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A Scenario-Based Stochastic Programming Model for the Optimization of Process Integration Opportunities in a Pulp Mill

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Abstract

Several studies show that substantial industrial energy savings can be achieved through process integration. The returns on such investments are, however, uncertain because of uncertainties in future energy prices and policies. This article presents a stochastic mixed-integer programming approach which enables the identification of robust process integration investments under uncertainty. The proposed approach is applied to the case of a pulp mill for which the complete optimization model is presented. The model is a scenario-based multistage stochastic programming model with the objective of maximizing the net present value of the investments. The model also enables the optimization of investment timing. We show as one important result that the probability distribution can be varied rather much without a change in the optimal solution. This implies that the stochastic programming approach is a valuable tool although the true probabilities for the future scenarios are not known.

Keywords: investment analysis, multistage stochastic programming, scenarios, decision support analysis, process integration.

1 Introduction

The cost-effectiveness of industrial investments in energy efficiency is strongly related to constantly changing energy market conditions, making decision-making regarding such investments a complicated task. In particular, the increased climate concern in society leads to higher CO_2 emissions charges. Although such an increase of the emissions charges makes investments in energy efficiency more profitable, uncertainty about the future development of climate policies might, however, make it more difficult to evaluate the investments. In the worst case, no energy efficiency investments are made, although they should be profitable, because of the difficulty of knowing what the outcome will be of the investment decision. Blyth et al. (2007) conclude that in order for policy-makers to promote low-carbon technologies, some long-term certainty about the future policy development should be provided.

For strategic investments especially, profitability depends on the future energy market. Electricity and fuel prices, emissions charges and taxes are all examples of energy market parameters that are highly uncertain, but directly influence the profitability of the investments. In a stochastic programming approach, the uncertainties are explicitly incorporated in an optimization model, and the investment planning is improved. In stochastic programming it is assumed that, like in reality, investments are made before the outcome of the uncertain parameters is revealed. The objective is to maximize the expected net present value of the investments over all future scenarios. This kind of approach will provide better information to base the decisions on. Several recent studies dealing with energy-related investment decisions confirm the importance of accounting for uncertainty and timing (Blyth et al., 2007; Laurikka, 2006; Wickart and Madlener, 2007; Yang et al., 2007).

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This article presents a multistage stochastic programming model for the investment planning of process integration measures in a pulp mill. The objectives are mainly to present a complete model for this kind of optimization, and to illustrate and discuss the modelling issues that arise. Since the decision problem is an engineering design problem, there are integer requirements on the variables. The objective is the net present value of the investments which can be made at multiple stages, making the economic evaluation more complicated than if all of the investments were made at the same time.

The term ‘process integration’ is a wide concept that refers to systematic methods for optimization of production systems, primarily with respect to energy efficiency and reduction of environmental effects. This paper gives a description of a stochastic programming model, which is also the mathematical framework used in previous work by the authors on the optimization of process integration investments under energy market uncertainty (Svensson E et al., 2008a,b). Here, some new angles for the analyses of results are provided which have not been presented in the previous papers. These new results include further analyses of the robustness of the optimal solution and an evaluation of the investment timing modelling.

In stochastic programming, the probabilities for different future scenarios are assumed to be known. For the kind of uncertainties that are dealt with here, assumptions can be made about probabilities, but there is no way to define a ‘true’ distribution. In this paper, we present a case study of a pulp mill for which we show that the optimal solution is actually not very sensitive to moderate changes in the assumed probability distribution. Hence, the stochastic programming approach can be adopted also when the probabilities for the future scenarios are not know for certain, and the optimal solution can be considered a robust solution. The robustness is closely related to the integer requirements on the decision variables, which result in the model having quite few feasible solutions. This relation between integrality and robustness is further discussed in Section 4.

2 The case study

The model used for this study is the same as the one used in previous work by the authors (Svensson E et al., 2008a). The focus then was to illustrate what kind of results can be achieved by using such a stochastic programming approach for the optimization of energy efficiency investments. Here, the focus is rather on the underlying mathematical model that constitutes the framework of the proposed optimization methodology.

The pulp and paper industry is the fourth largest industrial energy user in the world (IEA, 2007), which makes it an important sector in the progress to mitigate climate change. Cost-effective energy savings have been identified in the pulp and paper sector in several studies (Axelsson and Berntsson, 2008; Martin et al., 2000; Möllersten et al., 2003). The cost-effectiveness of the proposed measures is, however, depending on, for example, the electricity and wood fuel prices, which are uncertain.

The analyzed mill is a computer model of a typical Scandinavian pulp mill. It was originally developed for the Swedish national research programme ‘The Future Resource Adapted Pulp Mill’ (FRAM, 2005). The mill will increase its production by 25% in the near future, which renders the opportunity to make other changes in the process. The production increase case has previously been studied by Axelsson et al. (2006b).

When the pulp production is increased at a pulp mill, the recovery boiler is often a bottleneck. To the recovery boiler comes a process stream of black liquor from the pulp digester which contains, among other substances, pulp digesting chemicals, but also lignin which is a biomass by-product in the pulp production process. In the recovery boiler, the digester chemicals of the black liquor are recovered. In addition, the energy of the lignin is utilized to produce high-pressure (HP) steam. The production increase will lead to an increase of black liquor flow to the recovery boiler, but also an increased steam demand of the process.

Traditionally, a recovery boiler upgrade (RBU) has to be carried out–at substantial costs–to meet the new capacity requirements, see Figure 1a. However, since more HP steam can be produced in the upgraded boiler, there will be a possibility to increase the electricity production. Alternatively, the recovery boiler upgrade can be avoided by decreasing the load on the recovery boiler. This can be achieved by separating lignin from the black liquor (Axelsson et al., 2006b), see Figure 1b. The lignin can be exported as wood fuel. In this approach, the steam production cannot be increased, since the heat of the separated lignin is not utilized. Without any other process changes, the steam demand of the process will, however, still increase. Nevertheless, lignin extraction will remain an interesting option if enough steam savings are carried out to prevent the steam demand from increasing.

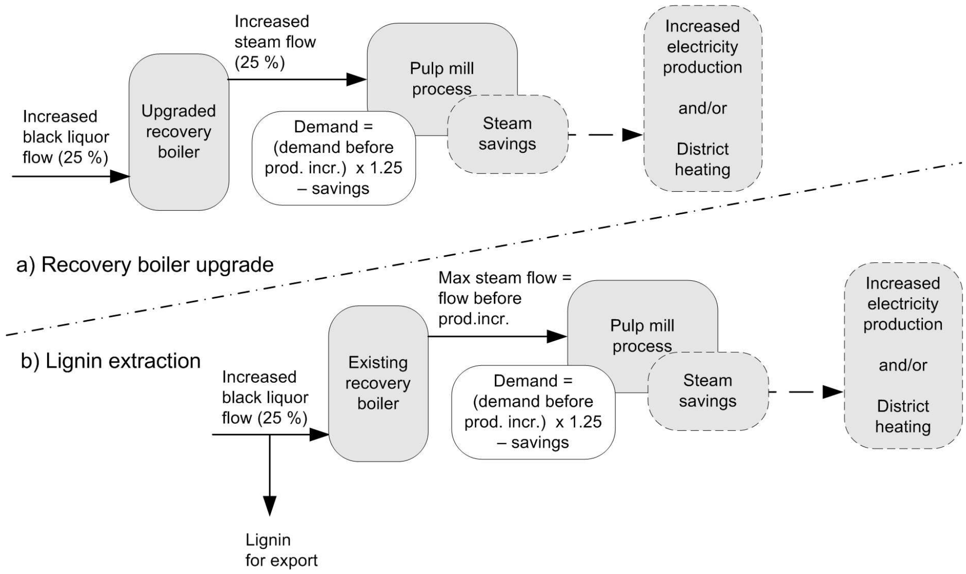

Figure 1: Two approaches to increased production. a) Upgrading the recovery boiler to the new capacity requirements. b) Debottlenecking the recovery boiler by separating lignin from the black liquor.

