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Abstract

In this paper a simple method is proposed to estimate extreme ship response, which is defined by the upcrossing spectrum of the responses at high levels. The method requires limited statistical information about shipping and the corresponding responses. Since the real ship responses are often non-Gaussian, a transformed Gaussian approach is employed. The parametric transformation is a function of the standard deviation, skewness, kurtosis and zero up-crossing frequency of a response which varies with changing sea conditions. Parameters needed in the transformation are computed from their relations with encountered waves, characterized by the significant wave height here. Finally, this method is compared and validated with the typical engineering approach, based on the full-scale measurements of a 2800 TEU container ship during the first six months of 2008. If no measured responses are available, the parameters of the transformation can be estimated using simple numerical analysis.

Keywords: Extreme response, up-crossing, non-Gaussian response, significant wave heights, skewness, kurtosis

1 Introduction

Estimation of extreme ship responses is usually the first step to evaluate the safety of a ship during design. If long records of ship responses are available and stationary shipping is assumed, then standard statistical procedures could be employed to predict the extreme value of ship responses. One such method is to extract the maxima of blocks of recorded responses, (*e.g.*, to find yearly maxima of the response), and then fit Gumbel or Generalized Extreme Value distribution to the maxima. Another popular method, known as the Peak Over Threshold (POT) method, is to employ Generalized Pareto Distribution to model the tail of the responses, for example see Coles [1].

The most commonly used method in the engineering community is to fit the long term cumulative distribution function (cdf) of the local maxima for the responses. Often, a two parameter Weibull cdf is used to model the long term cdf of the maxima, see DNV [2]. Here, the long term cdf fitting approach will be referred to as the "Method 1". The drawback of this method is that if the recorded responses are low or moderate, using the corresponding maxima to extrapolate the tails to extreme levels will lead to large prediction errors.

This paper focuses on an alternative method, referred to as "Method 2" in the following description, to predict the extreme response. The estimates of the extreme values are compared with those obtained using Method 1 (the standard approach). Actually, Method 2 uses the estimated expected number of upcrossings by the response to bound the long term cdf of the response maxima in Method 1. Method 2 is derived by mathematical modeling of the variability of the responses as stochastic processes. Typically, developing a model for response variability involves: (a) using some well established models for variability of encountered waves; (b) mathematical description of ship wave interaction to compute wave loading; and (c) a model for structure properties to compute structural stresses. In the simplest case, this procedure leads to Gaussian models for the responses, enabling analytical evaluation of Rice's formula, see Rice [3, 4], to estimate upcrossings of extreme levels by the responses. Subsequently, the extreme responses can be predicted for any encountered waves.

However, due to the complex interaction between ship structure and applied waves, the true ship responses are non-Gaussian, particularly under large sea states. In such an eventuality, the Gaussian model can lead to large prediction errors, e.g., severe underestimation (50%) was reported in Mao et al. [5]. Several more complex non-linear wave models have been discussed in the literature, e.g., a quadratic correction is added to Gaussian sea to handle large waves. The interaction between ship and waves is also modeled in more complex fashion. Although the variability of responses is then more accurately described, the evaluation of Rice's formula becomes more difficult. There are several numerical approaches proposed in the literature to compute Rice's formula for non-Gaussian responses, see some recent references Naess and Karlsen [6], Butler et al. [7] or Galtier et al. [8]. In this paper, the transformed Gaussian process will be used to model the response, see e.g., Rychlik et al. [9].

The transformed Gaussian method proposed by Winterstein et al. [10] is employed to model ship responses in this paper. It requires only the knowledge of variance, skewness and kurtosis of the responses. If the expected number of upcrossings of mean level is known then the expected number of upcrossing of any level can be computed analytically by Rice's formula. Hence, the extreme response can be predicted if the long term distribution of response variance, skewness, kurtosis and mean level upcrossing rate is known.

In Section 2, a brief review of the two methods is presented, and a discussion on the conditions under which both methods give identical extreme value predictions is also included. Subsequently, the extreme value of ship responses is estimated by Method 2, and further validated by the typical Method 1 based on the full-scale measurements. Section 3 starts with a brief review of Winterstein transformation method. This is followed by the validation of the transformation method, using the same measurements. Since typically measurements of stresses over long time periods are not available, a statistical method to find the long term distribution of parameters needed to determine the transformation is also discussed in Section 3. In Section 4, Method 1 and Method 2 are used to predict the extreme responses for a vessel for which the response measurements are available. Finally, the conclusions emerging from this study are presented in Section 5.

2 Extreme estimation by Method 1 &2

The variability of ship responses are mainly caused by the encountered waves, which can be modeled as "locally" stationary processes (sea states). A stationary sea state (usually from 20 minutes to several hours) is generated using, for example, a linear (Gaussian) or a Stokes (quadratic) wave model. Most often, it is defined by a vector of parameters, say W, whose elements could be the significant wave height H_s, wave period T_p, etc. The encountered wave environments are described by a sequence of sea states W_i, $i = 1, \ldots, K$, where K is the average number of sea states encountered during a long term period. The distribution of W is called *long term* distribution of sea states and "statistically" describes the variability of sea state W selected at random from the sequence. The probability density function (pdf) of the distribution, denoted by $f(W)$, depends on shipping routes. In this paper, ship responses are related to its structural stresses, which clearly depend on the encountered seas states.

