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UNCERTAINTY OF ESTIMATED VEHICLE DAMAGE FOR RANDOM LOADS

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ABSTRACT

This paper presents two methods to estimate the variation coefficient for the accumulated damage, when only one measurement of the load is available. The methods are applied to loads on heavy vehicles travelling on uneven roads. The accuracy and precision of the estimates of the variation coefficient is evaluated using both a standard road profile model (Gaussian with spectrum according to ISO 8608) and a more realistic model that includes irregular sections. Four examples are presented to illustrate the performance of the estimates. Finally, the coefficient of variation is computed as a function of road length, for a real measured gravel road. Such an analysis indicate how long one need to measure, before vehicle damage variability can be neglected.

KEYWORDS

Vehicle fatigue, random loads, damage variability, road roughness

INTRODUCTION

Travelling vehicles are exposed to dynamic loads caused by unevenness in the road. These loads induce fatigue damage in the vehicle, which may cause structural failures. Often, in statistical analysis of the fatigue damage $D(t)$, only the expected value $\mathrm{E}[D(t)]$ is considered. However, in many practical situations also the variability is of interest.

For example, decisions of measurement length of vehicle loads may be based on damage variability [1]. To decide if stochastic load models give the same fatigue damage as actual load measurements, it is necessary to know the uncertainty of the fatigue damage estimates [2]. Furthermore, computations of safety index for fatigue failure includes computations of the coefficient of variation $\mathrm{CoV}[D(t)] = \sqrt{\mathrm{V}[D(t)]}/\mathrm{E}[D(t)]$, see [3]. This paper is focused on computation of the coefficient of variation, CoV.

We restrict ourselves to stationary and ergodic loads. The ergodicity assumption is needed in order to be able to estimate the expected damage and covariance of the damage intensity from one sample path (measurement of a load). Here both Gaussian and non-Gaussian loads will be considered. Note that, for stationary random loads when the probabilistic model is known, one can obviously use Monte-Carlo methods to estimate the expected damage and the variance and the assumption of ergodicity is unnecessary. However, for vehicle loads, an accurate random model is usually not known, and only one load measurement is available.

VEHICLE FATIGUE DAMAGE

Vehicle fatigue damage is assessed by studying a quarter-vehicle model travelling at constant velocity, *v*, on road profiles. This very simple vehicle model can not be expected to predict loads on a physical vehicle exactly, but it will high-light the most important road characteristics as far as fatigue damage accumulation is concerned; it might be viewed as a 'fatigue load filter'. In this study the model comprises masses, linear springs and linear dampers; the only non-linearity is the ability to loose road contact. The parameters are set so that the dynamics of the model resembles a heavy vehicle.
The total force acting on the sprung mass of the quarter-vehicle model is rainflow-counted, i.e. each load maximum is paired with a load minimum selected by the rainflow method, see [4] . A load cycle is defined as the difference between a maximum and corresponding minimum. Damage caused by these load cycles s_j are evaluated with Palmgren-Miner's linear damage accumulation hypothesis and fatigue strength is described by Basquin's relation. Finally, the total damage is given by

$$D(T) = C\sum_{j=1}^{N_{rfc}} s_j^k \ . \tag{1}$$

Here, the value of *C* is unimportant, since only relative (pseudo) damage values will be studied. Hence, *C* will not affect the coefficient of variation and to simplify notation we let C=1. The other fatigue parameter, *k*, has a large influence on the coefficient of variation. For vehicle components, *k* is usually in the range 3 – 8.

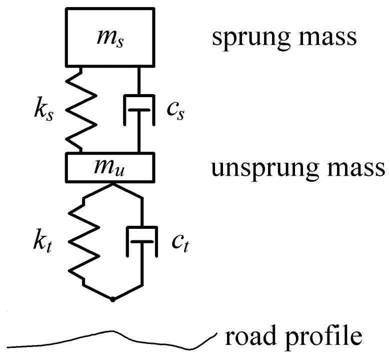

Fig. 1: The quarter vehicle.

