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Estimation of Extreme Ship Responses

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Abstract

In practice the severity of extreme ship response is measured by high quantiles of long term distribution of the response. The distribution is estimated by combining the short term distribution of the response with a long term probability distribution of encountered sea states. The paper employs an alternative approach, the so called Rice's method, based on estimation of expected number of up-crossings of high levels by stress during one year. The method requires description of long term variability of the standard deviation, skewness, kurtosis and zero up-crossing frequency of ship response. It is assumed that the parameters are functions of encountered significant wave height, heading angle and the vessel service speed. The relation can be estimated from the measured stresses or computed by dedicated software assuming rigid ship hull model. Then Winterstein's transformed Gaussian model is utilized to estimate the up-crossings rates of response during a sea state. The proposed method is validated using the full-scale measurements of a 2800 TEU container ship during the first six months of 2008. Numerical estimation of 4400 TEU container ship extreme response illustrates the approach when no measurements are available.

Key words: *Extreme Response, transformed Gaussian process, long term distribution, significant wave height.*

1, Introduction and some preliminaries

Evaluating the risk of the ship response exceeding the component strengths is an important consideration during ship design. Usually, the components are designed, such that, the probability of the design response exceeding the component strength is very low. For highly reliable designs, the ships are designed for responses with high return periods. In this paper, the design return periods that are considered are of at least 20 years or longer and the response quantity of interest is the stress at specified locations in the ship structure.

If long records of response measurements are available and if stationary shipping can be assumed, then standard statistical methods to predict the extreme values could be employed. The three approaches that have been classically used are based on selecting a suitable quantile of the fitted cumulative distribution function (cdf) as the design value. Specifically, these methods are as follows:

A. The most commonly used method is to fit the so called “long term” cdf to the observed peaks of the response. Often, the two parameter Weibull cdf is used; see DNV [1].
B. The second approach is to extract the maxima in blocks of measurements, e.g., to find hourly, yearly maxima of the response, and then fit Gumbel or Generalized Extreme Value distribution to the maxima, see Coles [2].
C. The third method is known as the Peak Over Threshold (POT) method, which is a systematic way to analyze the tails of a distribution by means of exceedances over some high levels. In the standard version, POT employs the so called Generalized Pareto Distribution to model the tails of a response, see Coles [2].

In the present paper, the so called Rice's method, which is similar to the approach B, will be used to estimate the design values. The proposed method uses the expected number of up-crossings of the response, across some fixed level, to bound the yearly response maxima. More precisely, if we denote the maximum stress X, say, during a chosen period of t years, by M, then the design extreme stress x_T is the t/T quantile in M-distribution, i.e., solution of the following equation

$$P(M > x_T) = \frac{t}{T}.$$

Let $N^+(x)$ be the expected number of up-crossings of a high level x by $X(\tau)$ during a period t years. Then the M-cdf can be bounded as follows

$$P(M > x) \le N^+(x) + P(X(0) > x),$$

see Cramer and Leadbetter [9] for a more detailed discussion. Now, since the probability of the stress at $t = 0$, i.e., $X(0)$ being larger than the design extreme stress x_T is negligible, Rice's method proposes to conservatively estimate the up-crossing of x_T as

$$N^+(x_T) = \frac{t}{T}. \tag{1}$$

With respect to the stress levels in a ship, estimation of $N^+(x)$ is not an easy task. Here, we shall follow a standard way of estimating $N^+(x)$ based on the assumption that the encountered wave environment is a series of stationary sea conditions, called sea states, lasting from 20 minutes to several hours. Most often, sea states are characterized by some parameters, such as significant wave height H_s, wave period T_p, etc., gathered in a vector W, say. Then the encountered wave environment is described by a sequence of sea states parameters W_i, $i = 1, \dots, K$. Here K is the average number of sea states encountered during t years. The distribution of W_i is called long term distribution of sea states parameters. It is a statistical description of the variability of encountered sea condition. Obviously, the cdf is dependent on the shipping routes, maritime decisions of the captains and many other factors. In the following, the pdf of the long term sea state parameters will be denoted by $f(W)$.

The variability of the ship response, caused by the encountered wave environment, can also be modeled as “locally” stationary processes. A stationary sea condition is often modeled by means of linearly interacting Gaussian cosine waves and for heavy seas, by non-Gaussian second order Stokes waves. For response analysis, one needs to model the interaction between the ship structure and the applied waves, and this can be very complex. However, for the moment, we assume that we can compute $\mu^+(x|W)$ - the frequency of up-crossings of level x by a stress X, for the sea conditions described by parameter W. Then, if the long term pdf, $f(W)$, of encountered sea states is available and the expected number of encountered sea states K during the period t has been estimated, then we can write

$$N^+(x) = K \cdot \Delta t \int \mu^+(x \mid W) f(W) dW \ ; \tag{2}$$

where, Δt is the duration of the (stationary) sea state. Often, $\Delta t = 30$ minutes.

The paper is organized as follows. In Sections 2 and 3, methods to compute the crossing frequencies $\mu^+(x|W)$ is presented. The estimation of the long term pdf $f(W)$ of the encountered sea states is addressed next in Section 4. Finally, in Section 5, the accuracy of the proposed method is validated using full-scale measurements of a 2800 TEU container. In addition, extreme stresses in another 4400TEU container ship are estimated in order to illustrate how to use the method when there are no stress measurements are available.

