	19	3	0000.00000	1754.340

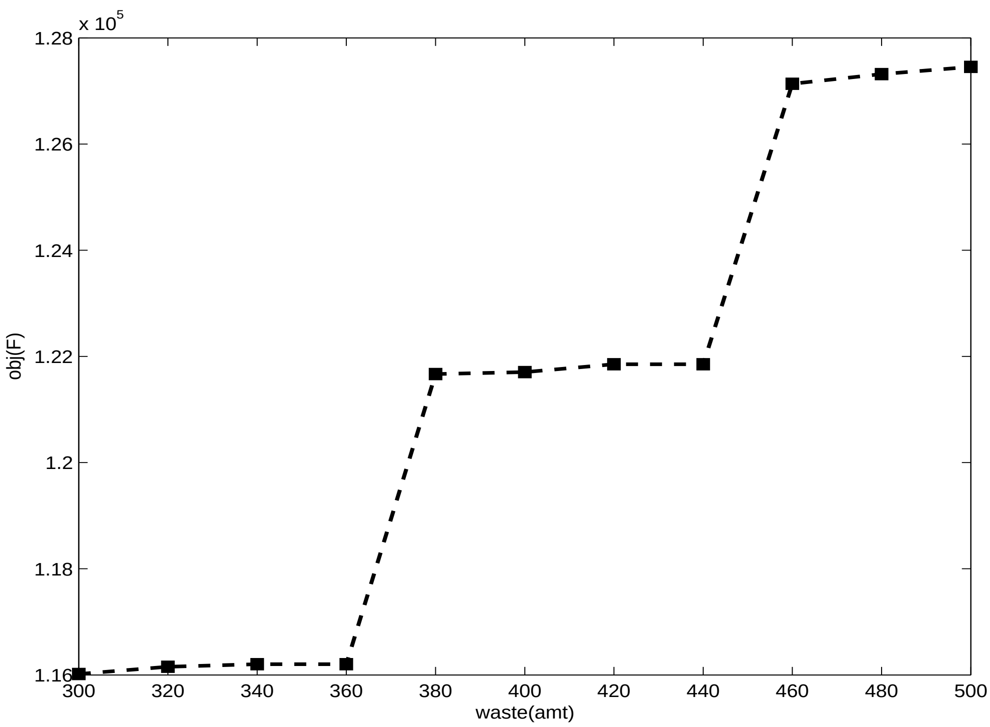

Figure 11: Total cost plotted against waste amount at Ntinda

no waste will deposited there and consequently any further increments in the cost will not affect the total cost.

Table 21: Total costs and transportation costs from Kiwatule waste source to Kiteezi landfill.

$c(soti)$	$obj(F)$	$T(tot)$	$RT(tot)$	B	$W(tot)$
0.10	121570.524	18	3	1496.98332	1659.710
0.30	121624.596	18	3	1496.98332	1659.710
0.60	121705.704	18	3	1496.98332	1659.710
0.80	121759.776	18	3	1496.98332	1659.710
1.00	121813.848	18	3	1496.98332	1659.710
10.00	124247.088	18	3	1496.98332	1659.710
15.00	125598.888	18	3	1496.98332	1659.710
17.00	125878.697	19	3	2993.96664	1629.670
19.00	125878.697	19	3	2993.96664	1629.670
21.00	125878.697	19	3	2993.96664	1629.670