2.1 Review of Method 1

Cumulative distribution function (cdf) of local maxima of responses is often used for extreme prediction. The cdf describes the extreme value of response maxima. It is often employed because the cdf of response maxima height can be evaluated by means of generalized Rice's formula, see Crammer and Leadbetter [11]. Here, we use the cdf of response crest height. In Fig. 1, the crests of a signal are marked with dots.

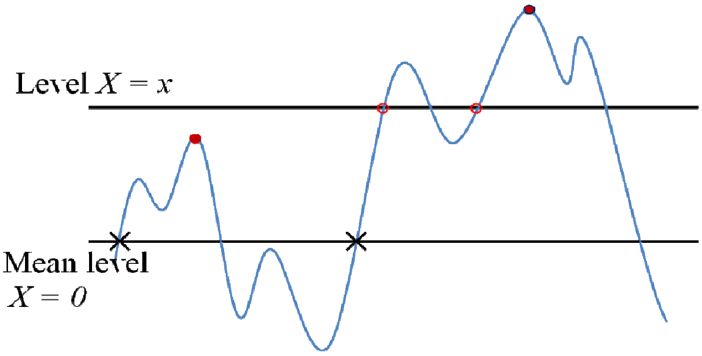


Fig.1: Example of up-crossings of mean level zero (crosses), up-crossings of level *x* (circles) and crests (dots).

Let *X* denote the height of the stress crests selected at random. The probability distribution of *X* is defined as the ratio of expected number of crests with height below *x* and the total number of crests. In the following, the mean value of stress heights is assumed to be a constant and without loss of generality, is set to be zero, for simplicity of expressions. Then, the distribution of crests *X* is given by

$$F(x) = \frac{\#(i : X_i \le x)}{n}, \tag{1}$$

where $\#(\bullet)$ denotes the number of points satisfying $(\bullet)$. Obviously, the distribution depends on the population of crests. For example, if one wants to estimate the extreme responses in time T, x_T, (T often equals to 20, 50 or 100 years), the number of crests during the period T should be determined first. However, assuming stationary shipping, a shorter period t, typically one year, can be used instead of T. (More precisely T is divided into T/t periods and one assumes that the stress crests have the same cdf in each of the T/t periods.)

Then the long term cdf of response crest height $F_t(x)$, say, during the period t, is given by

$$F_t(x) = \int F(x \mid W) f(W) dW \, . \tag{2}$$

Here, $f(W)$ is the pdf of the vector of parameters defining the sea states encountered in the period t and $F(x|W)$ is the so-called "short term" cdf of crest X computed for a stationary sea state W. Finally, the T-return stress value x_T is estimated by finding crest height that is exceeded once in $\frac{n \cdot T}{t}$ crests, i.e. it is the solution of the following equation

$$F_t(x_T) = 1 - \frac{t}{n \cdot T}, \tag{3}$$

Here, $T > t$ has the same unit as t.

2.2 Approximation of $F_t(x)$

Most often, when long records of stress measurements are available, a two-parameter Weibull distribution is employed to approximate the distribution $F_t(x)$. Then the long term cdf of crest X is given by

$$F_t(x) \approx 1 - \exp\left[-\left(\frac{x}{\alpha}\right)^m\right], \tag{4}$$

where α is the scale parameter, and m is the shape parameter of a Weibull distribution. The fitted Weibull distribution can usually well describe the distribution of the real ship responses, in particular for large responses. For example, ship response crests from half year's full-scale measurements of a 2800TEU container vessel are first presented in a Weibull probability paper as the left plot of Fig.2. Further, a logarithmic scale on Y-axis is chosen to present the exceedance probability, *i.e.,* $1-F_t(x)$, in the right plot of Fig.2. It shows that the long term distribution of crest height can be fitted well by a Weibull distribution. The Weibull distribution can be used to describe the distribution of the tail area with large responses.