With substantial steam savings it might even be possible to achieve an energy surplus at the mill. Different opportunities for energy efficiency can be identified by using process integration methods. Pinch technology (Kemp, 2007; Smith, 2007) is one such process integration method which was used by Axelsson et al. (2006a) to identify the potential for energy savings at the studied mill. The achieved steam surplus can be used to increase the electricity production, to further increase the lignin extraction, or for district heating. High- and/or medium-pressure steam can be used to produce electricity in a back-pressure turbine while low-pressure steam can be used in a condensing turbine.

District heating (DH) can be produced from low-pressure steam or from excess heat of a lower temperature, for example hot water. The district heating potential depends on the demand and the alternative district heating production in nearby communities. There is generally a larger potential for profitable excess heat cooperation between mills and energy companies in small district heating systems (Jönsson et al., 2008) than in larger systems. Hence, we assume here the presence of a small district heating system nearby, and use the data for the small system studied by Svensson IL et al. (2008c).

The overall system consequences of process integration are, as can be understood from the above description, complex to evaluate. The opportunities for electricity production, district heating, and lignin extraction are closely related to each other, as well as on how far the steam savings are taken. The optimization formulation of the problem enables a modelling of the system without knowing the overall consequences of every decision. There is, for example, no need here to know the exact amount of steam savings that are needed to avoid the recovery boiler upgrade. Instead, it is sufficient to set the required lignin extraction rate and to model the relation between steam savings and lignin extraction.

3 The optimization model

The objective of the optimization is to maximize the expected net present value (NPV) of energy efficiency investments at a pulp mill. The general assumption is that decisions are made ’here-and-now’, before uncertainties are resolved and any energy market changes occur. We assume that a point in time when investment decisions can be made is followed by a period of a couple of years, when no new investments can be made. This gives rise to two stages of the programming model. The cash flow of the second stage, that is the period when no investments can be made, is a function of the previous investment decisions, the energy prices, and the operative decisions. A model of this kind, with two types of decisions where the second one is a reaction to the first as well as on the realization of the uncertain parameters, is termed a recourse model. The model presented here is in fact a multistage model. This means that after each investment period, new investments can be made. After each point in time where investments are decided on follows a period with realizations of uncertain parameters and changed cash flows. The uncertain parameters, typically energy prices, are modelled using scenarios.

As the problem at hand is basically an engineering design problem, it typically involves simulations, experimental data, and catalogue selections to establish the functions connecting design variables and the dependent characteristics and attributes of the design. Because it is, in practice, impossible to express these relations as analytical continuous functions, the decision variables are typically binary, expressing a choice between discrete options, for which the dependent characteristics can be established in advance. Here, we additionally require the final optimization model to be linear, as the solver intended to be used is restricted to mixed-integer linear programming (MILP) models. The model was formulated in AMPL (Fourer et al., 2003) and solved using the MILP solver CPLEX (ILOG, 2006). The introduction of binary variables into the optimization model increases its computational complexity and thus the solution time. The scenario-based modelling of the random variables further increases the number of decision variables, making the model grow combinatorially with a corresponding considerable increase in computing time. However, the model will remain a MILP model, also in the presence of recourse, which is an advantage since there are algorithms for solving that kind of models. The theory of stochastic linear programming is covered in, e.g., Birge and Louveaux (1997) and more recently in Ruszczyński and Shapiro (2003) and Kall and Mayer (2005). Stochastic integer linear programming is described in, e.g., Louveaux and Schultz (2003) and Sen (2006).

The main parts of the model were developed for a general stochastic optimization of investments in energy efficiency measures. Those parts are not limited to the use of a specific mill or industry or a specific set of measures or ways of benefiting from the implemented energy savings. However, parts of the model have to be built upon assumptions about a specific case. The whole model is presented here for the case of a production increase in the model pulp mill described in Section 2. This enables clearer explanations of the different parts of the model, which means, for example, that energy savings are supposed to be in the form of steam savings. Wood input to a pulp mill is of course primarily a raw material for the pulp production, but wood by-products are also used to cover the energy demand of the process. The model mill studied here is, like many chemical market pulp mills of today, self-sufficient in energy supply from the wood by-products and no additional fuel is imported. Hence, energy savings will lead to a heat surplus at the mill since the wood input cannot be decreased if the production is to be kept constant. An obtained heat surplus will therefore, in such mills, enable an increased export of electricity, wood fuel, or heat. For many other industries, energy savings are primarily enabling a decreased import of fuel.

3.1 The scenario tree

A scenario model is developed to handle the uncertainties in future energy prices and policies such as taxes. The characteristics of the uncertainties make it, in practice, impossible to completely describe the set of possible future scenarios, but also to know the probabilities for different scenarios. The developed scenario model is therefore kept simple. Since electricity and fuel prices are strongly linked, a few consistent energy market parameter sets are being used as building blocks in a scenario tree. These parameter sets can represent, for example, different ambition levels for CO_2 reduction. These blocks are combined to form different scenarios, for which the probability should then be estimated.

The scenario model is based on a tool for generating consistent energy market parameter sets (Axelsson et al., 2007). The tool is used to create three different scenario building blocks, and present Swedish conditions are used to form a fourth one. The different blocks are further described below, and their data are given in Table 1.

Block I The Swedish energy market in the near future. Electricity and wood fuel prices as well as marginal power production technology and policy instrument conditions are based on data from the first quarter of 2006.

Block II A 'business as usual' (BAU) evolution of society.

Block III A 'moderate change' evolution of society where the CO_2 emissions charge is increased relative to the present value (corresponding to an assumed decrease on the CO_2 emissions cap). The green power certificates are however assumed to drop in price because of the higher CO_2 charge which also promotes green electricity production.

Block IV A 'sustainable' evolution of society, i.e. the CO_2 emissions charge is further increased relative to block III. Consequently, the green power certificates are further reduced in price as well.

Table 1: Energy market parameter sets for the four scenario building blocks.

	Scenario Block			
Price parameters [€/MWh]	I	II	III	IV
Electricity price	38.6	57.3	60.8	61.9
Green electricity certificates	21.7	16.0	10.6	5.3
Lignin price	19.5	22.9	26.9	31.0
District heating price	21.3	25.3	29.5	33.7

Based on these four blocks, a number of possible scenarios are constructed. The ideas follow the work by Ådahl and Harvey (2007). The parameters are assumed to be constant for periods of five years, and the total calculation horizon is 30 years, which in this case study corresponds to the economic lifetime of the investments. The final scenario tree is illustrated in Figure 2.

BAU denotes a 'business as usual' development, with minor attention to climate issues. M1 denotes moderate climate concern in the distant future while M2 denotes moderate concern in the near future. S1 and S2 describe a development towards sustainability in the distant and the near future respectively. Finally path E denotes an extreme development towards sustainability, where a radical change happens in the very near future. The probabilities for each node in the scenario tree can easily be calculated if the probabilities for each of the development paths are given. These probabilities are of course not known, but they can easily be changed to test different assumptions.