COMPUTATION OF THE COEFFICIENT OF VARIATION

When the model of a random load is fully specified, e.g. if the load is a stationary Gaussian process with a known mean and spectrum, then both expected damage as well as coefficient of variation can be estimated by means of the Monte Carlo method. More precisely, one may simulate a large number of loads, with fixed length, from the model and compute the accumulated damage for each of the loads. Then expected damage and variance can be estimated using standard statistical estimators. (Even the distribution of the damage can be studied in this way.) For very long sequences the variability of the observed damage values is approximately normally distributed and hence mean and coefficient of variation fully describe the variability. However, for high values of the parameter *k* the convergence to normality can be very slow.

In the situation when one has only one measured load then one could, based on data, fit a model and use Monte Carlo methodology to estimate mean and variance. However, this usually introduces a modelling error, which should not be neglected. Here, instead, we study two non-parametric approaches, where no particular model, for the measured load is specified. The loads are assumed to be stationary and ergodic. (Ergodicity means that one infinitely long realisation defines the random mechanism that generated the load.)

Method 1: The partial damage approach
The damage for each kilometre is computed separately, and the total damage is expressed as the sum of all partial damage values,

$$\tilde{D}(T) = \sum_{j=1}^{N} d_j = N\overline{d} \tag{2}$$

where $\overline{d}$ is the average partial damage. The length of each section must be long enough, so that the sum of all partial damage values approximates the damage computed from the whole sequence, i.e $\tilde{D} \approx D$. We suppose also that d_j , j = 1,…,N, is a stationary ergodic sequence, with the covariance $r_j = \operatorname{cov}[d_{n+j}, d_n]$. Then, the variance is simple to compute,

$$\operatorname{Var}[\tilde{D}(T)] = \operatorname{cov}\left[\sum_j d_j, \sum_n d_n\right] = N\left(r_0 + 2\sum_{j=1}^{N_0}(1-\frac{j}{N})r_j\right) = N\sigma^2 \tag{3}$$

Obviously σ^2 is a function of N as well. However, if r_j converges fast to zero as j increases then this dependence can be neglected for large values of N,

$$\sigma^2 \approx r_0 + 2\sum_{j=1}^{\infty} r_j \,. \tag{4}$$

Note that, in practice, r_j is unknown and has to be estimated from a load measurement.

Suppose that a load has been observed for a time period [0,t] and let $\hat{r}_j$ be the estimated covariance. For a fixed j, the estimate $\hat{r}_j$ can be very uncertain, unless j is much less than N. In addition, if we assume that r_j converges fast to zero, then a large part of $\hat{r}_j$, say above N_0, is not significantly different from zero. Consequently we propose to remove the nonsignificant $\hat{r}_j$ from the sum and estimate σ^2 by the following formula

$$\hat{\sigma}^2 = \hat{r}_0 + 2\sum_{j=1}^{N_0}(1-\frac{j}{N})\hat{r}_j \,. \tag{5}$$

Finally, the coefficient of variation is estimated by

$$\operatorname{CoV}[D(T)] \approx \frac{1}{\sqrt{N}}\frac{\hat{\sigma}}{\overline{d}} \,. \tag{6}$$

For the theoretical examples in the following sections, the covariance values r_j, $j > 0$ are close to zero. However, for (long) measured loads the covariance is usually correlated. Hence, in practical situations, N_0 is usually larger than zero.

Method 2: The cycle approach
In this section we use a similar approach as above, but use the original expression (1). We denote the damage due to the j:th maximum and corresponding minimum by $q_j = s_j^k$. Now the total damage is expressed as $D = N_{rfc}\overline{q}$, where $\overline{q}$ is the average of all q_j. Furthermore, we suppose that q_j , j = 1,…,N_{rfc}, is a stationary ergodic sequence, with the covariance

$\rho_j = \text{cov}[q_{n+j}, q_n]$. Now, in the same way as above, we express $\text{Var}[D(T)] = N_{rfc}\sigma^2$ and get the estimate

$$\hat{\sigma}^2 = \rho_0 + 2\sum_{j=1}^{N_1}(1 - \frac{j}{N_{rfc}})\rho_j \; . \tag{7}$$

where the covariance above N_1 is neglected. Finally, the coefficient of variation is estimated by

$$\text{CoV}[D(T)] \approx \frac{1}{\sqrt{N_{rfc}}}\frac{\hat{\sigma}}{\bar{q}}. \tag{8}$$

STOCHASTIC MODELS OF THE ROAD PROFILE

We will check the accuracy of the two CoV-estimates using a quarter-vehicle travelling on road profiles realized from two different stochastic models. These two road profile models are described in the following two subsections.