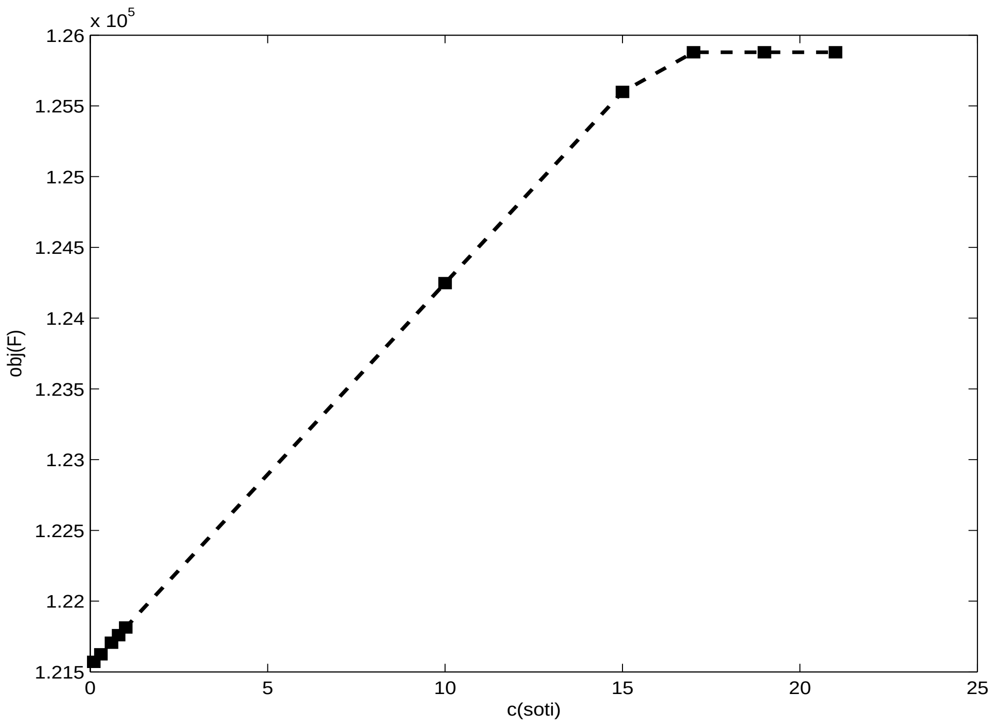

Figure 12: Total cost plotted against transportation costs from Kiwatule to Kiteezi

Table 22 shows the variation in the values of $obj(F)$ with respect to changes in the values of ρ in the interval [0.0, 0.6]. It is clear from the table that the lower the value of ρ, the lower the total costs. There are slight changes in the values of $T(tot)$ but $RT(tot)$ remains stable. There may be no benefits if the value of ρ is at least equal to 0.4; this is shown in the table and in Figure 13. A graphical relationship between $obj(T)$ and ρ is given in Figure 13; it is evident from the graph that the lower ρ is the lower the total cost. This means that the better the quality of the incinerators the

Table 22: Total costs and fractions of unrecovered waste at incinerators

ρ	$obj(F)$	$T(tot)$	$RT(tot)$	B	$W(tot)$
0.000	106803.046	17	3	10030.80660	1757.34
0.025	112685.992	18	3	9780.03643	1757.34
0.050	112936.762	18	3	9529.26627	1757.34
0.100	117459.517	18	3	4490.94996	1742.32
0.150	119587.758	18	3	3635.53092	1629.67
0.200	120769.988	18	3	2525.52288	1644.69
0.250	121291.242	18	3	1985.79420	1674.73
0.300	121705.704	18	3	1496.98332	1659.71
0.400	121803.010	18	3	0000.00000	1667.22
0.500	121803.010	18	3	0000.00000	1667.22
0.600	121803.010	18	3	0000.00000	1667.22