2, Expected number of up-crossings of levels by response

If the joint probability density function (pdf) of ship response X and its derivative $\dot{X}$ under a stationary sea state W is known, then the up-crossing frequency of the level x, $\mu^+(x|W)$, can be computed by means of Rice's formula(Rice [3, 4]) viz.

$$\mu^+(x \mid W) = \int_0^\infty z f_{X(0),\dot{X}(0)}(x, z \mid W) dz \,. \tag{3}$$

For Gaussian responses, the integral in Eq. (3) can be evaluated analytically if the standard deviations of the stress and its instantaneous time derivative, denoted respectively by σ_X and $\sigma_{\dot{X}}$, are known. (Estimating σ_X and $\sigma_{\dot{X}}$ is not easy. If no stress measurements are available, a dedicated software has to be used to compute these parameters for a fixed location in the ship and for sea conditions described by parameters W.) Obviously, when the relations $\sigma_X(W)$ and $\sigma_{\dot{X}}(W)$ are established, if the expected number of encountered sea states K is determined and the long term distribution of sea states $f(W)$ is estimated, then $N^+(x)$ can be computed by means of Eq. (2). The steps involved in calculating $N^+(x)$ in the case of Gaussian response, sketched above, can be generalized as follows:

A. Find a parametric family of pdfs $f_{X(0),\dot{X}(0)}(x, z \mid \Theta)$, where Θ is a vector collecting statistical parameters of ship response under sea states W;

B. Establish the relation between sea states W and the parameter vector Θ, i.e., a function $\Theta(W): W \rightarrow \Theta$;
C. Find the statistics for encountered wave environment, i.e., K and $f(W)$;
D. Estimate the design extreme stress x_T by means of Formulas (1, 2 and 3).

However, it is well known that in severe sea conditions, the stresses are non-Gaussian. In such situations, employing a Gaussian model for the stresses may lead to underestimation of the design values by 100%, see Mao et al. [5].

2.1 Modeling non-Gaussianity of response

A simple Gaussian model largely underestimates the up-crossings since the interaction between the ship structure and the encountered waves which result in a very complicated nonlinear dynamical system, is neglected. In this paper we shall not model the interaction but propose random models for the variability of stresses.

A new class of flexible models (includes the Gaussian responses) - the so-called Laplace Moving Average (LMA) processes, have been recently proposed to describe the non-Gaussian responses, see Åberg et al. [6]. In Mao et al. [7], Laplace processes were used to model the non-Gaussian ship responses and the Laplace model predicted well the up-crossing rates observed in measured stresses. The LMA models require knowledge of response power spectrum, skewness and kurtosis of the stresses. The spectrum of the response can be parameterized.

However, a limitation of the Laplace model, similar as for the second order Stokes waves, is that the pdf $f_{X(0),\dot{X}(0)}(x, z \mid \Theta)$ is not available in an analytical form. (The pdf is defined in the frequency domain by its characteristic function and has to be computed using numerical methods.) Since in order to evaluate $N^+(x)$ an integral in Eq. (2) has to be numerically computed, a simpler model for the non-Gaussian responses giving analytical expression for $\mu^+(x|W)$ would be preferable.

The transformed Gaussian model proposed in Winterstein et al. [10], is employed here. The transformation is defined by the third order Hermite polynomial which is calibrated so that the variance, skewness and kurtosis of the transformed Gaussian model match the corresponding moments of the responses X, viz. mean stress m, standard deviation σ_X, skewness $\alpha_3 = E\left[X(0)^3\right]/\sigma_X^3$, and kurtosis $\alpha_3 = E\left[X(0)^4\right]/\sigma_X^4$. Consequently, the vector Θ consists of (m, σ_X, $\sigma_{\dot{X}}$, α_3, α_4). Note that for some sea states W, the transformation is not defined for extreme levels and some modifications of the transformation is needed for such cases. However, this is a small problem in comparison to the advantage obtained by having an analytical expression for the crossing frequency $\mu^+(x|W)$.

Given the parameter Θ(m, σ_X, $\sigma_{\dot{X}}$, α_3, α_4), the transformed Gaussian process is defined by

$$x(\tau) = G(Y(\tau)) = mH_0 + \kappa\sigma_X \left[H_1(Y(\tau)) + c_2 H_2(Y(\tau)) + c_3 H_3(Y(\tau))\right], \qquad (4)$$

where, H_i are Hermite polynomials and Y is a standard Gaussian process (in what follows, the mean stress $m = 0$). The other parameters in Eq. (4) are given by

$$\kappa = \frac{1}{\sqrt{1 + 2c_2^2 + 6c_3^2}};$$

$$c_2 = \frac{\alpha_3}{6} \cdot \frac{1 - 0.015|\alpha_3| + 0.3\alpha_3^2}{1 + 0.2(\alpha_4 - 3)};$$

$$c_3 = 0.1c_4\left[(1 + 1.25(\alpha_4 - 3)^{1/3} - 1\right];$$

$$c_4 = \left(\frac{1 - 1.43\alpha_3^2}{\alpha_4 - 3}\right)^{1 - 0.1\alpha_4^{0.8}}. \qquad (5)$$

Let G^{-1} be the inverse function of G, then $Y(\tau) = G^{-1}(Y(\tau))$ and hence,

$$\mu^+(x|\Theta) = f_z \exp\left(-\frac{G^{-1}(x)^2}{2}\right), \quad f_z = \frac{1}{2\pi}\frac{\sigma_{\dot{X}}}{\sigma_X}. \qquad (6)$$

3, Estimation of relation $\Theta(W)$

In order to use the transformed Gaussian model, the parameters in Θ as a function of the sea state W, are necessary. In the following, the sea conditions are characterized by a single parameter - the significant wave height H_s. In the previous work by the present authors, Mao et al. [8] and Mao et al. [5], estimation of the standard deviation of response, $\sigma_X(W)$, and the zero up-crossing frequency, $f_z(W)$, were already discussed. Hence, here only the relation between skewness and kurtosis and the encountered significant wave height will be considered.

Skewness and kurtosis are measures of non-Gaussianity of a response. It is well known in ocean engineering that the effects of non-linear interactions between ship and waves are no longer negligible for large sea states. Therefore, we expect skewness and kurtosis to depend mostly on the encountered significant wave height. We investigate the relation using full scale-measurements of stresses of the 2800TEU container ship during first six months of 2008. The measured places are located at the *1/4* ship length forward of after perpendicular (denoted as after section), and amidship (denoted by mid section), respectively. The measurements contain both winter and spring voyages, which can be used to represent the variability of longer term wave environments. Since we are interested in crossings of high levels, only measured stresses under heavy seas will be considered, and hence only sea states with significant wave height H_s above 4 meters are considered.