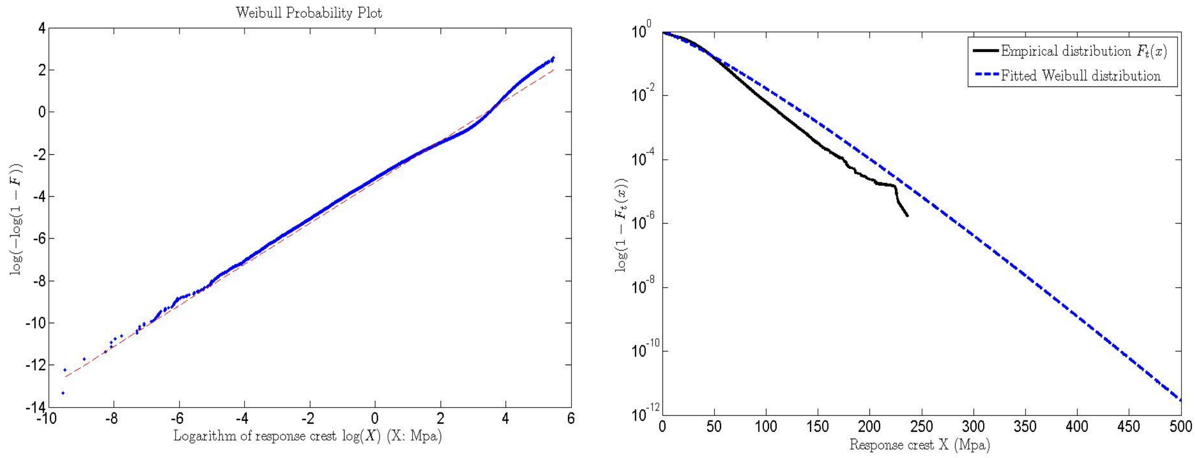


Fig.2: **Left**: Weibull plot of response crest heights measured during the period of first 6 months of 2008; **Right**: Empirical distribution and fitted Weibull distribution using the same records as the left plot.

However, the Weibull distribution does not always give a good fit to the tails of the long term distributions of ship response crests. In particular, for long periods with large number of calm sea states, the ship responses under each sea state can be modeled as Gaussian processes. Here, the Weibull distribution may be less appropriate to model the tails. This will be illustrated by the following numerical experiments.

Let the whole measured stresses be divided into a series of stationary parts (sea states), the corresponding spectrums of each sea state are easily computed. Assuming Gaussian responses for each sea state, the Gaussian stresses are then simulated from the related spectrums. Finally, the stress crests are extracted. The variability of the crests height is described by the empirical distribution function, i.e. $F_t(x)$ defined in Eq. (1). The extracted crests are then plotted on a Weibull probability paper, see Fig. 3 left plot, indicating that the distribution should be well modeled by the Weibull cdf. However a more careful inspection of the plot reveals that the fit is less accurate in the tails. In Fig. 3 right plot, the function $1-F_t(x)$ is estimated by Eq. (1), solid irregular line. It can be compared with the Weibull approximation $exp(-(x/a)^m)$ represented as the dashed dotted line. One can see that the design value based on Weibull cdf would be too much conservative.

An alternative approximation based on Eq. (2) is presented next. The short term cdf of X can be bound by the expected number of up-crossings as follows:

$$F(x\,|\,W) \geq 1 - \frac{\mu^+(x\,|\,W)}{\mu^+(0\,|\,W)}, \tag{5}$$

where $\mu^+(x|W)$ is the up-crossing intensity of level x by the responses during the sea state W and $\mu^+(0|W)$ is the zero up-crossings intensity, i.e. the frequency of crests. For example, in Fig. 4, the left plot presents the short term distribution, $F(x|W)$, bounded by Eq. (5) together with the empirical distribution. It shows that the bounded distribution converges fast to the empirical distribution at high response levels.

When the ship responses are Gaussian, Rice's formula can be evaluated analytically leading to the following expression for $F(x|w)$:

$$F(x\,|\,W) \geq \frac{\mu^{+}(x\,|\,W)}{\mu^{+}(0\,|\,W)} = \exp\left(-\frac{x^2}{\sigma^2}\right)$$

Here, σ^2 is the variance of the reponses. The above equation shows that the Rayleigh distribution is actually a lower bound for short term *X*-cdf for Gaussian responses. In the engineering literature, the method of replacing *F(x|W)* in Eq.(5) by the Rayleigh distribution, termed as the *narrow band approximation*, is often used. It is less well known that the approximation is actually the bound, which will give conservative estimates of the design extreme response, see Rychlik and Leadbetter [12]. By combining Eqs. (2) and (5), one obtains an upper bound for $1\text{-}F_t(x)$. In Fig. 3 right plot, the bound is plotted as the dashed line. One can see the excellent agreement between $1\text{-}F_t(x)$ computed by means of Eq. (1) and the bound.