Finally, we introduce some notation related to the scenario tree. The set of all nodes in the scenario tree is denoted by N, and R is the root node. A specific node in the scenario tree is referred to by its node number n, the parent of that node is given by $p(n)$, and the level of that node is given by $\ell(n)$, where $\ell(0) = 0$. The levels of the scenario tree represents a time scale where each level corresponds to five years.

We start by modelling the objective function which is to maximize the net present value. Then we model the investment costs. After that, we move on to the formulation of a function for the revenue. The necessary constraints, as well as the input data that are needed are presented along the way.

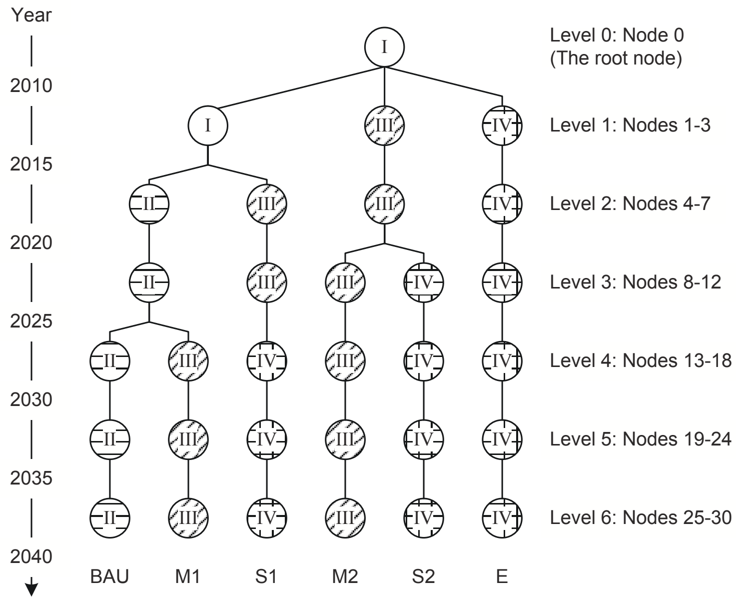

Figure 2: Scenario tree.

3.2 The objective function: The expected net present value

The multistage model deals with successive decisions on energy efficiency investments, which generate a revenue by enabling a decreased import of fuel or, as in this case study, an increased export of electricity, heat, or wood fuel. The revenue is determined by the energy price levels of the different scenarios and by the exports from the different options, that is, the power generated by the turbines, the exported lignin, and the district heating deliveries. Adjustments are made for the residual value of investments at the end of the analyzed time horizon. The objective is to maximize the expected net present value, which is defined by the following formula

$$\text{maximize} \quad \text{E[NPV]} := \sum_{n \in N} \mathrm{pr}^n \left(\sum_{k=1}^{\tau} \frac{f_R(\alpha^n, \xi^n)}{(1+r)^{(\ell(n)-1)\tau+k}} - \frac{\left((1+r)^{T-\ell(n)\tau} - 1\right) f_C(\hat{x}^n, \hat{y}^n, \delta^n)}{(1+r)^T - 1} \right),$$

where pr^n is the probability of node n, f_R is the yearly revenue, which is a function of the vector of the exports from the different options, α^n, and the uncertain parameters, ξ^n, in node n, and f_C is the total capital expenditure, which is a function of the vectors of investment decisions, $\hat{x}^n$, $\hat{y}^n$, and δ^n, in node n. Further, r is the discount rate, τ is the time difference in years between scenario tree levels, and T is the calculation horizon in years corresponding to the economic lifetime of the investments. The parameters r, τ, and T can be chosen freely. Here, $\tau = 5$ years, while $T = 30$ years and $r = 0.093$. These chosen values of T and r correspond to an annuity factor of 0.10, which has been identified as a reasonable value for strategic decisions concerning energy-efficiency investments in industry (FRAM, 2005).

3.3 Investment costs

The investment cost of process equipment as a function of size or capacity is in many cases given by a non-linear concave function. Since we desire a linear model, such investment costs have to be linearized. The idea of the linearization procedure is to divide the equipment capacity into intervals in which the cost function is approximately linear. The investment cost is thus modelled according to

$$\text{cost}_u^n = c_{u0}\hat{y}_u^n + \sum_{i \in I_u} \frac{c_{ui} - c_{u,i-1}}{k_{ui}} \delta_{ui}^n, \qquad u \in U,\ n \in N,$$

where U is the set of different technologies for energy exports such as the back-pressure turbine or the lignin extraction, and I_u is the set of linearization intervals for technology u. Further, $\hat{y}_u^n$ is a binary variable with value 1 if investments in technology u are made in node n, and 0 else, and c_{u0} denotes the base cost for technology u. The parameter c_{ui} denotes the investment cost parameter for technology u at the end of capacity interval i, and k_{ui} is the size of that interval. Finally, δ_{ui}^n denotes the installed capacity within interval i for technology u in node n. The variables $\hat{y}_u^n$ and δ_{ui}^n then have to fulfil the following constraints for all options $u \in U$ and all nodes $n \in N$.

$$\begin{aligned} z_{u0}^n &= \hat{y}_u^n, && \\ z_{ui}^n k_{ui} \le \delta_{ui}^n &\le z_{u,i-1}^n k_{ui}, && i \in I_u, \\ \hat{y}_u^n, z_{ui}^n &\in \{0,1\}, && i \in I_u, \end{aligned}$$

where z_{ui}^n is an auxiliary binary variable that orders the intervals i for technology u.

The linearization is implemented as an AMPL script, giving values for I_u, k_{ui} and c_{ui}. The user only needs to provide the original cost function, error tolerances for the linearization, and the range for which the linearization should be valid. Here, an absolute error tolerance of 0.05 M€ and a relative error tolerance of 3% was employed for the difference between the piecewise linear approximation and the original function. This input data results in the interval sizes shown in Table 2. The different technologies of the set U are denoted by: BP (back-pressure turbine), CT (condensing turbine), LIG (lignin separation plant), DH60 (district heating from low quality (60°C) excess heat), DH100 (district heating from medium quality (100°C) excess heat), and DHLP (district heating from low-pressure steam). The

Table 2: Interval sizes (k_{ui} [MW]) of the investment cost linearization intervals.