The Gaussian model

In the standard ISO8608 a stationary Gaussian road profile model is suggested. A stationary zero-mean Gaussian model is uniquely defined by its spectrum. In ISO 8608 the following spectrum is proposed

$$R_{ISO}(\xi) = \begin{cases} 10^q \left(\dfrac{\xi}{0.1}\right)^{-w}, 0.01 \le \xi \le 10, \\ 0, \qquad \text{otherwise.} \end{cases} \tag{9}$$

where ξ is the spatial frequency in m^{-1}.

The superposition-model

The standard model is not an accurate road model [5]. A more complex model, which includes sections with increased roughness, is more similar to actual roads [2, 6]. The main variability in the road profile is described by the stationary Gaussian process $Z^{(0)}(x)$, with spectrum

$$R_0(\xi) = \begin{cases} 10^{a_0}\left(\dfrac{\xi}{\xi_0}\right)^{-w_1}, 0.01 \le \xi \le 0.2, \\ 10^{a_0}\left(\dfrac{\xi}{\xi_0}\right)^{-w_2}, 0.2 \le \xi \le 10, \\ 0, \qquad \text{otherwise,} \end{cases} \tag{10}$$

where $\xi_0 = 0.2$ m^{-1}. In order to add rough parts, irregularities of two types, long-wave and short-wave, are superimposed to $Z^{(0)}(x)$. The two types occur independently of each other.

To exemplify, a 300 m long road is generated with three long-wave and two short-wave irregularities, see Figure 2. Note that, as the example shows, long-wave and short-wave irregularities may overlap. Moreover, the *i*:th long-wave irregularity and the *j*:th short-wave irregularity are described by $Z_k^{(1)}(x)$ and $Z_j^{(2)}(x)$.

To avoid discontinuities at the start and end of the rough sections, the added irregularities starts and ends with two values equal to zero. Thus, the irregularities are non-stationary and hence it is impossible to assign a spectral density to them. However, an irregularity reaching from -∞ to +∞ is stationary. The spectral densities of such infinite length long-wave and short-wave irregularities are given by

$$R_1(\xi) = \begin{cases} \left(10^{a_1} - 10^{a_0}\right)\left(\dfrac{\xi}{\xi_0}\right)^{-w_1}, & 0.03 \le \xi \le 0.20, \\ 0, & \text{otherwise,} \end{cases} \tag{11}$$

$$R_2(\xi) = \begin{cases} \left(10^{a_2} - 10^{a_0}\right)\left(\dfrac{\xi}{\xi_0}\right)^{-w_2}, & 0.20 \le \xi \le 2.0, \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

For finite length irregularities see [6].

Furthermore, the location and length of the sections with added roughness is random. More precisely, the distance between the end of a long-wave irregularity and the start of the next is exponentially distributed with mean θ_1. Similarly, the distance between end and start of short-wave irregularities is exponentially distributed with mean θ_2. The length of long-wave and the length of short-wave irregularities are exponentially distributed with mean d_1 and d_2, respectively. Table 1 comprises all road parameters from the superposition model.