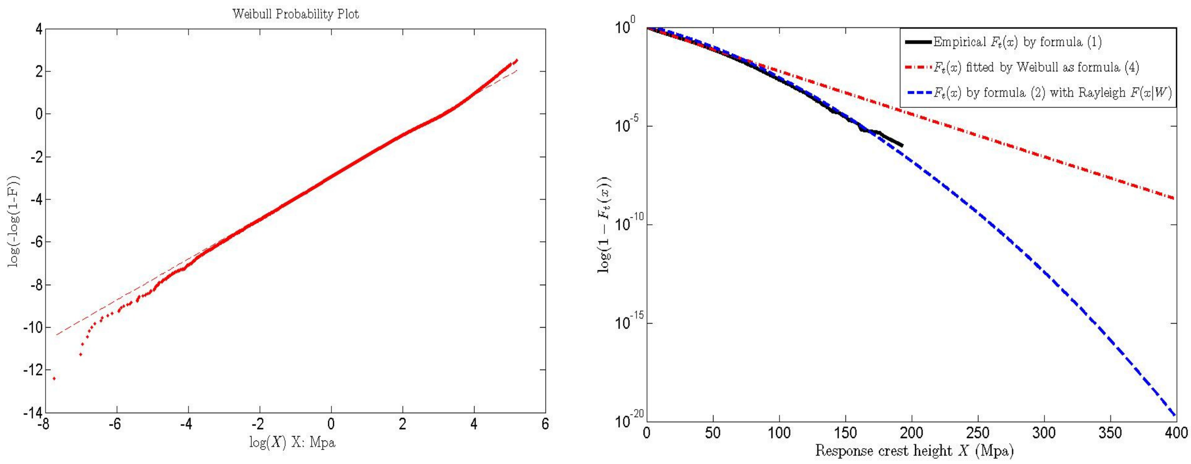


Fig. 3: **Left**: Weibull plot of crests of all simulated Gaussian responses; **Right**: Empirical distribution $F_t(x)$, Eq. (1), of the crest heights from a series of simulated Gaussian processes; $F_t(x)$ fitted by Weibull distribution Eq. (4), together with $F_t(x)$ computed by Eqs. (2) and (5), i.e. with short term Rayleigh cdf.

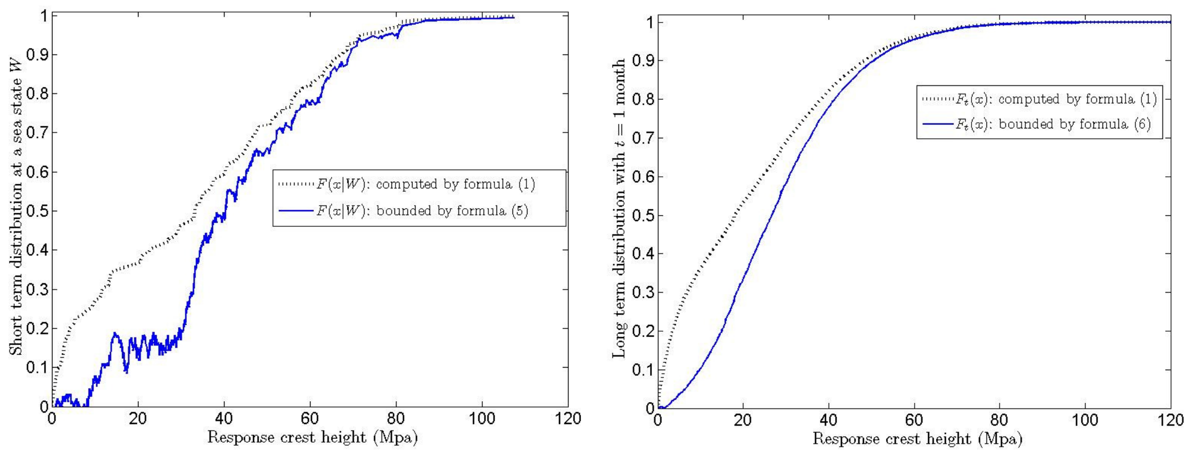


Fig. 4: **Left**: short term distribution *F(x|W),* bounded by Eq. (5)(solid line) and empirical distribution as Eq. (1)(dotted line) from full-scale measurements at a stationary sea state; **Right:** long term (*t = 1* month) distribution $F_t(x)$, bounded by Eq. (6) (solid line) and empirical distribution (dotted line).

Actually, the bound can be evaluated for non-Gaussian responses and can be proved by means of Longuet-Higgins series (Longuet-Higgins [13]), that for very high levels *x*, *1-F(x|W)* is very close to the bound, see the left plot of Fig. 4. However, the bound in Eqs. (2) and (5) can be further simplified if the mean stress can be assumed to be constant during the period *t*. (For simplicity, the mean can be assumed to be zero.) For any period *t*, let $N^+(x)$ denote the expected number of up-crossings of level *x* by the response in time *t*, and $N^+(0)$ denote the expected number of zero up-crossings. The long term cdf has the following relation,

$$F_t(x) \geq 1 - \frac{N^+(x)}{N^+(0)}. \tag{6}$$

Again the bound is close to $1\text{-}F_t(x)$ for high values of *x*. Numerically it is illustrated in the right plot of Fig. 4, where $1\text{-}F_t(x)$ *is* computed by means of Eq. (1) and the bound by Eq. (6). Here, $N^+(x)$ is estimated from the measured stress for $t = 1$ month.