	Interval									
	1	2	3	4	5	6	7	8	9	10
BP	0.1	0.1	0.3	0.9	2.3	4.1	6.5	9.6	13.4	12.7
CT	0.2	0.3	0.8	1.4	2.3	3.4	4.8	5.8		
LIG	1	1	3	5	8	12	17	23	30	25
DH60	100									
DH100	16.3									
DHLP	125									

base costs and investment costs for each interval are given by the following formulas.

$$\begin{aligned} c_{\text{BP},0} &= 274, & c_{\text{BP},i} &= 1090\left(\sum\nolimits_{j=1}^{i} k_{\text{BP},j}\right)^{0.6}, & i &= 1,\ldots,10, \\ c_{\text{CT},0} &= 746, & c_{\text{CT},i} &= 1960\left(\sum\nolimits_{j=1}^{i} k_{\text{CT},j}\right)^{0.6}, & i &= 1,\ldots,8, \\ c_{\text{LIG},0} &= 882, & c_{\text{LIG},i} &= 882\left(\sum\nolimits_{j=1}^{i} k_{\text{LIG},j}\right)^{0.6}, & i &= 1,\ldots,10, \\ c_{\text{DH60},0} &= 0, & c_{\text{DH60},1} &= 109 k_{\text{DHLP},1}, & & \\ c_{\text{DH100},0} &= 400, & c_{\text{DH100},1} &= 400, & & \\ c_{\text{DHLP},0} &= 570, & c_{\text{DHLP},1} &= 30\left(k_{\text{DHLP},1} + 19\right). & & \end{aligned}$$

All investment options that are not included in the set U are instead included in the set M, which in this case study includes steam-saving measures and investments that are not improving the energy efficiency of the mill. The recovery boiler upgrade (RBU) is an example of the latter type of measure. The RBU does not yield any steam savings, but to meet the planned production increase, the investment is required if not enough lignin is extracted. The measures of the set M are assumed to be discrete options, which can either be carried out at a fixed cost, generating a fixed steam saving, or not be carried out at all. For these investments in measures $m \in M$, the investment costs are simply constants, such that the investment cost in each node $n \in N$ is given by

$$\text{cost}_m^n = b_m \hat{x}_m^n, \\ \hat{x}_m^n \in \{0, 1\},$$

where b_m is the investment cost parameter for measure m, and the binary variable $\hat{x}_m^n$ equals 1 if measure m is implemented in node n, and 0 else. The possible measures included in the set M are presented in Table 3 along with their respective investment costs and resulting steam savings.

Table 3: Investment costs and steam savings for energy-efficiency measures.

Measure		Cost (b_m) [k€]	LP steam ($s_{m,\mathrm{LP}}$) [tonnes/h]	MP steam ($s_{m,\mathrm{MP}}$) [tonnes/h]	HP steam ($s_{m,\mathrm{HP}}$) [tonnes/h]
1.	New 3-stage flash	3500	3.1	16.1	0.0
2.	New HWWS	600	12.5	0.0	0.0
3.	Wood yard	0	2.6	0.0	0.0
4.	Shoe press	6000	11.8	0.0	0.0
5.	Blow out	0	13.9	0.0	0.0
6.	Blow down	0	-78.4	-39.5	102.5
7.	Convap*‡	9300	59.5	-19.3	9.3
8.	Convap*§	9700	59.5	-19.3	9.3
9.	PIvap†‡	11700	63.9	-14.3	9.3
10.	PIvap†§	10900	63.9	-14.3	9.3
11.	Convap¶	4400	0.0	0.0	0.0
12.	RBU	29800	0.0	0.0	0.0
13.	DH piping	6600	0.0	0.0	0.0

* A modern conventional evaporation plant
† A process integrated evaporation plant
‡ Not adapted for lignin extraction
§ Adapted for lignin extraction
¶ Least-cost evaporation plant yielding no steam savings

Investments can only be made in the first levels of the scenario tree. The reason is that late investments will not be analyzed correctly since the resulting cash flows will be calculated for a too short time period. Hence, investment decision variables are restricted to be zero (or they are simply not defined) for high levels, according to

$$\hat{y}_u^n = \hat{x}_m^n = 0, \qquad u \in U,\ m \in M,\ n \in N_A,$$

where N_A is the set of nodes in which investments cannot be made. From this follows that also δ_{ui}^n and z_{ui}^n are zero for $n \in N_A$.

To summarize, the total capital expenditure f_C is given by:

$$f_C(\hat{x}^n, \hat{y}^n, \delta^n) = \sum_{m\in M} b_m \hat{x}^n_m + \sum_{u\in U} c_{u0}\hat{y}^n_u + \sum_{i\in I_u} \frac{c_{ui} - c_{u,i-1}}{k_{ui}} \delta^n_{ui}, \qquad n \in N. \tag{1}$$

3.4 The revenue function

The total yearly revenue that is generated in each node is a function of the exports for the different technologies and the net export revenues. It can be expressed as

$$f_R(\alpha^n, \xi^n) = \sum_{u\in U} f_{E,u}(\xi^n) e \alpha^n_u, \qquad u \in U,\ n \in N,$$

where $f_{E,u}(\xi^n)$ is the net export revenue for option u in node n and $\alpha^n_u \geq 0$ is the export for technology u in node n. The factor e is simply a unit conversion, which in this case equals 7.8 (GWh/year)/MW. The function $f_{E,u}$ represents the export revenues minus operating costs; it is a function of the energy prices presented in Table 1. In this case, there is an operating cost only for lignin extraction and for the heat pump. The functions $f_{E,u}$ for different technology options U and all nodes $n \in N$ are given by

$$\begin{aligned} f_{E,\text{LIG}}(\xi^n) &= \xi^n_{\text{lignin}} - (5.72 + 0.0162\xi^n_{\text{elec.}}), \\ f_{E,\text{BP}}(\xi^n) = f_{E,\text{CT}}(\xi^n) &= \xi^n_{\text{elec}}, \\ f_{E,\text{DH60}}(\xi^n) &= \xi^n_{\text{heat}} - 0.357\xi^n_{\text{elec.}}, \\ f_{E,\text{DH100}}(\xi^n) = f_{E,\text{DHLP}}(\xi^n) &= \xi^n_{\text{heat}}. \end{aligned}$$

3.5 Mass and energy balances and capacity limitations

The amount of generated power, extracted lignin, and delivered district heating is contained in the export vector α^n_u, which is limited by the available steam surplus and the capacity of the equipments. In this section, we formulate constraints to handle steam balances as well as capacity limitations for the different options.

3.5.1 General constraints relating steam to power, heat, and lignin exports

The general constraint stating the required steam surplus to achieve a specific output of at least α^n_u of power, heat, or lignin is given by

$$\alpha^n_u \leq \sum_{p\in P} q_{up}\rho^n_{up} + \gamma^n_u, \qquad u \in U\backslash L,\ n \in N. \tag{2}$$

Here, P denotes the set of steam pressure levels (LP, MP, and HP steam) and the parameter q_{up} relates the steam surplus of pressure p to the power, lignin, or heat output for technology option u. The parameter q_{up} was calculated from process and equipment data such as efficiencies, enthalpies, and so on. The values that were obtained are given in Table 4. Further, the variable ρ^n_{up} denotes the flow of steam with pressure p used for technology u in node n. Finally, the variable γ^n_u is the possible additional output for option u in node n, which can be achieved without any steam input.

In constraint (2), L denotes the set of the condensing turbine option and the district heating options. For these options, constraint (2) above is replaced by the constraints (3)–(6), which are described in Section 3.5.2. The reason for this is that the possibility to deliver district heating is limited by the district heating demand which varies over the year. This requires a finer time resolution for the different district heating options, but also for the condensing turbine. Low-pressure steam can either be used for district heating or for the condensing turbine, which causes the district heating demand variations to affect also the condensing turbine option.

Table 4: Production per steam surplus (q_{up}) [MW/(t steam/h)].

Option	LP steam	MP steam	HP steam
BP	0	0.0406	0.0976
CT	0.1243	0	0
LIG	0	0	0.8110
DH60	0	0	0
DH100	0	0	0
DHLP	0.647	0	0

3.5.2 District heating constraints

The district heating demand curve is modelled as a finite number of time steps of given length and district heating demand. We define J as the set of time steps, t_j as the length of time step j, and v_j as the external demand in step j. Here, the demand curve is divided into 20 intervals, representing the twelve months and eight additional time periods with an unusually high or low demand. The numbers of the parameters are given in Table 5.