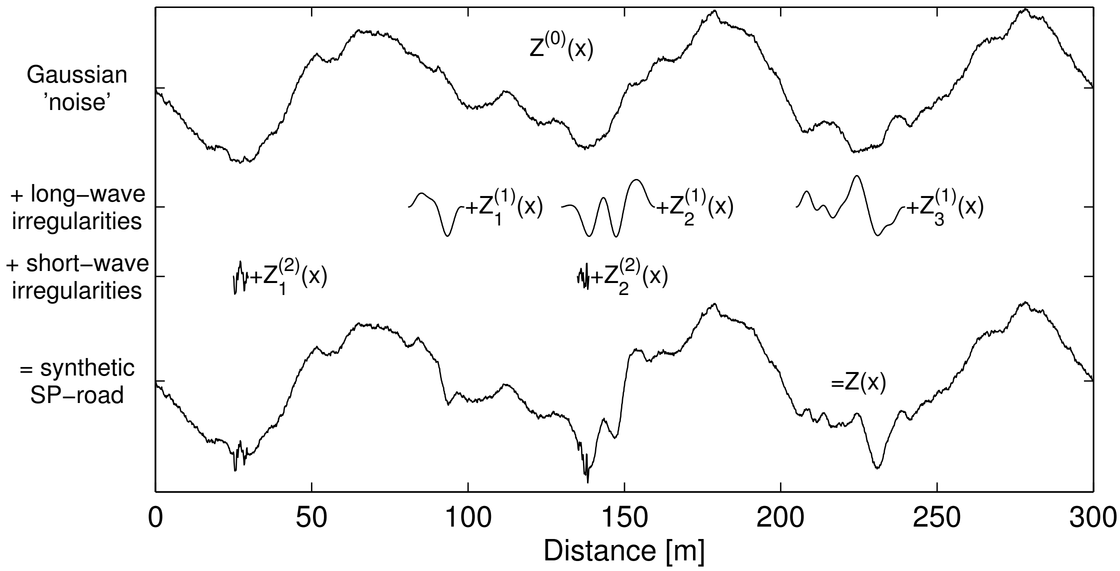

Fig. 2: A synthetic profile $Z(x)$ from the superposition-model.

Table 1: Parameters in the road model, LWI = long-wave irregularity, SWI = short-wave irregularity.

	Description		Description
a_0	Severity level, 'regular' road	θ_1	Mean distance between LWI:s
a_1	Severity level for LWI:s	θ_2	Mean distance between SWI:s
a_2	Severity level for SWI:s	d_1	Mean length of LWI:s
w_1	Spectrum 'slope', long-wave region	d_2	Mean length of SWI:s
w_2	Spectrum 'slope', short wave region		

EVALUATION OF THE ESTIMATION PROCEDURES

In this section N=1000 roads are simulated from one of the road models and the vehicle fatigue damage corresponding to each road is computed. The ratio between the empirical standard deviation and average damage, gives the empirical coefficient of variation. Since N is large we can treat this as the true CoV. Also, from each load sequence the CoV is estimated using each of the two methods. Hence, N CoV-estimates are obtained for each method, which are compared to the true value.

Coefficient of variation: Gaussian model

In the first example we simulate N = 1000 stationary Gaussian roads of length L = 45 km with $q = 10^{-4}$ and w = 2. The vehicle velocity is set to 70 km/h and the fatigue exponent k = 4. Figure 3 shows that the average estimate from Method 1 is closer to the true value than the estimate of Method 2. However, the variability of the estimate from Method 2 is lower than the estimate from Method 1. Note also that the empirical CoV is low (0.042) already at L = 45 km. However, it is well known that this road profile model is not accurate [6]. In the coming examples a more realistic road model is studied.