2.3 *Method 2 for extreme prediction*

Method 2, also called Rice's method here, starts with somewhat different, although equivalent definition, of the extreme stress x_T. First, one defines the maximum stress *M* during a long period *t*. Then, the design extreme stress x_T is taken to be the *t/T* quantile in *M*-distribution, i.e. solution of the following equation

$$P(M > x_T) = \frac{t}{T}.$$

Next, the *M*-cdf is bounded by the expected number of up-crossings of the level *x* during the period *t*, $N^+(x)$. More precisely, for any response process *X(t)*

$$P(M > x) \leq N^+(x) + P(X(0) > x),$$

see Cramer and Leadbetter [12] for more detailed discussion. Now, since the probability of the stress at the first moment being larger than the design extreme stress x_T is negligible, Method 2 proposes to conservatively estimate x_T by a solution of the equation

$$N^+(x_T) = \frac{t}{T}. \tag{7}$$

The estimate, i.e. solution of Eq. (7) has exactly the same value as the one obtained using Eq. (2), with $F_t(x)$ bounded as in Eq. (6).

Method 2 can only be employed if the expected number of up-crossings of a high level *x*, $N^+(x)$, during the long term period *t*, can be evaluated. This can be achieved by a similar method as was used in Eq. (2). Suppose that the long term pdf of encountered sea states, *f(W)*, has been found and that the expected number of encountered sea states *K* during the period *t* is known, then the expected number of up-crossings is computed viz

$$N^+(x) = K \cdot \Delta t \int \mu^+(x \mid W) f(W) dW, \tag{8}$$

where, $\mu^+(x|W)$ is the up-crossing intensity of level *x* for a sea state *W*, and Δt is the short term period length of stationary sea states. It has been assumed in this study that $\Delta t = 30$ minutes.

3 The up-crossing intensity

Up-crossing intensity $\mu^+(x|W)$ for all encountered sea states during a long term period *t*, should be first computed to get the expected number of up-crossings for the long term responses. If the joint probability density function (pdf) of ship responses *X(t)* (zero mean stresses) and its derivative $\dot{X}(t)$ under a stationary sea state W is known, the up-crossing intensity of the level x under such a sea state can be computed by the Rice's formula (Rice [3, 4]) viz.

$$\mu^+(x \mid W) = \int_0^\infty z \cdot f_{X(t),\dot{X}(t)}(x, z \mid W) dz.$$

However, the joint pdf $f(\cdot)$ is often not known or hard to compute, *e.g.*, for quadratic responses see Butler et al. [7], Jensen and Pedersen [14], Naess [15] and Machado [16]. An alternative approach to compute $\mu^+(x|W)$ is presented in the following subsections for both Gaussian and non-Gaussian processes. The observed up-crossings are subsequently employed to check the accuracy of up-crossings computed using these approaches. Further, they are compared with the up-crossings computed by the typical Method 1 based on full-scale measurements.

3.1 Up-crossing intensity of Gaussian processes

If ship responses at a sea state W are Gaussian, the up-crossing intensity can be computed by Rice's formula for Gaussian loads, viz.

$$\mu^+(x\,|\,W) = \frac{1}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_0}}\exp\left(-\frac{x^2}{2\lambda_0}\right), \tag{9}$$

where λ_0, λ_2 are the zero-order and second-order spectral moments of responses $X(t)$. The first two terms on the right hand side of Eq. (9) are often referred to the zero up-crossing response frequency f_z as follows,

$$f_z = \mu^+(0\,|\,W) = \frac{1}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_0}}\,. \tag{10}$$

The obtained up-crossing intensities are then combined with encountered waves to compute the expected number of up-crossings during the long term period t (see Eq. (8)). However, ship responses are known to be non-Gaussian processes as the real environmental loads, e.g. ocean waves, show considerable non-Gaussian features, such as a skewed marginal distribution with heavy tails. Further, the non-linear interaction between ships and wave loads can no longer be neglected for extremely large sea states. Gaussian assumption of ship responses largely underestimates the extreme values based on the investigation of full-scale measurements. Hence, an alternative method is needed to model the expected up-crossings for the non-Gaussian responses.

3.2 Winterstein's transformed Gaussian processes

In our previous work Mao et al. [17], the so-called Laplace Moving Average (LMA) is shown to be able to model the up-crossing intensities of non-Gaussian ship responses obtained by both measurements and numerical analysis. The LMA models require knowledge of response power spectrum, skewness and kurtosis of the stresses. The spectrum of the response can be parameterized.

However, a limitation of the Laplace model, similar to the second order Stokes Waves, is that the pdf $f_{X(t),\dot{X}(t)}(x,z\,|\,W)$ is not available in an analytical form (the pdf is defined in the frequency domain by its characteristic function and has to be computed using numerical methods). In this paper, the transformed Gaussian model proposed in Winterstein et al. [10], is employed to model the non-Gaussian ship responses. Then the expected up-crossings are simply computed by Rice's formula given in Eq. (9).

The transformation is defined by the third order Hermite polynomial which is calibrated so that the variance, skewness and kurtosis of the transformed Gaussian model match the corresponding moments of the responses $X(t)$, viz. mean stress m, standard deviation σ_X, skewness $\alpha_3 = E\lfloor X(t)^3 \rfloor / \sigma_X^3$, and kurtosis $\alpha_3 = E\lfloor X(t)^4 \rfloor / \sigma_X^4$.