Table 5: Data for the district heating demand parameters.

Step (j)	Month	Demand (v_j) [MW]	Length (t_j) [h]	Step (j)	Month	Demand (v_j) [MW]	Length (t_j) [h]
1	Jan	21	720	11	Jul	1	720
2	Jan peak	41	24	12	Jul low	1	24
3	Feb	21	648	13	Aug	3	720
4	Feb peak	41	24	14	Aug low	3	24
5	Mar	21	720	15	Sep	10	720
6	Mar peak	41	24	16	Oct	10	744
7	Apr	10	720	17	Nov	21	696
8	May	3	744	18	Nov peak	41	24
9	Jun	3	696	19	Dec	21	720
10	Jun low	3	24	20	Dec peak	41	24

We define X as the set of options directly limited by the external demand, that is, the district heating options. The variable μ_{uj}^n denotes the heat or power output of option $u \in L$ in step j in node n, where L includes the condensing turbine in addition to the district heating options. Finally, we let $\tilde{\rho}_{uj}^n$ be the low pressure steam flow of option $u \in L$ in step j in node n (replacing ρ_{up}^n for $u \in L$). The following constraints can then be formulated for all nodes $n \in N$:

$$\alpha_u^n \sum_{j \in J} t_j \leq \sum_{j \in J} \mu_{uj}^n t_j, \qquad u \in L, \tag{3}$$

$$\mu_{u,j}^n \leq q_{u,\mathrm{LP}} \tilde{\rho}_{uj}^n + \gamma_u^n, \qquad u \in L,\ j \in J, \tag{4}$$

$$\sum_{u \in X} \mu_{uj}^n \leq v_j, \qquad j \in J, \tag{5}$$

$$\mu_{uj}^n, \tilde{\rho}_{uj}^n \geq 0, \qquad u \in L,\ j \in J. \tag{6}$$

Constraint 6, ensuring positive values of μ_{uj}^n and $\tilde{\rho}_{uj}^n$, is based on no district heating being delivered today and hence, the deliveries cannot decrease. As a result of the above formulation, α_u^n will express a yearly average of the district heating deliveries and of the electricity produced in the condensing turbine.

3.5.3 Steam balance constraints

The steam used to produce electricity, heat, or lignin is of course limited by the available steam surplus. Before we go on to the formulation of the steam balances, we need to introduce two sets of binary variables. The previously introduced $\hat{x}_m^n$ controls the investment cost and takes the value 1 only in the node where investments are made. The variable x_m^n controls the available steam surplus and takes the value 1 in all nodes where the steam surplus for that measure is available. The same type of distinction holds between y_u^n and $\hat{y}_u^n$. While $\hat{y}_u^n$ controls the investment cost, y_u^n controls the availability of the technology and is 1 if option u is available to use in node n, and 0 else. The relations between these variables are stated in the following constraints, where also the variables $\breve{x}_m^n$ and $\breve{y}_u^n$ are introduced for the possibility to deactivate investments:

$$x_m^0 = y_u^0 = 0, \qquad m \in M,\ u \in U, \tag{7}$$

$$x_m^n = x_m^{p(n)} + \hat{x}_m^{p(n)} - \breve{x}_m^{p(n)}, \qquad m \in M,\ n \in N \backslash R, \tag{8}$$

$$y_u^n = y_u^{p(n)} + \hat{y}_u^{p(n)} - \breve{y}_u^{p(n)}, \qquad u \in U,\ n \in N \backslash R, \tag{9}$$

$$y_u^n \geq \breve{y}^n, \qquad u \in U,\ n \in N, \tag{10}$$

$$\hat{y}_u^n \geq \breve{y}_u^n, \qquad u \in U,\ n \in N, \tag{11}$$

$$0 \leq x_m^n, y_u^n \leq 1, \qquad m \in M,\ u \in U,\ n \in N, \tag{12}$$

$$\breve{x}_m^n, \breve{y}_u^n \in \{0, 1\}, \qquad m \in M,\ u \in U,\ n \in N. \tag{13}$$

The variables x_m^n and y_u^n need not be integrality constrained. In an optimal solution these variables will possess binary values, due to the above constraints and the fact that $\hat{x}_m^n$, $\breve{x}_m^n$, $\hat{y}_u^n$, and $\breve{y}_u^n$ are required to be binary. The deactivation possibility is used, for example, in the case of reinvestments in technologies that have already been invested in.

The steam balance on each steam pressure level P is simply stating that the use of steam for different options (ρ_{up}^n or $\tilde{\rho}_{uj}^n$) must equal the acquired steam surplus plus steam that is passed from higher pressure levels, minus steam that is passed to lower pressure levels. Let s_{mp} denote the steam saving at pressure level p for measure m. The steam savings for each measure are given in Table 3. Further, the set of options that reduce pressure (the back-pressure turbine but also pressure-drop valves) is denoted by Q, and the parameter h_{up} is a factor that is introduced due to the fact that when steam is passed from higher to lower pressures, it will be superheated, and water will be added to saturate the steam. This factor will take the values $h_{\mathrm{BP,MP}} = 1.040$, $h_{\mathrm{BP,LP}} = 1.010$, $h_{\mathrm{V,MP}} = 1.174$, and $h_{\mathrm{V,LP}} = 1.027$, where the index V denotes the pressure-drop valves. For high- and medium-pressure steam, the constraints for all nodes $n \in N$ are then

$$\sum_{u \in (U \cup Q) \backslash L} \rho_{u,\mathrm{HP}}^n \leq \sum_{m \in M} x_m^n s_{m,\mathrm{HP}},$$

$$\sum_{u \in U \backslash (Q \cup L)} \rho_{u,\mathrm{MP}}^n \leq \sum_{m \in M} x_m^n s_{m,\mathrm{MP}} + \sum_{u \in Q} \left(\rho_{u,\mathrm{HP}}^n - \rho_{u,\mathrm{MP}}^n\right) h_{u,\mathrm{MP}}.$$

The steam balance for the low-pressure steam is expressed by the similar constraint:

$$\sum_{u \in L} \tilde{\rho}_{u,j}^n \leq \sum_{m \in M} x_m^n s_{m,\mathrm{LP}} + \sum_{u \in Q} \rho_{u,\mathrm{MP}}^n h_{u,\mathrm{LP}}, \qquad j \in J,\ n \in N.$$

Positive values of the steam flows are of course required, except for rebuilt or replaced equipment, where a decrease (relative to the present situation) can occur. The possible decrease of steam flow with pressure p for the option u is denoted by d_{up}. Here, a decrease is possible for the back-pressure turbine option only. The values (which correspond to the current use of steam) are $d_{\mathrm{BP,HP}} = 201$ tonnes/h and $d_{\mathrm{BP,MP}} = 157$ tonnes/h respectively. For all other options, the value of d_{up} is zero (see also the constraint (6)).

$$\rho_{up}^{n} \geq -d_{up}, \qquad u \in (U \cup Q)\backslash L,\ p \in P,\ n \in N.$$

For some technology options it is possible to increase the output α_u^n without using any extra steam. This is the case for the new back-pressure turbine, for which the output is increased compared to the old one, simply because it has a higher efficiency. This is also true for the two district heating options that use low quality excess heat and not steam. The maximum 'steam-free' output is denoted w_u and the variable γ_u^n, which was introduced in the constraints (2) and (4), denotes the actual utilization of the 'steam-free' output according to

$$\gamma_u^n \leq w_u y_u^n, \qquad u \in U,\ n \in N.$$

For the new back-pressure turbine $w_{\mathrm{BP}} = 1.3$ MW. For DH60, the value is $w_{\mathrm{DH60}} = 83.6$ MW, corresponding to the maximum heat delivery from a heat pump that uses all available excess heat of $60°$C. Finally, $w_{\mathrm{DH100}} = 16.3$ MW, corresponding to the available excess heat of $100°$C. For all other options the value of w_u is zero.