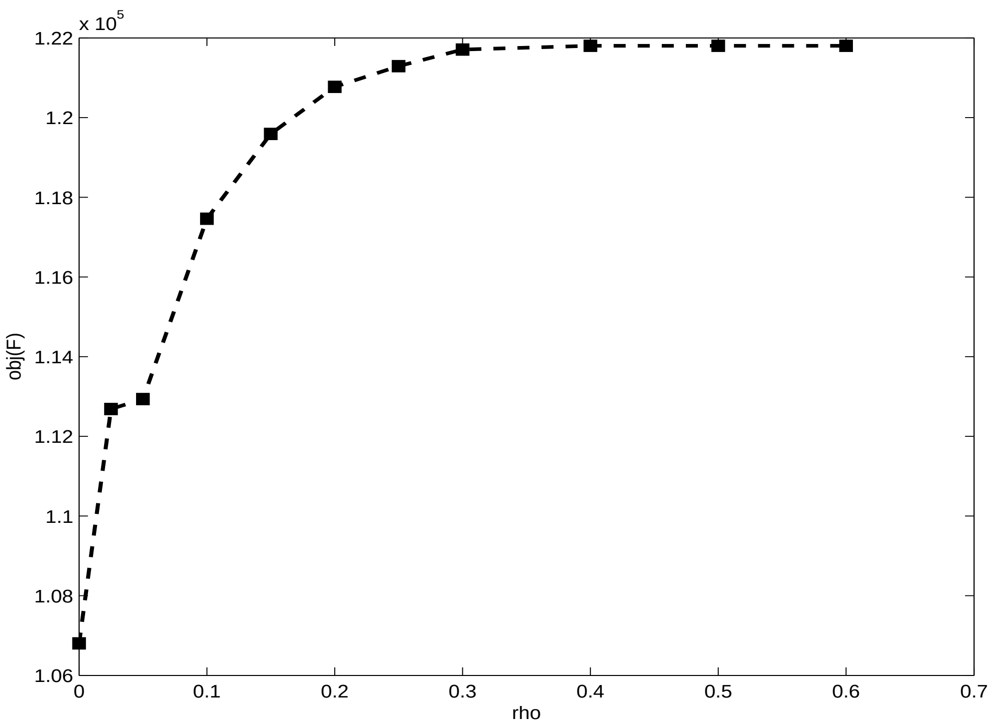

Figure 13: Total costs plotted against fractions of unrecovered waste

lower the expenditure; eventually if the quality of the incinerators is very bad then there will be no gain at all.

These tests and the graphical observations have indicated that the model is satisfactorily robust. The nature of the graphs in figures 10, 11, 12, and 13 is not surprising since the objective function of the model described in Section 3.1 is a linear combination of convex functions defined over convex sets. The variables in the model are integer; if they were to be continuous then a piecewise linear appearance would be depicted in the graphs. This would be so because of the piecewise linearity of the functions involved (see Murty [50]) in the objective function. The graphs in Figures (10)-(13) are composed of discrete points that have been joined by lines in order to show the trend of the plots.

4.3 Solution, Validity, and Robustness of the Second Model

In this section we present the solution to the second model, as well as the findings from the sensitivity analysis tests conducted on the first model described in Section 3.1 and the second model described in Section 3.3.

The solution to the model has been obtained using a Pentium IV 2.66 GHz computer in less than five seconds. Sensitivity analysis tests have been conducted on the second model over the same intervals of data like in the case of the first model. The findings are summarized in Tables 23-26; the number of trucks given in these tables is, in general, the same as that given in Tables 19-22 for the first model. The graphical comparison of the two models is given in Figures 14-17. The second model gives superior values of the total cost and the amount of waste at the collection points.

Table 23 gives the variation of the total cost with respect to the change in the revenue from the incinerator at Kiwatule, over the interval [4.78, 9.78]. The total cost falls with an increase in the revenue; this is also the case for the first model. Figure 14 compares the differences in the total cost for the two models; the second model gives much better total cost values.

Table 23: Total costs and benefits from Kiwatule.

$rev(kiw)$	$obj(F)$	$T(tot)$	$RT(tot)$	B	$W(tot)$
4.78	121504.482	18	3	0000.00000	1514.2
5.28	121504.482	18	3	0000.00000	1514.2
5.78	121504.482	18	3	0000.00000	1514.2
6.28	121504.482	18	3	0000.00000	1514.2
6.78	121427.152	18	3	1496.98332	1514.2
7.28	121316.755	18	3	1607.38032	1514.2
7.78	121206.358	18	3	1717.77732	1514.2
8.28	121095.961	18	3	1838.17432	1514.2
8.78	120985.564	18	3	1938.57132	1514.2
9.28	120875.167	18	3	2048.96832	1514.2
9.78	120764.770	18	3	2159.36532	1514.2

Table 24 gives the relationship between the total cost and the waste amount at Ntinda, as the waste amount is varied over the interval [300, 500]. It is evident from the table and Figure 15 that the total cost increases with the waste amount. It is clear from the Figure that the two models have very close total cost values.