3.1 Regression model for Skewness

In Fig. 1, the skewness of responses under large sea states against the encountered significant wave height Hs is plotted. In the figure the linear regression model is also presented. As shown in Fig. 1, the value of skewness will increase with the encountered H_s.

The regressed models for both sections are,

$$\alpha_{3\_aft} = 0.11H_s - 0.35\,,$$
$$\alpha_{3\_mid} = 0.063H_s - 0.39\,. \qquad (7)$$

where, α_{3_aft} and α_{3_mid} represent the skewness of after section and mid section, respectively. In the study, one has tried to regress skewness on other parameters, e.g. heading angle, but the more complex models did not explain the variability of H_s any better than the simple regression on H_s.

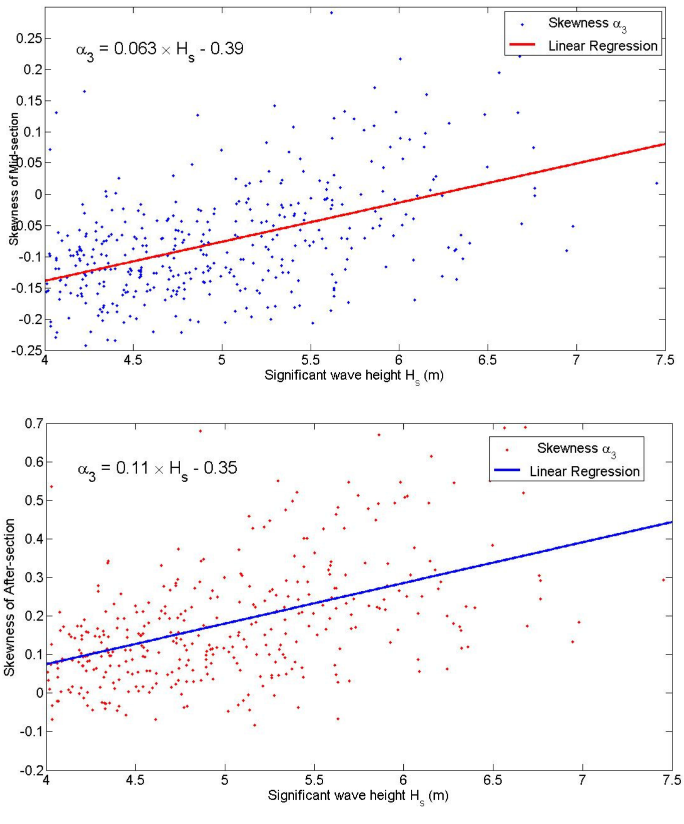

Fig. 1: Linear Regression of Skewness as a function of significant wave heights. Upper plot: Results for After-section; Lower plot: Results for Mid-section.

3.2 Estimation of Kurtosis

In Fig. 2, estimates of kurtosis α_4 for sea states with $H_s \geq 4$ m, are presented. There is no significant trend in the data and hence we propose to model the kurtosis by its mean value, here 3.5. For some pairs of parameters (α_3 , α_4) the Winterstein's transformation was not defined in the tail area. One has resolved this problem by alternating the value of kurtosis. This approach is motivated by an observation that the computed expected number of up-crossings $N^{+}(x)$ was not very sensitive for small variations in the kurtosis.

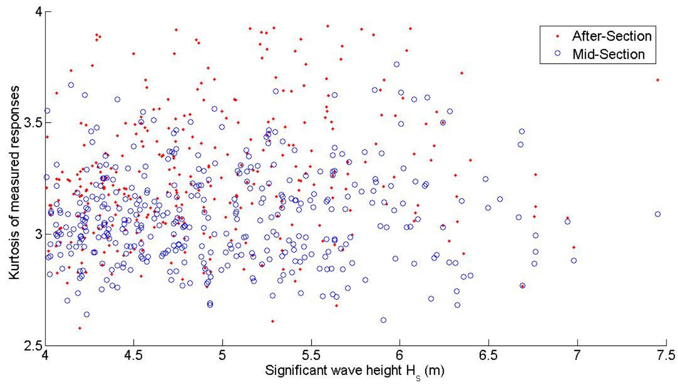

Fig. 2: Kurtosis of measured responses for both mid-section and after-section.

Note that estimates of skewness and kurtosis from 30 minutes long records are quite uncertain, i.e. the statistical error is significantly large. In order to illustrate the error, we have simulated the 30 minutes Gaussian signals with spectrums estimated in the response; see Fig. 3. In Fig. 3, we observe that estimates of skewness and kurtosis vary around their expected values - zero and three. We expect that statistical uncertainty of the estimates is even larger for non-Gaussian signals.