3.5.4 Capacity constraints

The production α_u^n of power, lignin, and district heating for the different technology options is of course limited to a maximum of the installed capacity. We define the variable β_u^n, as the available capacity in node n for option u. The constraints are then for all nodes $n \in N$:

$$\begin{aligned} \alpha_u^n &\leq \beta_u^n - g_u y_u^n, && u \in U\backslash L, \\ \mu_{u,j}^n &\leq \beta_u^n, && u \in L,\ j \in J. \end{aligned}$$

The parameter g_u states the current capacity for technology option u at the mill. Here, $g_{\mathrm{BP}} = 24.7$ MW and $g_u = 0$ MW for all the other options. (The existing back-pressure turbine has a capacity of 24.7 MW.) The reason for subtracting $24.7 y_u^n$ in the case of the back-pressure turbine is that only the power output exceeding this level will contribute to an added incoming cash flow. The relation between β_u^n and the variable δ_{ui}^n which is used in the cost function (1) can be stated similar as the constraints (7)–(13) above which relate x_m^n to $\hat{x}_m^n$ and y_u^n to $\hat{y}_u^n$. For all options $u \in U$, we then have

$$\begin{aligned} &\beta_u^0 = 0, \\ &\beta_u^n = \beta_u^{p(n)} + \sum_{i \in I_u} \delta_{ui}^{p(n)}, && n \in N\backslash R. \end{aligned}$$

3.6 Combination constraints

Except for the specific data needed for this case, and the adaptations of some constraints to handle the varying demand of district heating, a number of constraints have to be added to the model. These are constraints that specify how process integration measures can be combined. The measures in the set M are numbered according to Table 3. Technology options of the set U are denoted by: BP (back-pressure turbine), CT (condensing turbine), LIG (lignin extraction), DH60 (district heating from low quality excess heat), DH100 (district heating from medium quality excess heat), and DHLP (district heating from low-pressure steam).

Some investments have to be made immediately because of the production increase, which is planned to be implemented in the beginning of the analyzed time period. For the production increase, either the recovery boiler has to be upgraded (RBU) or lignin has to be separated. For the RBU option, the only opportunity to invest is from start. Later, the opportunity for RBU is foregone due to the long production down-time. If RBU is not carried out, at least 53.6 MW of lignin has to be separated in order not to overload the existing recovery boiler. Furthermore, a new evaporation plant is also necessary from the start due to the production increase.

$$\hat{x}_{12}^n = 0, \qquad n \in N \backslash R, \tag{14}$$

$$\alpha_{\mathrm{LIG}}^n \geq 53.6(1 - x_{12}^n), \qquad n \in N \backslash R, \tag{15}$$

$$\sum_{m=7}^{11} \hat{x}_m^0 = 1. \tag{16}$$

Large steam savings can be achieved by rebuilding the evaporation plant, but a number of constraints limits the actual potential. There can only exist one evaporation plant at each point in time, either the cheapest one with no actual steam saving (Convap¶), or a more modern but conventional evaporation plant (Convap*), or a process integrated evaporation plant (PIvap) (see Table 3). To make it possible to install PIvap, the hot and warm water system (HWWS) has to be rebuilt. Another constraint is that PIvap cannot be combined with the new three-stage flash. This gives, for all nodes $n \in N$ the constraints

$$\begin{aligned} \sum_{m=7}^{11} x_m^n &\leq 1, \\ x_9^n + x_{10}^n - x_2^n &\leq 0, \\ x_1^n + x_9^n + x_{10}^n &\leq 1. \end{aligned}$$

Lignin separation cannot be implemented without investments in a new evaporation plant and the evaporation plant has to be adjusted for lignin extraction. If the evaporation plant is in fact designed for lignin extraction, lignin has to be extracted with an amount that equals the design capacity. This is because the evaporation plant that is designed for lignin extraction is designed for different flows than a conventional evaporation plant. We then have the constraints

$$\begin{aligned} y_{\mathrm{LIG}}^n - (x_8^n + x_{10}^n) &\leq 0, && n \in N, \\ \lambda^0 &= 0, && \\ \lambda^n &= \lambda^{p(n)} + \hat{\lambda}^{p(n)}, && n \in N, \\ -M_{\mathrm{LIG}}(\hat{x}_8^n + \hat{x}_{10}^n) \leq \hat{\lambda}^n &\leq M_{\mathrm{LIG}}(\hat{x}_8^n + \hat{x}_{10}^n), && n \in N, \\ \lambda^n - M_{\mathrm{LIG}}(1 - (x_8^n + x_{10}^n)) \leq \alpha_{\mathrm{LIG}}^n &\leq \lambda^n, && n \in N \backslash R. \end{aligned}$$

We have introduced M_{LIG}, a 'big enough' parameter, in the above constraints.

For an existing back-pressure turbine, the load cannot change independently for the high-pressure part and the medium-pressure part, but the steam flow change has to be equal for both parts. When the turbine is exchanged for a new one, the steam flow can change freely. Also, if investments are made in a new back-pressure turbine, the existing back-pressure turbine has to be replaced. We introduce M_{BP}, which is a 'big enough' parameter for the following constraints:

$$\left(\rho_{\mathrm{BP,HP}}^n - \rho_{\mathrm{BP,MP}}^n\right) - \left(\rho_{\mathrm{BP,HP}}^{p(n)} - \rho_{\mathrm{BP,MP}}^{p(n)}\right) \leq M_{\mathrm{BP}} \hat{y}_{\mathrm{BP}}^{p(n)}, \qquad n \in N \backslash R, \tag{17}$$

$$\left(\rho_{\mathrm{BP,HP}}^n - \rho_{\mathrm{BP,MP}}^n\right) - \left(\rho_{\mathrm{BP,HP}}^{p(n)} - \rho_{\mathrm{BP,MP}}^{p(n)}\right) \geq -M_{\mathrm{BP}} \hat{y}_{\mathrm{BP}}^{p(n)}, \qquad n \in N \backslash R, \tag{18}$$

$$\sum_{i \in I_{\mathrm{BP}}} \delta_{\mathrm{BP}i}^n \geq g_{\mathrm{BP}} \left(\hat{y}_{\mathrm{BP}}^n - \check{y}_{\mathrm{BP}}^n\right), \qquad n \in N. \tag{19}$$