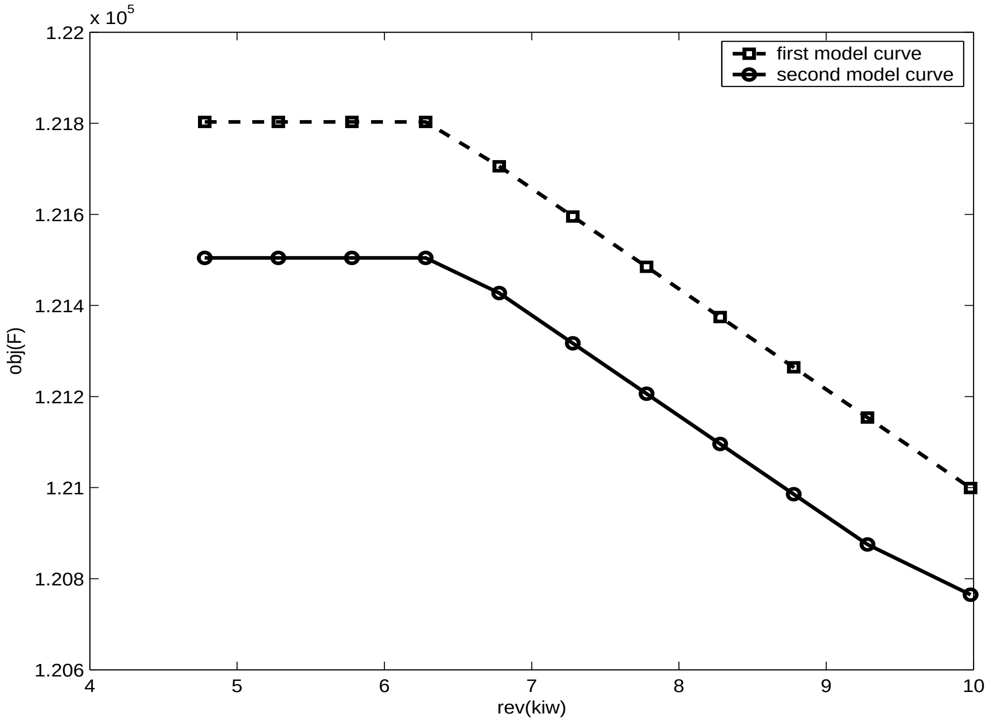

Figure 14: Total cost plotted against benefits from incineration

Table 24: Total costs and waste amounts at Ntinda.

$waste(amt)$	$obj(F)$	$T(tot)$	$RT(tot)$	B	$W(tot)$
300	115724.443	17	3	1544.50660	1414.2
320	115871.682	17	3	0000.00000	1434.2
340	115904.882	17	3	0000.00000	1454.2
360	115938.082	17	3	0000.00000	1474.2
380	121357.118	18	3	1544.50660	1494.2
400	121427.152	18	3	1496.98332	1514.2
420	121537.682	18	3	0000.00000	1534.2
440	121570.882	18	3	0000.00000	1554.2
460	126896.975	19	3	1544.50660	1574.2
480	127023.243	19	3	1544.50660	1594.2
500	127170.482	19	3	0000.00000	1614.2