Consequently, if given the stochastically parameters of the responses $X(t)$ at a sea state W, denoted by Θ = (m, σ_X, $\sigma_{\dot{X}}$, α_3, α_4), the transformed Gaussian process is defined by

$$X(t) = G(u(t)) = m + \kappa\sigma_X \cdot \left[H_1(u(t)) + c_2 H_2(u(t)) + c_3 H_3(u(t))\right], \tag{11}$$

where, H_i are Hermite polynomials and $u(t)$ is a standard Gaussian process (in what follows, the mean stress $m = 0$). The other parameters in Eq. (11) are given by

$$\kappa = \frac{1}{\sqrt{1 + 2c_2^2 + 6c_3^2}};$$

$$c_2 = \frac{\alpha_3}{6} \cdot \frac{1 - 0.015\,|\,\alpha_3\,| + 0.3\alpha_3^2}{1 + 0.2(\alpha_4 - 3)};$$

$$c_3 = 0.1c_4\left[(1 + 1.25(\alpha_4 - 3))^{1/3} - 1\right];$$

$$c_4 = \left(\frac{1 - 1.43\alpha_3^2}{\alpha_4 - 3}\right)^{1 - 0.1\alpha_4^{0.8}}. \tag{12}$$

Let G^{-1} be the inverse function of G, and then $u(\tau) = G^{-1}(X(\tau))$. Hence,

$$\mu^+(x\,|\,W) = f_z(W)\exp\left(-\frac{G^{-1}(x)^2}{2}\right), \quad f_z(W) = \frac{1}{2\pi}\frac{\sigma_{\dot{G}^{-1}(X)}}{\sigma_{G^{-1}(X)}} \approx \frac{1}{2\pi}\frac{\sigma_{\dot{X}}}{\sigma_X}, \tag{13}$$

Note that $f_z(W)$ is defined for the Gaussian process $u(t)$. However it can be approximated by the zero upcrossing frequency of $X(t)$ since the two processes have the same ratio of the first two spectral moments. When the distribution is fitted by a Weibull distribution as Eq. (4), combing with Eq. (6), the expected number of up-crossings is approximated by viz.

$$N^+(x) \approx N^+(0)\exp\left(-\left(\frac{x}{\alpha}\right)^m\right), \tag{14}$$

where $N^+(0)$ is the number of response crests, n, during the long term period t.

The performance of the above discussed method is illustrated through the following illustration. The time history of the responses of full scale full-scale measurements at two places of a 2800TEU container ship is available. The measured places are located at the *1/4* ship length forward of after perpendicular (denoted as after section), and amidships (denoted by mid section), respectively. The detailed information about the measurements can be referred to Storhaug et al. [18].

Firstly, assuming the ship responses under each sea state to be Gaussian processes, the expected number of upcrossings is computed by Rice's formula in Eq. (9). The long term upcrossings are then computed by integrating the upcrossings of all sea states as Eq. (8). Secondly, instead of Gaussian assumption, the upcrossings of each sea states are estimated by the transformed Gaussian approach as Eqs. 11 to 13 (also refer to the Method 2 in this paper). In addition, when the time history of the responses is available, the fitted Weibull distribution is then used to approximate the long term upcrossings as Eq. (14). Fig. 5 presents the upcrossings computed by the above 3 approaches, together with the observed up-crossings. It can be seen that the Gaussian assumption of ship responses largely underestimates the expected number of up-crossings for high levels x. For the real non-Gaussian ship responses, both fitted Weibull approach (Method 1) and transformed Gaussian approach (Method 2) give almost identical results, also close to the observed upcrossings. Further, the expected numbers of up-crossings computed by these two methods also converge to each other when extrapolating to even higher levels. Therefore, both methods can be employed to estimate the extreme responses with good accuracy for the given set of data.

However, large amount of data of the responses is needed for Method 1 to fit correct parameters of the Weibull distribution. It also limits the application of this method since the ship response data from both measurement and correct numerical analysis is expensive and time consuming. Method 2 (with transformed Gaussian approach to model non-Gaussian response) may be applicable with less detailed information, since the parameters of the model can be obtained easily through their relation with the encountered waves, see our previous investigations Mao et al. [5]. More elaborate discussion on this issue is presented in the following subsection.

***Remark*:** For the used full-scale measurements, the Weibull distribution with a shape parameter approximately equal to unity ($m \approx 1$ in Eq. (14)) can be used to fit the long term cdf of ship response crests. Then its scale parameter α is estimated by the mean value of standard deviation of ship responses in each sea state. Based on our current investigations, we conclude that this simple method can approximate the long term cdf of the ship response quite well.