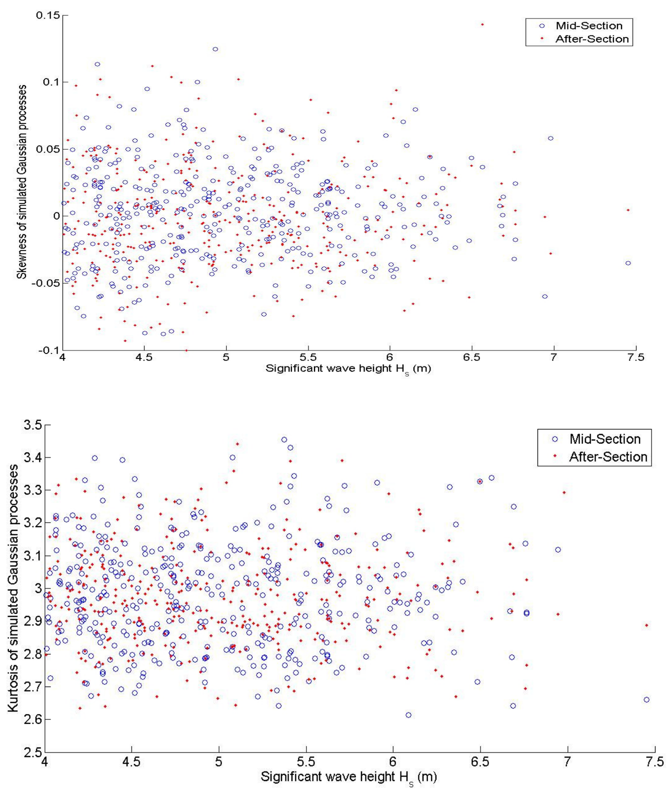

Fig. 3: Skewness (Upper) kurtosis (Lower) estimated from 30 minutes Gaussian responses with power spectrum estimated from the measured stresses for both mid-section and after-section.

4, Estimation of long term distribution of H_s

The statistical parameters (collected in the vector Θ) of a ship response under a sea state W, are denoted as $\Theta = (\sigma_X, f_z, \alpha_3, \alpha_4)$ here. The sea state W is described by its significant wave height H_s. Hence, the relation $\Theta(W)$ is actually a mapping $H_s \to (\sigma_X, f_z, \alpha_3, \alpha_4)$. For the computation of up-crossings, apart from the relation $\Theta(W)$, one also needs estimates of the expected number of encountered sea states K and the long term distribution of significant wave height H_s. The first quantity is related to the expected sailing time while the second depends on the shipping. Since the available stress measurements are from North Atlantic, this region will be considered in what follows.

The variability of sea environments, mainly H_s, has been extensively studied and many databases are available. In the following, 3 different estimates will be compared: (a) fitted distribution to encountered sea states measured by onboard radar on 2800TEU container ship during the first six months of 2008 (in Figure 4 the routs are presented); (b) distribution based on Spatio-temporal modeling of satellite measurements of H_s along the routs undertaken by 2800TEU container ship, see Baxevani et al. [14] and the appendix in Mao et al. [8], and (c) Weibull distribution recommended by DNV [12] for the North Atlantic.

In Fig. 5 one can see that the long term cdf estimated from the onboard radar measurements agrees well with the cdf determined from the Spatio-temporal model for small and moderate values of H_s; when Hs lies between 2 and 6m. However, the two cdf differ for higher H_s values. (The observed significant wave-heights are significantly smaller than the one found in the satellite data.)

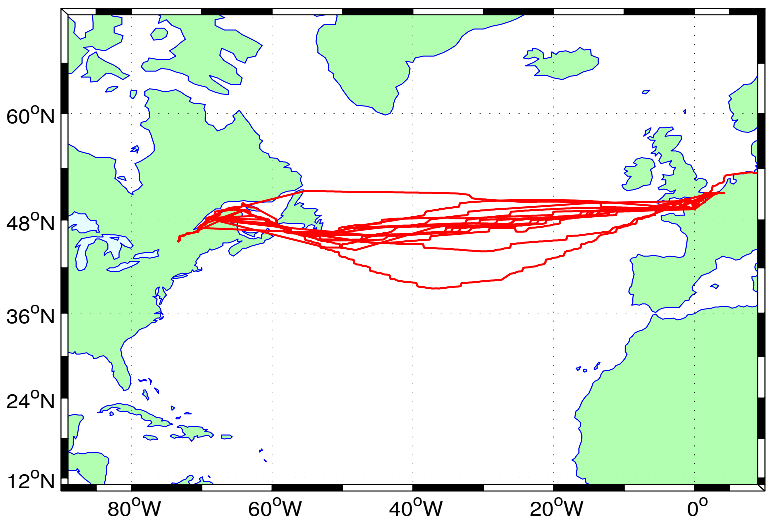

Fig. 4: Ship routes between EU to Canada from full-scale measurements during first 6 months of 2008.

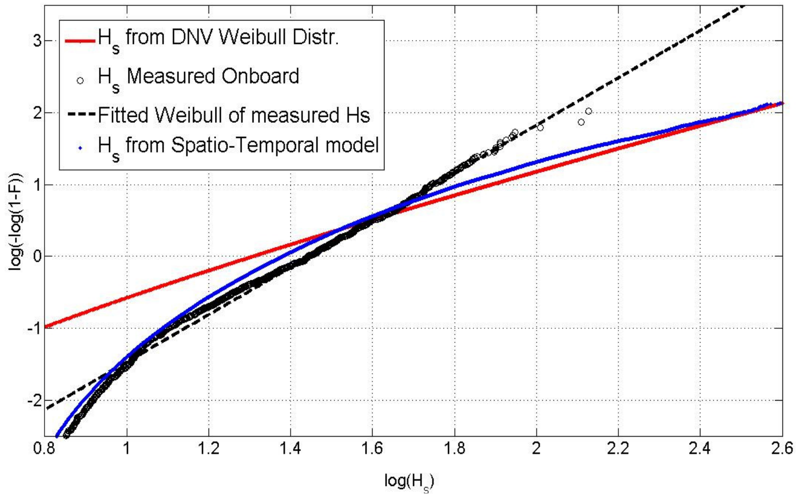

Fig. 5: Empirical distribution of Hs measured by onboard radar and long term distribution of Hs recommended by DNV [12] and obtained from the spatio-temporal model for H_s presented in Baxevani et al. [14].

The DNV recommended long term Weibull cdf agrees well with the one derived from the spatio-temporal model for high H_s values, which are essential for the estimation of extreme response. (The two distributions differs for small and moderate H_s values.) Consequently, the prediction of the extreme responses based on the DNV recommended Weibull cdf and the one derived from Spatio- temporal model will give similar values, while the long term cdf of H_s estimated from on-board measurements will lead to smaller design stresses x_T, see Table 2 and 3.