Finally, there are a number of constraints related to the opportunities for district heating deliveries. If excess heat is used internally in the mill, either for process-integrated evaporation or for a new three-stage flash, excess heat is not available for district heating. This means that PIvap cannot be combined with DH100 nor DH60, and the flash cannot be combined with DH100. If any of the district heating options are chosen, investments have to be made in district heating piping. This cost for district heating piping is assumed to partially be taken by the energy company. Thus, once district heating is decided on, the exports are not allowed to decrease. This yields the constraints

$$
\begin{aligned}
x_m^n + y_u^n &\leq 1, && m \in \{9, 10\},\ u \in \{\mathrm{DH60}, \mathrm{DH100}\},\ n \in N, \\
x_1^n + y_{\mathrm{DH100}}^n &\leq 1, && n \in N, \\
x_{13}^n &\geq y_u^n, && u \in \mathrm{X},\ n \in N, \\
\sum_{u \in \mathrm{X}} \alpha_u^n &\geq \sum_{u \in \mathrm{X}} \alpha_u^{p(n)}, && n \in N \backslash R, \\
\sum_{u \in \mathrm{X}} \alpha_u^{n_1} &= \sum_{u \in \mathrm{X}} \alpha_u^{n_2}, && n_1 \in N \backslash R,\ n_2 \in N \backslash R:\ p(n_1) = p(n_2).
\end{aligned}
$$

4 Results and discussion

In order to solve the model, a discrete probability distribution has to be assumed for the scenarios described in Section 3.1. There are, however, no statistics or logic to base such an assumption on. Instead, the assumed probability distribution will represent the decision-maker's opinion or beliefs regarding the future development of the energy market. This should, of course, be based on sound judgement as well as insights regarding the political agenda and the planned development of energy market policy instruments. Nevertheless, this is a difficult assumption to make. Thus, in order for the model to be of any use, it is important to evaluate the robustness of the solution with respect to the probability distribution.

Here, we will begin with a uniform probability distribution to arrive at some general results. In Section 4.2, we then test how much this distribution can be changed without altering the optimal solution. Then, in Section 4.3, the modelling of investment timing will be further elaborated.

The model was, as mentioned earlier, formulated in AMPL and solved using CPLEX, which is a solver for mixed-integer linear programs (MILP). The computation time was 58 CPU seconds on a computer with a 2.0 GHz AMD Athlon processor for the basic case presented below.

4.1 General results

The optimal solution, under the assumption of a uniform probability distribution (illustrated as alternative B in Figure 3), is characterized by a lignin extraction rate that is just enough to avoid that the recovery boiler is upgraded, that is 53.6 MW of lignin (see the constraint (15)). The remaining steam surplus is used for electricity generation and district heating. Steam savings are achieved through energy-efficiency measures 2–6, 8, and 13 (these numbers referring to Table 3). Furthermore, all investments are made in node 0 and the lignin extraction, the electricity generation, and the district heating deliveries are kept at a constant level throughout the analyzed time horizon.

We also compute the optimal solution for each scenario, that is, the solutions that are achieved by setting the probability to 100% for one scenario at a time. None of the obtained solutions implies any investments after the initial ones are made. Interestingly, the scenarios M2 and S1 both render the same optimal solution as the one achieved with a uniform probability distribution (alternative B in Figure 3) and scenario S2 renders a very similar solution (alternative C in Figure 3). However, in the optimal solution for scenarios BAU and M1 no lignin is extracted and all steam surplus is used for electricity production and district heating (alternative A in Figure 3). The optimal solution for scenario E is on the contrary to use the steam surplus for maximum lignin extraction and only to slightly increase the electricity production (alternative D in Figure 3).

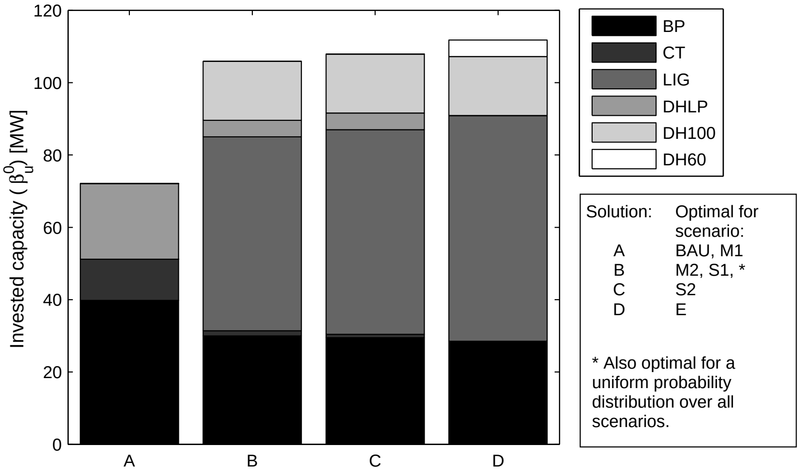

Figure 3: The main investment alternatives and their characteristics.

There are of course more solutions than those illustrated in Figure 3, although they are not optimal for any of the scenarios. Nevertheless, it is quite obvious that the number of feasible solutions is limited and in fact rather few. This is of course a consequence of the integer requirements on, mainly, the decision variables for the energy-efficiency measures. The constraints on how the different measures can be combined, and the requirement on the immediate production increase also limit the number of feasible solutions.

4.2 Sensitivity of the optimal solution to probability distribution variations

To analyze how robust the solution is to changes in the assumed probability distribution, we sought to find how much the probability could deviate from a uniform distribution without altering the optimal solution. Obviously, the probabilities for scenarios M2 and S1 can be up to 100%. However, for BAU, M1, S2, and E, there exists a probability level (between 17% and 100%) for which the solution switches from alternative B to some other solution.

We found that, if the probabilities for the rest of the scenarios were kept equal, the probability for scenario BAU can increase to 80% before the solution switches from alternative B to A. Whether this number indicates robustness with respect to the probability assumption for the BAU scenario has to be judged by those who will make the decision on investment. It is, however, reasonable to believe that someone who believes in a higher probability than 80% for a 'business as usual' development would not carry out this kind of analysis at all. Table 6 shows the breakpoint probabilities for all scenarios.

As can be seen in Table 6, the breakpoint probabilities for scenario E are substantially lower than the breakpoint probabilities for the other scenarios. The solution switches first, at 42% probability for scenario E, from solution B to solution C, and then, at 51%, from C to D. These levels of breakpoint probabilities might seem low, indicating that the solution is not robust to changes in the probability of this scenario. On the other hand, considering the extreme properties of this scenario, already the value of 17%, which is the probability in the uniform distribution, should be regarded as quite high. In fact, zero probability could be reasonable for this scenario.

Here, we have varied the probability distribution by increasing the probability of one scenario, while decreasing the probabilities of the other scenarios uniformly. The probability distribution may, however,

Table 6: Breakpoint probabilities for the respective scenarios at which there is a change in the optimal solution. The optimal solutions are denoted by letters A–D according to Figure 3.

Scenario	Breakpoints (solutions)	
BAU	80% (B→ A)	
M1	99% (B→ A)	
M2	(no switch)	
S1	(no switch)	
S2	85% (B→ C)	
E	42% (B→ C)	51% (C→ D)

be varied in several other ways. Nevertheless, our tests show that it is difficult to find other solutions than the ones illustrated in Figure 3. As an example, all probability distributions having an equal probability for scenario BAU and E while the other scenarios also are changed uniformly results in the optimal solution B.

To summarize, the optimal solution seems to be robust to changes in the assumed probability distribution. The analysis is straight-forward and can easily be repeated for each new case study. It is also important that the results of the analysis is evaluated by the decision-makers of each case.