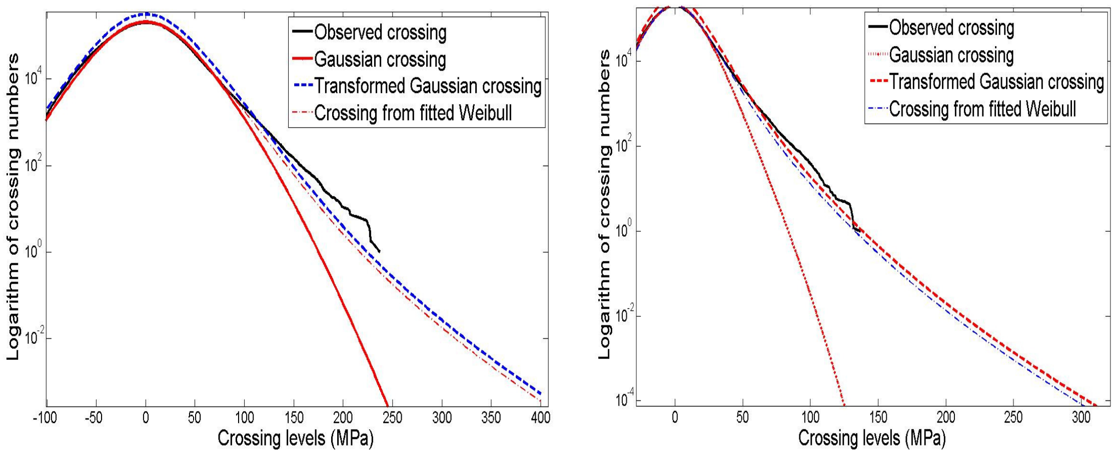


Fig.5: Up-crossings by transformed Gaussian approach Eqs. (11-13) (referred to Method 2), fitted Weibull distribution Eq. (14) (referred to Method 1), and Rice's formula assuming Gaussian responses as Eq. (9), together with the observed up-crossings for both after section and mid section of a container ship by full-scale measurements.

3.3 Estimation of parameters in transformed Gaussian model

The parameters in Θ needed for transformed Gaussian model are functions of the encountered sea state W. The parameters of the non-Gaussian ship responses were widely studied in Sikora [19] and Mansour and Wasson [20], and more recently in Jensen and Mansour [21]. The estimations in these literatures are efficient for applications at ship's conceptual design stage, since the parameters can be derived quite easily from the ship's main dimensions, encountered waves, and operational parameters.

In order to further consider the detailed ship properties, for example hull shapes and weight distributions, a simple linear numerical analysis is introduced for our estimations and it is illustrated to be accurate enough in comparison with the full scale measurements. As is known, wave environments are generally described by the significant wave height H_s and wave period T_z. Due to the difficulty in determining the strongly uncertain parameter T_z, its long term distribution conditional on H_s given in, for example DNV [22], can be used to simplify the estimations. Hence, in the following, the sea conditions are characterized by a single parameter - the significant wave height H_s. In the previous work by the present authors, Mao et al. [5], the standard deviation of response, $\sigma_X(W)$, is estimated from H_s by a simple linear relation, which is in terms of ship speed and heading angles. The zero up-crossing frequency, $f_z(W)$, is approximated by the encountered wave frequency.

Finally, only the relation between skewness (and kurtosis) and the encountered significant wave height should be further established. Skewness and kurtosis are measures of non-Gaussian property of the responses. It is well known that the effects of non-linear interactions between ship and waves are no longer negligible for large sea states. Therefore, we expect skewness and kurtosis to depend mostly on the encountered significant wave height, see also Jensen and Mansour [21].

The following investigation is based on the previously used full scale-measurements. The measurements contain both winter and spring voyages. Hence they can be used to represent the variability of longer term wave environments. Here only measured stresses under heavy seas are of interest for the extreme estimation. Hence only sea states with significant wave height H_s above 4 meters are considered.

The values of kurtosis α_4 for sea states with $H_s \geq 4\,m$, are presented in Fig. 6. It shows that there is no significant trend between kurtosis and H_s. Similar conclusion is also derived in Mansour and Wasson [20]. Therefore, we propose to model the kurtosis by its mean value, which is taken to be 3.5. For some pairs of parameters (α_3, α_4), the transformed Gaussian approach as Eqs. (11-13) is not defined in the tail area of the corresponding upcrossings. One can resolve this problem by using alternative values of kurtosis. This approach is motivated by an observation that the computed expected number of up-crossings $N^+(x)$ was not very sensitive for small variations of the kurtosis.

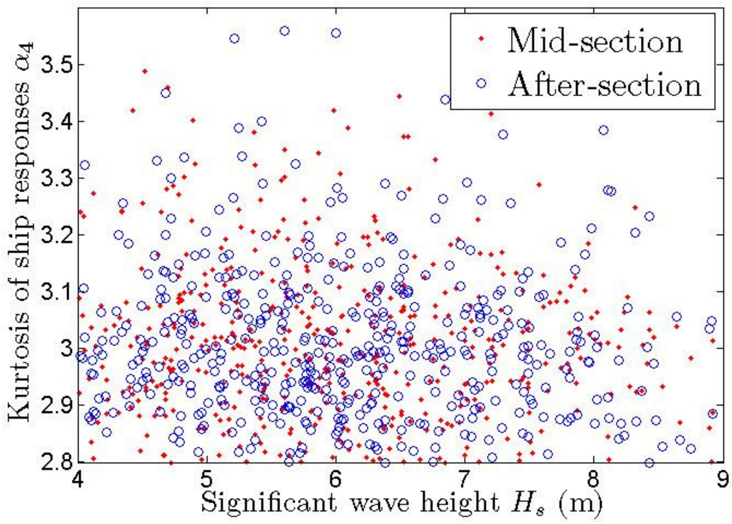


Fig. 6: Kurtosis of measured responses for both mid-section and after-section.