The large difference between the statistics of Hs encountered by the ship and the one recommended by DNV or derived from Spatio-temporal model could be a consequence of the routing plan system installed in the measured ship. However, the difference could also be due to just statistical error since the first estimate is based only on half year measurements while the second and third estimates are based on data collected over many years.

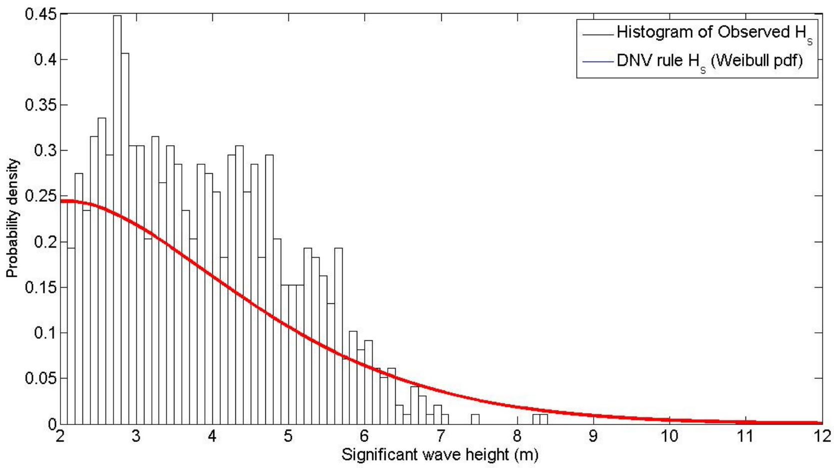

Fig.6: Empirical distribution of H_s measured by onboard radar, and long term distribution of Hs from DNV Rule.

The distribution of on-board observed significant wave heights (and also derived from the Spatio-Temporal) contains much more moderate seas than the cdf recommended by DNV, see Figure 6. This region of cdf is important for fatigue estimation. In Mao et al. [8], it was reported that the predictions of accumulated fatigue damage in the ship detail based on DNV recommended Weibull cdf for significant wave height, underestimated the observed damage by 50%. On the other hand, the long term cdf derived from spatio-temporal model gave predictions of the fatigue life well agreeing with the one estimated from the measured stresses.

5, Estimation of extreme responses

In this section, two examples of estimation of extreme responses are presented. The first example is carried out for a 2800TEU container vessel based on the full-scale measurements of two locations on the ship during the first 6 months of 2008. The measured places are located at the *1/4* ship length forward of after perpendicular (denoted as after section), and amidship (denoted by mid section), respectively. The measurements contain both winter and spring voyages, which can be used to represent the variability of longer term wave environments. The second example is performed on the basis of the numerical analysis of a similar 4400TEU container ship. The locations, chosen as the similar places of the previous ship and also denoted as after section and mid section, are used for estimations. The expected up-crossings of both after section and mid section are computed to predict the extreme response, e.g. 100-year stress x_{100}.

In both examples, we use the relations $\Theta(W)$ i.e. $H_s \rightarrow (\sigma_X, f_z, \alpha_3, \alpha_4)$, and three long term distributions of H_s presented in the previous section. The 100 years return values will be computed by means of Formulas (1, 2 and 3).

5.1 Example 1: Extreme prediction with measurements

5.1.1 Parameters of mean up-crossing models

In Section 3.1 the relations between skewness α_3 and H_s were derived; see Eq. (7). The parameter α_4 (kurtosis) is taken to be a constant and equal to 3.5 as before. The expected number of zero up-crossings by the stress process X, is approximated by a relation derived in Mao et al. [8] viz.

$$f_z = \left| \frac{1}{4.2\sqrt{H_s}} + \frac{2\pi U \cos(\beta)}{17.64 g H_s} \right| , \tag{8}$$

where, U is the service speed of the vessel, β the heading angle and g the gravity acceleration constant.

Finally, the standard deviation of ship responses σ_X, is determined through its relation with H_s viz $\sigma_X = C(\beta,U)H_s$. The values $C(\beta,U)$ of are computed by a linear strip theory. Table 1 lists its value in terms of heading angle β under service ship speed $U = 10$ m/s for the 2800TEU container ship.

Table 1: The expected relation $C(\beta,U)$ computed using a linear strip software with ship speed U = 10m/s, for both after and mid sections. Note that 0 for head sea, and 180 for following sea.

β	0	10	20	30	40	50	60	70	80
$C_{mid}(\beta)$	25.66	25.77	25.58	25.10	24.37	23.47	22.47	21.48	20.65
$C_{aft}(\beta)$	12.73	12.76	12.62	12.35	11.96	11.46	10.99	10.51	10.13
B	90	100	110	120	130	140	150	160	170
$C_{mid}(\beta)$	20.10	19.92	20.16	20.77	21.65	22.66	23.67	24.56	25.24
$C_{aft}(\beta)$	9.89	9.84	9.99	10.32	10.78	11.29	11.79	12.23	12.55

5.1.2 Estimation of extreme responses

Fig. 7 presents the expected number of up-crossings by different methods (de- noted as Method 1 to 4), together with the observed up-crossings. Methods 1 to 4 are all based on Winterstein's transformed Gaussian approach while using various parameter H_s as input. Method 1 uses observed H_s along ship voyages, see Fig.4; Method 2 uses H_s of the fitted Weibull distribution from observed H_s; Method 3 uses H_s from the Spatio-temporal model proposed by Baxevani et al. [14]; Method 4 uses H_s of the Weibull distribution recommended by DNV [12] for the North Atlantic Ocean.

Finally, the extreme response x_T is estimated by means of Equations (1 - 5) (note that $t = 0.5$ year in Eq. (1)). For extreme response prediction, only the modeling of right tail areas in Fig. 7 is of interest. The intersection between the estimated expected up-crossings and horizontal lines with the value of Y-axis equal to 0.025, 0.01, and 0.005 represent the predicted value of extreme stresses x_{20}, x_{50}, x_{100} respectively. The values of the quantiles are given in Table 2.