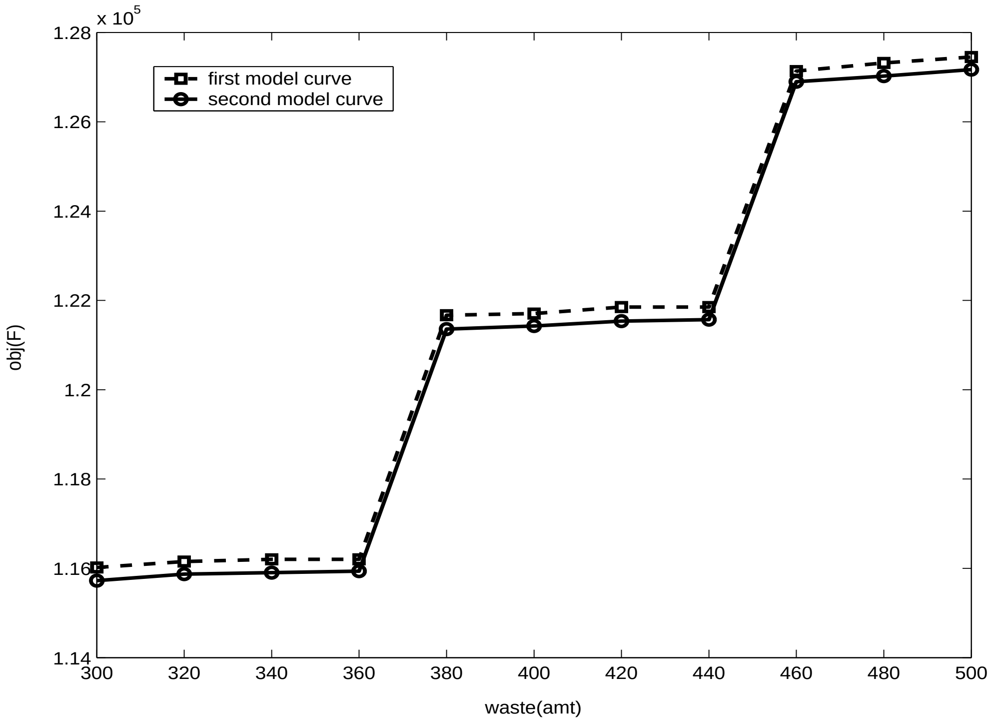

Figure 15: Total cost plotted against waste amount at Ntinda

Table 25 gives the relationship between the total cost and the changes in the transportation cost between a waste source at Kamwokya and a landfill at Najeera, over the interval [0.10, 21.00]. The table and Figure 16 show that the total cost increases with the transport cost. Again in this case, the second model gives much better total cost values than the first model. The gap between the total costs of the two models increases with the transportation cost. This is because the waste amounts are steadily inflated in the integer model (see Tables 21 and 25) so that as the transportation cost increases the gap between the two objective functions inevitably increases. The realistic transportation cost values lie between 0 and 1.5; the values of the interval between 10 and 21 were only considered to check whether the model does what is expected of it as the cost grows.

Table 25: Total costs and transportation costs from Kiwatule waste source to Kiteezi landfill.

$c(soti)$	$obj(F)$	$T(tot)$	$RT(tot)$	B	$W(tot)$
0.10	121302.152	18	3	1496.98332	1514.2
0.30	121352.152	18	3	1496.98332	1514.2
0.60	121427.152	18	3	1496.98332	1514.2
0.80	121477.152	18	3	1496.98332	1514.2
1.00	121527.152	18	3	1496.98332	1514.2
10.00	123777.152	18	3	1496.98332	1514.2
15.00	124677.971	19	3	2376.16400	1514.2
17.00	124677.971	19	3	2376.16400	1514.2
19.00	124677.971	19	3	2376.16400	1514.2
21.00	124677.971	19	3	2376.16400	1514.2