It should be noticed, however, that the robustness property only holds when the assumed probability for each of the scenarios is far from its respective breakpoint value. This is to a large extent a consequence of the integer requirements on the decision variables, which is the reason that there are quite few solutions to the optimization model. Close to the breakpoints, the optimal solution is naturally more sensitive to a variation of the probability assumptions. At these breakpoints, the optimal solution changes rather drastically, for example from a solution with no RBU to one with RBU but no lignin extraction. Remember that the investment cost for the RBU alone is almost 30M€, which means that there are high values at stake if the wrong decision is made.

4.3 Timing of investments

The multistage modelling of the investment decision problem enables the study of the timing of investments. In the above example, however, the optimal solution turned out to involve only immediate investments made at the root node of the scenario tree. In order to verify that the timing issue has been modelled correctly and to render the achievement of a solution where investments are allowed at more than one node possible, the above example was changed such that the production increase was assumed to be planned for year 2020 instead of year 2010.

A production increase the year 2020 corresponds to making the production increase investments in level 2 of the scenario tree. The increased production will then be in effect from level 3. Some parameters will possess different values before and after the production increase. This applies to the steam savings, s_{mp}, and the costs, b_m, of the energy-efficiency measures, but also the maximum 'steam-free' output of the export options, w_u. A node index, n, is therefore introduced for these parameters.

The steam savings, s^n_{mp}, after the production increase ($\ell(n) > 2$) are the same as those given in Table 3. The steam savings before the production increase ($\ell(n) \leq 2$) are given in Table 7 below.

Since the purpose here is to illustrate the modelling of the timing issue, detailed input data is not a requirement. To simplify, most of the investment costs for the measures are therefore assumed to be the same before and after the production increase, which can be reasonable if the investments are made with the planned production increase in mind. The evaporation plant, however, needs to be rebuilt for the production increase, and hence the evaporation plant measures (m = 7–10) are included with different values for both costs and steam savings before and after the production increase. The costs, b^n_m, after the production increase ($\ell(n) \geq 2$) are the same as those given in Table 3. The costs before the production increase ($\ell(n) < 2$) are given in Table 7 below.

Table 7: Investment costs and steam savings for measures before the production increase.

Measure		Cost (b^n_m) ($\ell(n) < 2$) [k€]	LP steam ($s^n_{m,\mathrm{LP}}$) ($\ell(n) \leq 2$) [tonnes/h]	MP steam ($s^n_{m,\mathrm{MP}}$) ($\ell(n) \leq 2$) [tonnes/h]	HP steam ($s^n_{m,\mathrm{HP}}$) ($\ell(n) \leq 2$) [tonnes/h]
1.	New 3-stage flash	3500	2.5	12.9	0.0
2.	New HWWS	600	10	0.0	0.0
3.	Wood yard	0	2.1	0.0	0.0
4.	Shoe press	6000	9.4	0.0	0.0
5.	Blow out	0	11.1	0.0	0.0
6.	Blow down	0	-30.7	-22.3	41.8
7.	Convap*‡	6400	47.6	-15.4	7.4
8.	Convap*§	6800	47.6	-15.4	7.4
9.	PIvap†‡	7700	51.1	-11.4	7.4
10.	PIvap†§	7300	51.1	-11.4	7.4
11.	Convap¶	4400	0.0	0.0	0.0
12.	RBU	29800	0.0	0.0	0.0
13.	DH piping	6600	0.0	0.0	0.0

*†‡§¶ See Table 3

Finally, the ‘steam-free’ output of the export options, w^n_u, will not change for the back-pressure turbine. For DH100 and DH60, the values will be $w^n_{\mathrm{DH100}} = 13$ MW and $w^n_{\mathrm{DH60}} = 66.9$ MW for $n \in N$ such that $\ell(n) \leq 2$. The values for $\ell(n) > 2$ were given at the end of Section 3.5.3 above.

Perhaps the most obvious model change when moving the production increase is that the constraints (14)–(16) will change, according to

$$
\begin{aligned}
\hat{x}^n_{12} &= 0, && n \in N : \ell(n) \neq 2,\\
\alpha^n_{\mathrm{LIG}} &\geq 53.6(1 - x^n_{12}), && n \in N : \ell(n) > 2,\\
\sum_{m=7}^{11} \hat{x}^n_m &= 1 && n \in N : \ell(n) = 2.
\end{aligned}
$$

The optimal solution now involves investments in two stages. In the root node, investments are made in steam-saving measures, in a new back-pressure turbine, in a condensing turbine, and in district heating from low-pressure steam as well as from $100°$C heat. In nodes 4–7 (at level 2) additional investments are made. For all these nodes, investments are made in a lignin separation plant, in an evaporation plant with increased capacity adapted for lignin extraction, but also in the shoe press, which was not invested in from the start. As is the case for the immediate production increase, lignin extraction turns out to be a better alternative than a recovery boiler upgrade.

At this level, after the production increase, investments have been made in exactly the same energy-efficiency measures and export options as was made in the optimal solution for the immediate production increase. The total capacity of the turbines and the district heating is, however, substantially higher, which means that not all the capacity is used. Remember that the steam load of the turbines will decrease when lignin is extracted, which will be the case at the time of the production increase. The potential for using steam for district heating will also be decreased. The results thus imply that, in this case, it is beneficial to make early investments in turbines although they will be used to their full capacity only for ten years. Here, there is no reason to wait with energy efficiency until the production increase.

Moreover, the steam load of the back-pressure turbine changes rather dramatically when lignin is extracted. Due to the constraints (17)–(19), a new investment in the turbine is thus required. Since the load decreases, however, a capacity increase is not necessary and the investment will be minimal. Although this is obviously a shortage in the turbine cost modelling, we will not make any definite changes to the model or input data here. A simple analysis shows that the investment cost for the changed steam load conditions could be around 5000 k€ before the optimal solution changes. It could be argued, that since the turbine is rather new (10 years), and the production increase was known when the turbine was first installed, the possibility of controlling the steam flows independently could have been built in from start, making the constraints (17)–(19) redundant for the new turbine. The discussion makes it clear, however, that the modelling, solving, and analyzing process may need to be iterated.

Although the investments are similar for all the nodes 4–7 (at level 2), the investments in node 7, which is part of the scenario E, differ slightly. In this scenario, the wood fuel prices are higher compared to the electricity prices, and lignin extraction is more profitable. Hence, the lignin extraction capacity is a few MW higher in this scenario than in the others. This affects the electricity production and the district heating deliveries which are slightly decreased.

To summarize the latter results, they show that with the proposed model, it is possible to arrive at optimal solutions where investments are made at more than one point in time, and these investments are evaluated correctly. The importance of employing an iterative procedure to update the model based on achieved results is also illustrated.

5 Conclusions

This article presents a multi-stage stochastic programming model for the optimization of process integration investments under economic uncertainty. The proposed approach enables optimization of combinations of measures for which the outcome is directly and indirectly affected by the implementation of the other measures as well as on uncertain market conditions.

Uncertainties are modelled in a scenario-based approach. We show that the probabilities for the different scenarios can be substantially changed without altering the optimal solution. This implies a robustness of the solution obtained with respect to the assumed probability distribution, which definitely is an advantage in dealing with the kind of uncertainties that are present in this model. Robustness, however, is not a general property. The model should therefore always be solved for some different probability distributions around the one the decision maker believes in, in order to check for robustness in the probability range of interest.

Furthermore, the model enables the optimization of the timing of investments, although for the case study presented here, we find that investments should be made immediately. A change of the conditions for the case study results in a model for which the optimal solution involves investments at more than one point in time.

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