In Fig. 7, the expected number of up-crossings computed by Method 1 and 2 are shown to be very close to the observed up-crossings. The deviations are much larger for the up-crossings computed by Methods 3 and 4. This can be explained by the fact that the significant wave heights H_s that have been used in methods 3 and 4 are obtained from the Spatio-temporal model proposed by Baxevani et al. [14] and Weibull distribution recommended by DNV [12]. It must be noted that the latter two models predict much higher values of H_s than encountered during half year of measurements, see Fig. 5. Obviously, adequate estimate of the long term cdf encountered sea states is very important when extreme response are studied.

Note that the values of the design extreme stresses presented in Table 2 do not appear to be realistic. It must be noted, however, that the estimation method is somewhat crude. Experimentally, one has been measuring the strains; the stresses were subsequently computed by an elastic strain-stress relation with a stress concentration factor 2. This crude method may overestimate the stress here due to the plasticity characteristics of materials with extreme stresses. Obviously, one could predict extreme strains instead of extreme “computed” stresses, but the present paper is focusing only on the methodology of extreme prediction rather than the engineering problems.

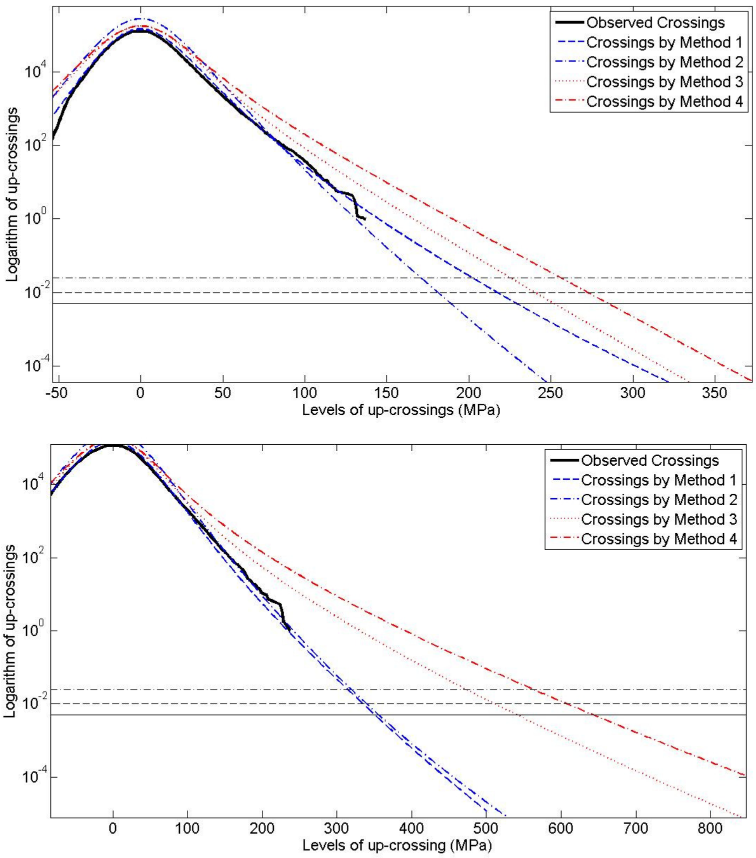

Fig. 7: Expected number of up-crossings computed by Winterstein's transform Gaussian approach on the basis of 6 months' full-scale measurements: Method 1 uses observed H_s; Method 2 uses H_s of the fitted Weibull distribution from the observed H_s; Method 3 uses Hs from Spatio-temporal model; Method 4 uses H_s of Weibull distribution recommended by DNV [12]. Horizontal dash-dotted lines represent the expected number of up-crossings related to x_{20}, x_{50}, and x_{100} respectively. Upper plot: Results for After-section; Lower plot: Results for Mid-section.

Table 2: Estimation of extreme stresses, i.e. the so-called 20-year, 50-year and 100-year stresses denoted by x_{20}, x_{50}, and x_{100} respectively, based on the full-scale measurements. Method 1 to 4 represent different methods to compute the expected number of up-crossings, same as Fig. 7.

Method No.	After section (Mpa)			Mid section (Mpa)		
	x_{20}	x_{50}	x_{100}	x_{20}	x_{50}	x_{100}
1	201.8	216.7	228.9	313.2	334.1	350.0
2	171.0	180.9	188.6	318.4	339.0	355.4
3	249.3	265.6	277.9	423.5	456.1	480.8
4	255.6	272.3	285.0	561.2	607.1	642.2

5.2 Example 2: Extreme prediction without measurements

The 4400 TEU container ship is used as an illustration of how one could use the method introduced in this paper when no measurements of responses are available. In order to compute the expected up-crossings using Winterstein's transformed Gaussian approach, we need to first compute the parameters of this model, i.e. standard deviation of responses σ_X, zero up-crossing response frequency f_z, and the skewness and kurtosis of the response α_3, α_4. As in the previous example, kurtosis α_4, which do not appear to influence the up-crossing frequencies, is assumed to be constant at 3.5. Further, f_z is approximated by the encountered wave frequency; see Eq. (8). Once this information is available, one needs to estimate the relations between significant wave height H_s, the standard deviation σ_X and the skewness α_3 of the response.

An estimate of the required relations can be achieved by carrying out some simulations. In performing the simulations, one assumes a stationary sea state of durations 3 hours. Only the mission condition with heading angle $\beta = 0$ and service speed $U = 10$m/s are considered. The stress is studied at two sections, similar as in the 2800TEU container vessel. Further, it was assumed that stress is only caused by vertical bending moment.