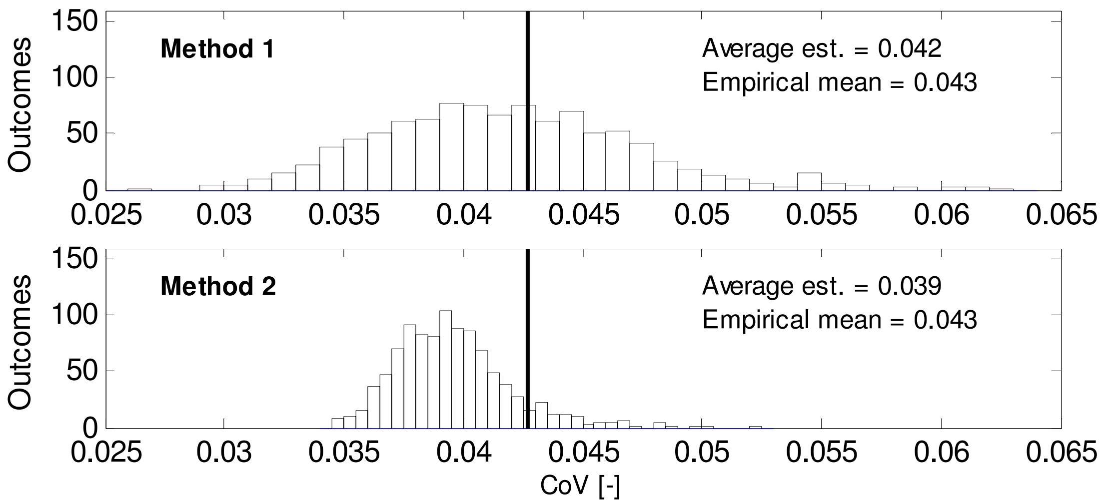

Fig. 3: Accuracy and precision of CoV-estimates (k=4, v=70 km/h, L = 45 km, Gaussian roads). The vertical solid line represents the 'true' value 0.043, the empirical CoV.

Coefficient of variation: superposition (SP) model

In the second example we simulate N = 1000 roads from the superposition model of length L=45 km. The parameters are set to a_0 = -5.4, a_1 = -4.3, a_2 = -3.5, w_1 = 3.4, w_2 = 2.2, θ_1 = 400 m, θ_2 = 1500 m, d_1 = 32 m and d_2 = 5 m. As the above example, the vehicle velocity v = 70 km/h and the fatigue exponent k = 4. In this example, both methods show a similar performance.

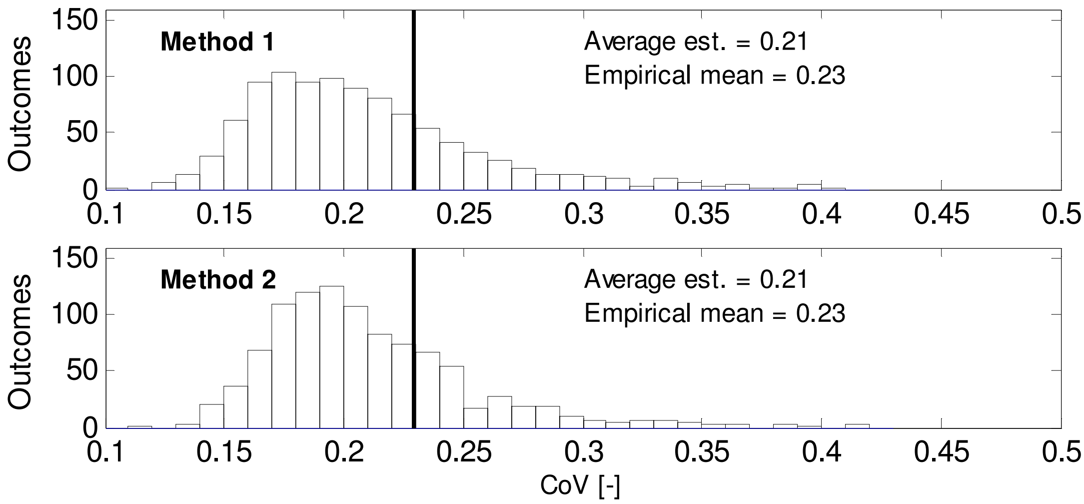

Fig. 4: Accuracy and precision of CoV-estimates (k=4, v=70 km/h, L = 45 km, SP-roads).

Figure 5 shows the result with the same settings (45 km long SP-roads, v=70 km/h) as the above example, but for $k = 6$. As expected the accuracy is worse. When the fatigue exponent is high, only high load peaks contribute significantly to the total damage, and the load sequence must be long enough, so that many high load peaks have occurred. In this example, the road length is too short, and the estimates are based on a (too) small number of high peaks.