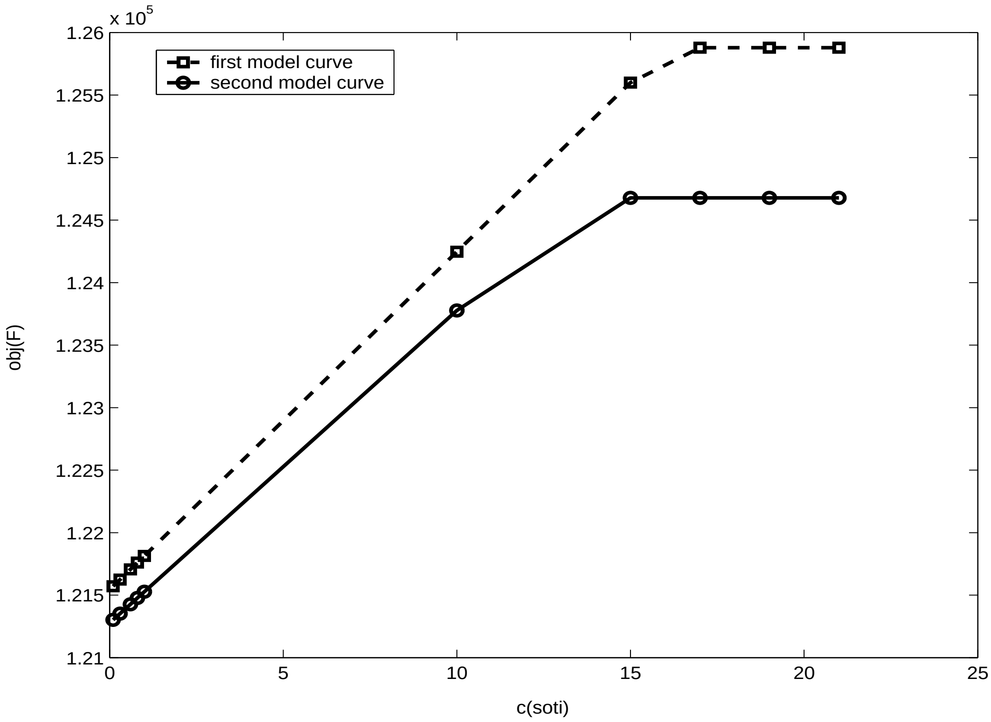

Figure 16: Total cost plotted against transportation costs from Kiwatule to Kiteezi

Tables 26 relates the total cost and the fraction ρ of waste that remains at the incinerator at Kiwatule over the interval [0.0, 0.6]. As the value of ρ increases in this interval, the value of the total cost grows; this means that the total cost is lowered with high quality incinerators. This is also the case for the first model. The two model solutions are compared in Figure 17; in general they are seen to be close. The integer model appears to do better than the mixed integer model for the first three values of ρ. This is false; the reason is that when there are benefits at some node there is a tendency to move as much waste as possible to that node as long as there is space on the truck. This is a weakness owing to the fact that all variables are integer; this weakness is reduced in the mixed integer model.

Table 26: Total costs and fractions of unrecovered waste at the incinerators

ρ	$obj(F)$	$T(tot)$	$RT(tot)$	B	$W(tot)$
0.000	107430.258	17	3	8911.63200	1514.2
0.025	113210.882	18	3	8688.84120	1514.2
0.050	113491.507	18	3	8466.05040	1514.2
0.100	116976.126	18	3	4582.60200	1514.2
0.150	118536.617	18	3	2885.34200	1514.2
0.200	120372.842	18	3	2647.72560	1514.2
0.250	120983.030	18	3	1985.79420	1514.2
0.300	121427.152	18	3	1496.98332	1514.2
0.400	121504.482	18	3	0000.00000	1514.2
0.500	121504.482	18	3	0000.00000	1514.2
0.600	121504.482	18	3	0000.00000	1514.2