5.2.1 Estimation of σ_X and α_3

The real non-Gaussian ship responses (stresses) usually contain wave frequency responses and high frequency responses. The wave frequency responses are related to wave induced loadings, and high frequency responses are referred to wave induced vibrations. The responses caused by wave induced loading are usually dominating under moderate and calm sea states. However, in the presence of big storms, the transient loading, such as the slamming, will induce high frequency vibration (also known as whipping) of the ship structure. The high frequency vibrations can be very close to the ship's natural vibration frequency, and can lead to the resonance of ship structures, also known as springing. Both whipping and springing make it very hard to correctly model the ship responses. Additionally, a numerical analysis considering these two phenomenons can be extremely time consuming. Therefore, estimation of the skewness of the ship responses is not an easy task when no measurements are available.

First, we use the available stress measurements on 2800 TEU container ship to investigate the dependence of the parameters σ_X, α_3 on the high frequency vibrations. This was done by (a) extracting from the signal the wave induced stress, (b) evaluating variance and skewness of the stress and (c) comparing with the estimated values from the original stress measurements. Since the estimates of skewness was only marginally affected by “smoothing”, i.e. removing the high frequency response, one decided to model the relation between H_s and σ_X, α_3 by the general non-linear numerical analysis, considering only wave induced loading. The above approach requires significantly less computational efforts in comparison to the full stress analysis to reach sufficient accuracy.

In the nonlinear hydrodynamic analysis, the ship hull is assumed to be a rigid body. Its dynamic response, when operated in the ocean, can be computed by the spring-mass system as

$$(\boldsymbol{M}+\boldsymbol{A})\ddot{\boldsymbol{X}}(\mathrm{t})+\boldsymbol{D}\dot{\boldsymbol{X}}(\mathrm{t})+\boldsymbol{C}\boldsymbol{X}(\mathrm{t})=\boldsymbol{F_h}+\boldsymbol{F_d}+\boldsymbol{F_g}, \tag{9}$$

where $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{A}$ is the added mass, $\boldsymbol{D}$ and $\boldsymbol{C}$ are the coefficients of damping and stiffness, respectively, and $\boldsymbol{X}$ denotes the displacement of ship hull elements nodes. On the right hand side of Eq. (9), $\boldsymbol{F}_h$ represents the hydrodynamic wave excitation force by the income waves and the hydrostatic restoring force. $\boldsymbol{F}_d$ is the diffraction force. $\boldsymbol{F}_g$ is the inertial force due to the gravity acceleration.

Wave surface elevations, modeled as a random field, are usually described by the sum of a series of harmonic waves. The amplitude of each regular wave can be computed from the wave spectrum in terms of significant wave height H_s and wave period T_p. Wave phase is then chosen as a random variable to model the random wave elevations. If the wave amplitude is very small, the Froude- Krylov and hydrostatic pressure can be integrated over the mean free surface and mean wetted surface. This linear approach is a good approximation when the ship operates in relatively calm waters, with small motions. In this paper, we study the large storm phenomena to predict the extreme response. The assumption of mean wetted surface may make the analysis too crude. Instead, the exact wetted surface is used to compute the correct wave excitation force and restoring force. Hence, in the time domain analysis, the relative motion in Eq. (9) is computed to determine the wetted surface for each time step.

Further, when ship operates with a high forward speed, the effect of diffraction and radiation cannot be neglected. After getting the hydrodynamic loads for each time step, structural responses can be computed using either the FEM method or a simplified beam model calculation.

In Fig. 8 the skewness α_3 estimated from the simulated responses (at both sections) are plotted against significant wave height H_s. The fitted linear regression on H_s is also shown in the figure. We observe that the relation between skewness of ship responses and significant wave heights H_s is quite linear. The responses at the after section are more skewed than the one at the mid section; a similar trend was observed for the 2800 TEU container ship. The following skewness models, i.e. α_{3_aft} for after section and α_{3_mid} for mid section, is used to estimate mean up-crossings by transformed Gaussian approaches:

$$\begin{aligned} \alpha_{3\_aft} &= 0.032 H_s + 0.332\,, \\ \alpha_{3\_mid} &= 0.078 H_s - 0.231\,. \end{aligned} \tag{10}$$

In principle, the relation between standard deviation of responses and significant wave heights H_s can be simply computed by a linear numerical analysis, as it was done for the 2800 TEU container. However, in this example, additionally the nonlinear responses used in skewness computations are available and could also be used to determine the relation between σ_X and H_s. If the nonlinear responses are used to compute the standard deviation, then the relation may no longer be linear, i.e. $\sigma_X \neq C(0,10) H_s$. In other words, C could be a function of H_s. However, as shown in Fig. 9, the fraction $C = \sigma_X / H_s$ does not deviate too much from 4 for after section, and 7.5 for mid section. Hence, in the current analysis, the standard deviations of responses of both midsection and after section are estimated by means of $\sigma_{X\_aft} = 4H_s, \sigma_{X\_mid} = 7.5H_s$, respectively.

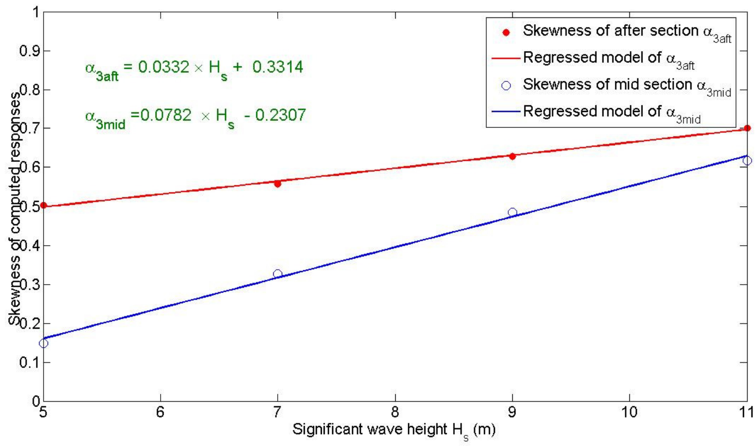

Fig. 8: Skewness models from the computed responses.