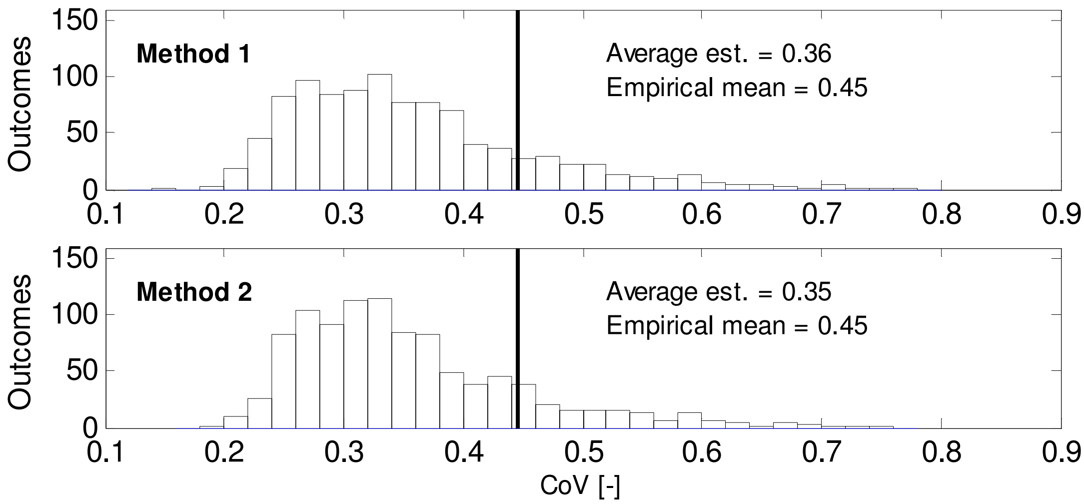

Fig. 5: Accuracy and precision of CoV-estimates (k=6, v=70 km/h, L = 45 km, SP-roads).

In the next example, the length is increased: 1000 SP-roads of length 400 km is simulated. The results in Figure 6 show that the average CoV-estimates from both methods have almost converged to the true value (indicated by the vertical solid line).

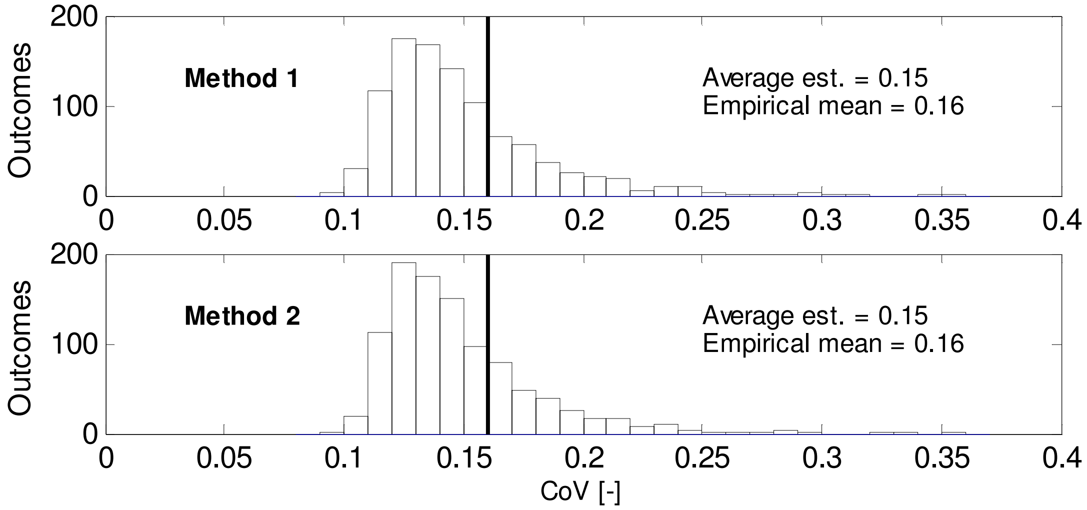

Fig. 6: Accuracy and precision of CoV-estimates (*k*=6, *v* =70 km/h, *L* = 400 km, SP-roads).