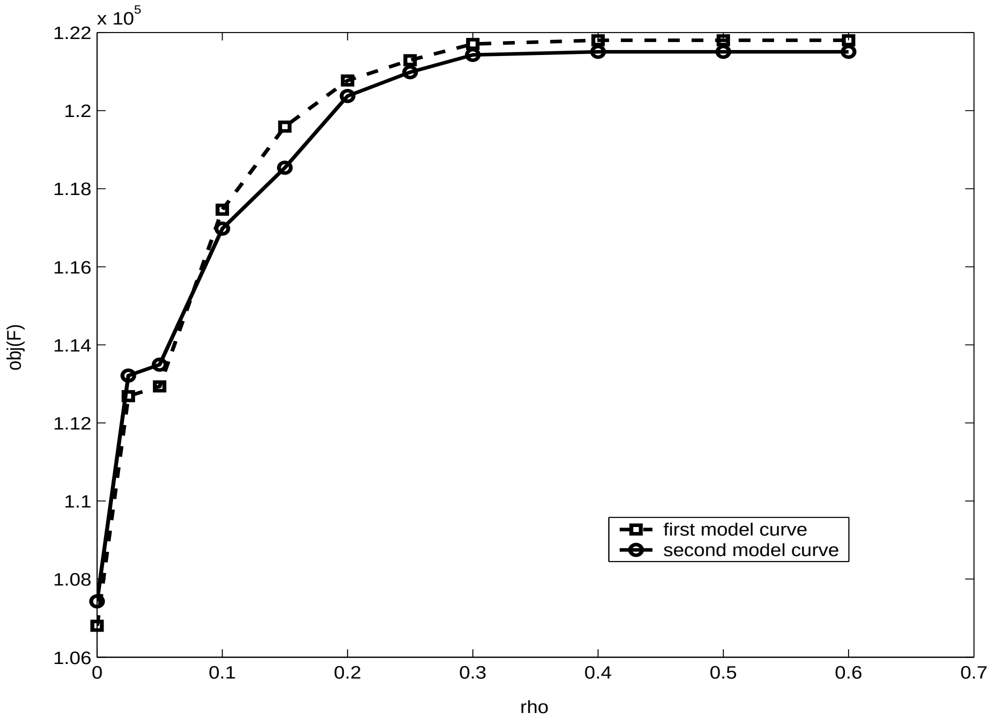

Figure 17: Total costs plotted against fractions of unrecovered waste

5 AMPL and CPLEX

The AMPL modelling system is a comprehensive, powerful, algebraic modelling language for problems in linear, non-linear, and integer programming. With AMPL models with maximum productivity can be created, and by using AMPL's natural algebraic notation, very large, complex models can be stated in a concise, understandable form. Since its models are easy to understand, debug, and modify, AMPL also makes model maintenance easy (see Fourer et al [23]). AMPL-compatible solvers include CPLEX, FortMP, MINOS, MINLP and others (see AMPL website: http://www.ampl.com). CPLEX is designed to solve linear programs, integer programs, mixed integer programs, and quadratic programs.

5.1 Linear Programming

CPLEX employs either a simplex or a barrier method to solve linear programming problems. Four distinct methods of optimization are incorporated in the CPLEX package:

- A primal simplex algorithm that first finds a solution feasible in constraints (Phase I), then iterates towards optimality (Phase II).
- A dual simplex algorithm that finds a solution that satisfies the optimality conditions (Phase I), then iterates towards feasibility (Phase II).
- A network primal simplex algorithm that uses logic and data structures tailored to the class of pure network linear programs.
- A primal-dual-barrier (or interior point) algorithm that simultaneously iterates toward feasibility and optimality, optionally followed by a primal or dual crossover routine that produces a basic optimal solution.

CPLEX normally chooses one of these algorithms, but its choice can be overridden by the directives described in the ILOG AMPL CPLEX system [41], pp 30-32. These directives (or specific options) apply to the solution of linear programs, including network linear programs. There are also directives for processing, controlling the simplex algorithm, controlling the barrier algorithm, improving stability, starting and stopping, controlling output described in the ILOG AMPL CPLEX system [41], pp 32-44.

CPLEX is highly robust and has been designed to avoid problems such as degenerate stalling and numerical inaccuracy that occur in the simplex algorithm. However, some linear programs can benefit from the adjustments to the stability directives if difficulties arise.

5.2 Integer Programming

For programs that contain integer variables, CPLEX uses a branch and bound approach. Because a single integer program generates many integer program sub problems, even small instances can be very computation-intensive and require significant amounts of memory. In contrast to solving linear programming problems, where little user intervention is required to obtain optimal results, some of the directives for preprocessing (see the ILOG AMPL CPLEX system, pp 47-50) may have to be set to get satisfactory results on integer programs. Either the way that the branch and bound algorithm works can be changed, or the conditions for optimality can be relaxed.

Other directives (see the ILOG AMPL CPLEX system [41], pp 50-60) include those for algorithmic control, relaxing optimality, halting and resuming the search, and controlling output. All processing directives that apply to linear programming are also applicable to problems that specify integer-valued variables. The directives on pp 47-50 control additional preprocessing steps that are applicable to certain mixed integer program only.

In dealing with a difficult integer program, it may be better to settle for a "good" solution rather than a provably one. Directives for relaxing optimality, described on pages 56-57, offer various ways of weakening the optimality criterion for CPLEX's branch and bound algorithm.

The most common problems faced in solving mixed integer programs with CPLEX are due to running out of memory, failure to reach optimality, difficult mixed integer program sub problems. The problems and the ways to overcome them are described in the ILOG AMPL CPLEX system [41], pp 60-63.