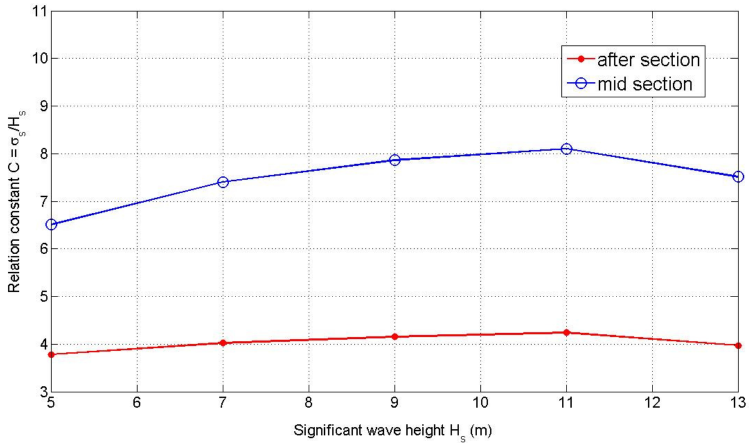

Fig. 9: Relation between standard deviation and significant wave height $C = \sigma_X / H_s$ for both after section and mid section, by means of non-linear numerical analysis.

5.2.2 Encountered wave environment H_s

For computing the expected up-crossings of the ship response during 6 months period using Winterstein's Transformed Gaussian approach, we need to calculate the encountered significant wave heights H_s. Estimates of H_s are obtained using the three approaches outlined in Section 4.

The 4400TEU container ship is assumed to be operated in the same routings as measured in Fig. 4. The measured wave environments are then applied to compute the expected up-crossings for the 4400TEU container ship. Consequently, we consider the same long term cdf as in the previous example.

5.2.3 Results of the extreme prediction

The design values are computed and compared in the same way as in Section 5.1.1. The expected numbers of up-crossings estimated for Methods 1 to 4, defined in Section 5.1.1, are presented in Fig. 10. The values of extreme responses, x_{20}, x_{50}, and x_{100} are listed in Table 3. The results of these extreme stresses also follow the same trend as the computed mean up-crossings in Fig. 10. Note again that the values of the extreme responses appear to be too high. This can be attributed to the crude model of elastic stress-strain relation and assuming the stress concentration factor (SCF=2). The estimates of the extreme stresses for the two vessels are very close. Additionally, the simplicity of the approach gives hope that the method could be useful in real engineering applications.

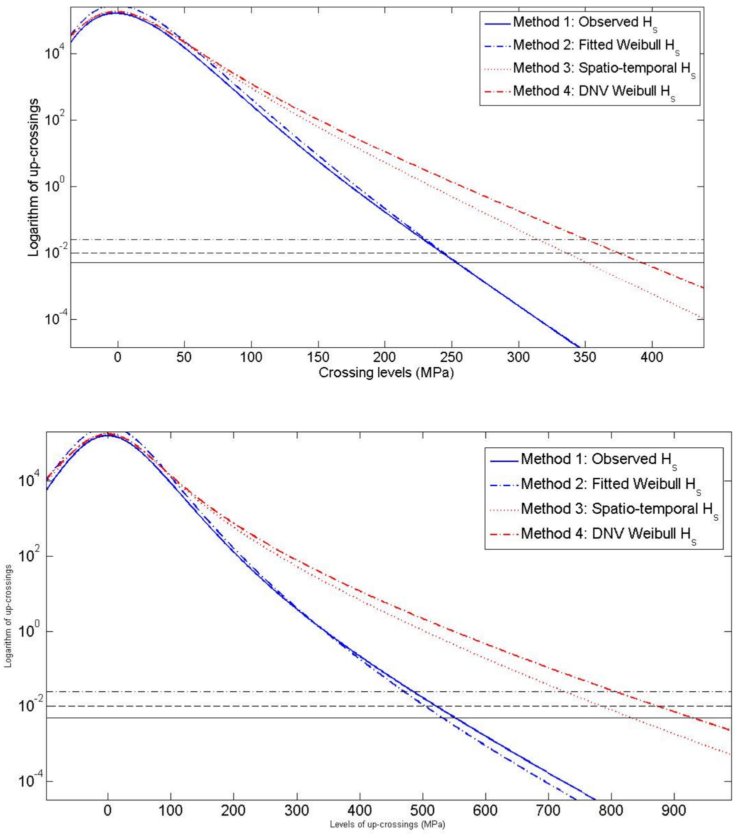

Fig. 10: Expected number of up-crossings and prediction of T-year extreme stresses x_T, computed by Winterstein's transform Gaussian approach on the basis of non-linear numerical analysis. Methods here represented are defined in Fig. 7. Upper plot: Results for After-section; Lower plot: Results for Mid-section.

Table 3: Estimation of extreme stresses on the basis of non-linear numerical analysis of the 4400TEU container ship. The methods used here are the same as Table 2.

Method No.	After section (Mpa)			Mid section (Mpa)		
	x_{20}	x_{50}	x_{100}	x_{20}	x_{50}	x_{100}
1	228.1	230.1	253.1	484.6	520.3	549.2
2	242.4	243.1	253.2	470.0	502.0	530.0
3	314.4	334.0	349.9	724.3	785.2	831.9
4	349.9	374.0	392.2	803.4	873.3	927.5

6, Discussions and Conclusions

This paper presented a simple approach for the prediction of extreme response, e.g. 100-year stress x_{100}, by means of Rice's method combined with Winterstein's transformed Gaussian model for stresses. The accuracy of this model is validated by the full-scale measurements of a 2800TEU container ship. The parameters in the Winterstein transformation are given by analytical functions of significant wave height only. The functions are estimated by means of a simple nonlinear numerical analysis assuming the ship hull as a rigid body.

The proposed method is also used to estimate extreme responses of a 4400 TEU container ship, for which no measurements are available. Results were similar to the 2800 TEU ship. Further validation of this method should be carried out for other types of container ships and other locations in the ship. Hopefully, for the ship the measurements would be available.

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