COEFFICIENT OF VARIATION FOR A MEASURED ROAD

This section presents the analysis of a 45 km long measured gravel road. The vehicle is simulated at 70 km/h and the coefficient of variation is estimated, using both methods. The table below show the estimates for $k = 3,\dots,8$.

Table 2: Estimated CoV for a 45 km long rough gravel road in northern Sweden.

Method	***k* = 3**	***k* = 4**	***k* = 5**	***k* = 6**	***k* = 7**	***k* = 8**
1: CoV (split)	0.11	0.15	0.19	0.23	0.27	0.33
2: CoV (cycle)	0.08	0.12	0.17	0.21	0.26	0.31

In Figure 7 the CoV is plotted as a function of road length, for $k = 6$. The function is obtained from the CoV-estimate of Method 1, 0.23, and (6). The measured length should be approximately 250 km in order to obtain a CoV below 0.1. But in practice, to obtain such a long measurement, one must measure several gravel roads (with similar properties). This will add to the variability, and the total measurement should be even longer.

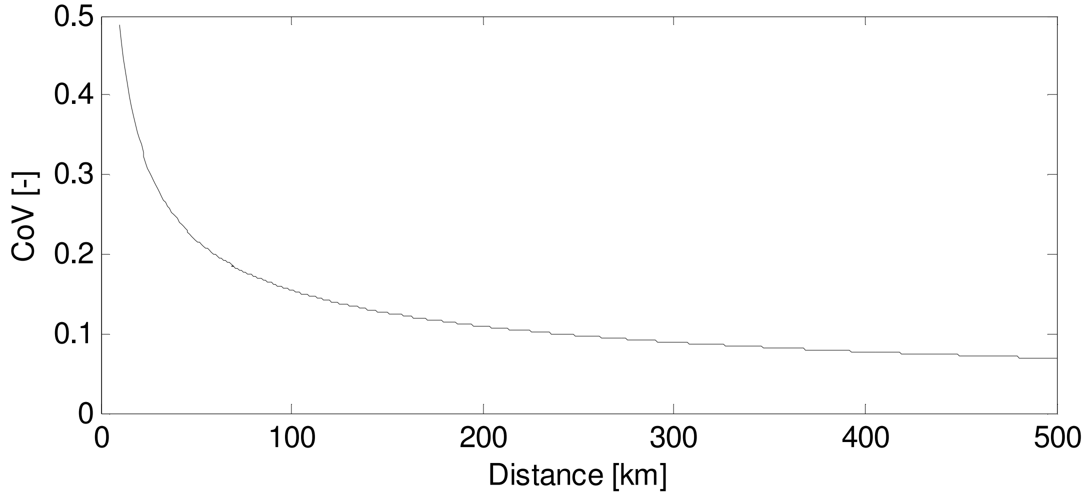

Fig 7: Estimated CoV as a function of measured distance ($k = 6$, $v = 70$ km/h).

DISCUSSION

From the examples above, we can see that, in general, the two methods perform similarly. Both methods have a tendency to underestimate. However, Method 1 is often easier to use in practical situations, when (more complex) measured vehicle loads are studied. For example, a construction vehicle often travels back and forth on the same road stretch. A measurement from such an operation will show correlated damage values, corresponding to twice the length of this road stretch. Also, for very long measurements (order 100 km), from changing conditions (e.g. changing road types, vehicle speed, etc), the covariance sequences (r and ρ) will converge slowly to zero. In such scenarios, it is easier to use the more crude division used in Method 1, in order to describe correlation for long distances. However for very short measured loads, when the Method 1 can not be applied, the Method 2 can still give useful estimates of the coefficient of variation.

CONCLUSION

Accurate estimation of damage variability from only one load sequence is difficult. Here, two quite simple methods to estimate the coefficient of variation of the accumulated rainflow damage are compared. Method 1, the slightly easier method to use, is proposed. In this method the variability is analysed by division of the total load sequence into subsequences. Here, each subsequence corresponds to 1 km long road sections. A damage value is assessed for each subsequence. Then, the covariance sequence of the damage values is estimated and, finally, the CoV can be estimated. The proposed method has a tendency to underestimate the CoV, especially for high values of the fatigue exponent, k.

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