6 Conclusions and future developments

An integer linear programming model and a mixed integer linear programming model have been proposed, and confirmed to be valid and robust. Their performance has been studied using a hypothetical case study, and other smaller models using AMPL/CPLEX. The mixed integer linear model has been found to be more precise in measuring waste flow amounts among various nodes in the model and total daily costs incurred in the management of waste. However, the integer linear model cannot be discarded because the choice between the two models depends on the interest of the user. One user prefers to measure transportation costs in terms of costs per trip from a waste source while another user wants to measure the transportation costs in terms of costs per unit mass of waste moved from a waste source. The technology in place can as well influence the choice of the model to apply. For example in the Ugandan situation, where it is not possible at the moment to measure waste from waste sources, the integer linear model is more appropriate. In this case we replace the coefficients of the variables X and Y in the objective function with the total costs per trip from the waste collection point. At the same time, instead of measuring the amount of waste using the number of trucks used multiplied by their capacities, continuous variables can be introduced to measure directly the amount of waste that goes to the plants and landfills. The integer linear problem is then transformed into a mixed integer problem that gives better total cost estimates and more precise waste amount measurements.

Through these models it is not only possible to obtain waste amounts transported to various facilities, but also obtain the number of trucks as well as replacement trucks used in doing so. The main pitfall of the integer linear model is due to the use of the number of trucks used to measure the amount of waste transported; the values of the waste amounts, the total cost, and the values of the benefits from the plants may be inflated, since the trucks are assumed to be fully loaded upon leaving the waste collection points. Some of the trucks may in reality be partially full, and this unexploited capacity leads to errors in computing the total costs, measuring waste amounts, and the benefit values. The weakness common to both models is that the estimated values of the parameters may not reflect the true behaviour of the parameters at the time of estimation. The findings indicate that both models can be useful decision support tools in the planning and management of municipal solid waste collection, transportation, incineration, recycling, composting, and disposal programs. They can as well be used as design tools for setting up plants, truck depots, and landfills.

The models may be useful in other areas of application. For instance, suppose an investor is opening an industry to produce various products. He also wants to open warehouses and find agents for his products in some potential areas. He intends to use trucks of various capacity to transport the goods to the warehouses and to his agents. The problem may be that of opening and running warehouses, and distributing the goods among the agents at minimum transportation costs. The first model can be adapted by considering collection points as the industry and its branches, the plants can be taken for the warehouses, and the landfills can be the agents.

In the order of importance, the work is to be extended or modified in the following ways;

1. Robustness issues are to be examined more globally by studying the performance of the models, under the changes in all key parameters. For instance we have studied the changes in the total costs with respect to the changes in the fraction of waste that remains at the incinerators, and these fractions have been assumed to be dependent on the incinerators only. We have not considered the dependence of these fractions on the type of waste.

2. Another consideration is that of extending the deterministic models to stochastic ones by treating the data as a random variable because of random fluctuations in some of the key parameters, like the amounts of waste at the collections points. In many real world problems, the yield or total expenditure, etc. is almost never known with certainty. It is a random variable that is subject to many random fluctuations that are not under our control. For example the daily total expenses depend on transportation costs per unit waste, the quality of the plants, the amount of waste at the collection points, etc. To analyse the problem treating the yield as a random variable requires the use of stochastic programming models (Murty [51]). The construction of stochastic models can be done by making assumptions about the nature of the probability distributions of the random data elements, or by estimating these distributions from the past data. The challenge is that the closeness of the optimum solution obtained from the model may depend on how close the selected probability distributions are to the true ones (Murty [51]).

3. The element of time (dynamic element) is to be introduced into the general models; for instance we can consider daily activities within time period $t = 1, \dots, T_0$, where some parameters can change with time t. Planning involves time, and if an application is concerned with a situation that lasts for days or months or years, the same types of decisions may have to be made everyday, for example (Murty [51]).

   When planning a multi-period horizon (say T_0), and there is no change in the data at all from one period to the next, then the optimum solution for the first period found from the static model for that period , will remain optimal for each period in the planning horizon.

   In most multi-period problems, data changes from period to the next are significant, and the optimum decisions for the various periods may be different, and the sequence of decisions will be interrelated. Designing a dynamic model with the aim of finding a sequence of decisions (one for every period) that is optimal for the planning horizon as a whole, requires reasonably accurate estimates of data for every period of the planning horizon. This is a challenge, but if such data is available, a dynamic model tries to find the entire sequence of interrelated decisions that is optimal for the model over the entire planning horizon (Murty [51]).

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