In order to compute the values of skewness of responses at all different sea states W_i, the relation between α_3 and H_s should also be established. When measurements of ship responses are available, the relation can be easily regressed by, for example, a least square method. Alternatively, a numerical analysis is usually used to get the responses when no measurements are available. As is known that the high frequency responses, such as whipping and springing, are very important for the extreme analysis, ship hull should be modeled as a flexible body. However, this makes the numerical analysis extremely time consuming and expensive. Based on the investigation of full-scale measurements, Fig. 7 tells us that only using wave induced responses are good enough to compute the skewness, since the skewness of wave induced responses is almost identical with that of the whole responses. Hence, ship hull is then modeled as a rigid body and even some commercial software is able to conveniently compute the wave induced responses.

Remark: Some authors derived that the high frequency response, for example, whipping, can increase the value of skewness of the wave induced responses, see Jensen and Mansour [21]. But here the results from our current full-scale measurements are not affected by the high frequency responses.

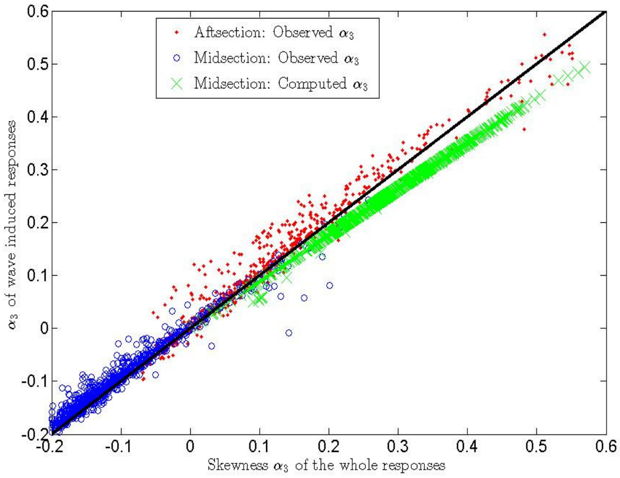


Fig.7: Skewness of wave induced responses and the whole response inducing high frequency vibration such as springing and whipping, computed from observations and by the formula proposed in Jensen and Mansour [21].

4 Example of Extreme prediction

In this section, an example will be used to illustrate the extreme estimations by the introduced Method 2. The non-Gaussian ship responses and corresponding upcrossings are computed by Winterstein's transformed Gaussian approach as Eqs. (11-13). The previously referred full-scale measurements are used to regress the relation between α_3 and H_s. The typical Method 1 is also used for estimation as a validation of Method 2. Further, as the significant wave height is an important parameter in modeling the expected number in Method 2, two different sources of encountered waves is used for comparison.

4.1 Estimation of parameters based on measurements

The standard deviation of ship responses σ_X, is determined through its relation with H_s , *i.e.*, $\sigma_X = C(\beta;\ U)H_s$. The values of $C(\beta;\ U)$ are computed by a linear strip theory. For the 2800TEU container ship, its value in terms of heading angle β under service ship speed $U = 10\ m/s$ is given in Mao et al. [23]. And zero up-crossing frequency of responses f_z, can be estimated by the encountered wave frequency. Again, kurtosis is assumed to be 3.5 in the example.

Skewness of responses under large sea states against the encountered significant wave height H_s is plotted in Fig. 8. It shows that the value of skewness will increase with the encountered H_s. The linear regression method gives the value of skewness as a function of H_s for both after section, α_{3_aft} , and midsection, α_{3_mid} , as follows

$$\alpha_{3\_aft} = 0.11H_s - 0.35\text{,}\ \ \alpha_{3\_mid} = 0.063H_s - 0.39\,. \tag{15}$$

In the current study, the skewness models in terms of other parameters, e.g., heading angle, are also tested by a linear regression method. But the more complex models do not explain the variability of the skewness any better than the simple regression model as Eq. (15). Note that for the midsection of the ship, the skewness of ship responses can be also computed by the closed formula in terms of significant wave height and operational profiles (ship speed and heading angle) in Jensen and Mansour [21]. But as is shown in the right plot of Fig. 8, there is a big gap of skewness computed by these two approaches. In particular for the full-scale measurements, there are a lot of sea states with negative skewness, and some of them are Gaussian even for very high sea states. However, in order to check if the model can be applicable for extreme prediction, the skewness regressed from the full-scale measurements will be used for the following investigations.

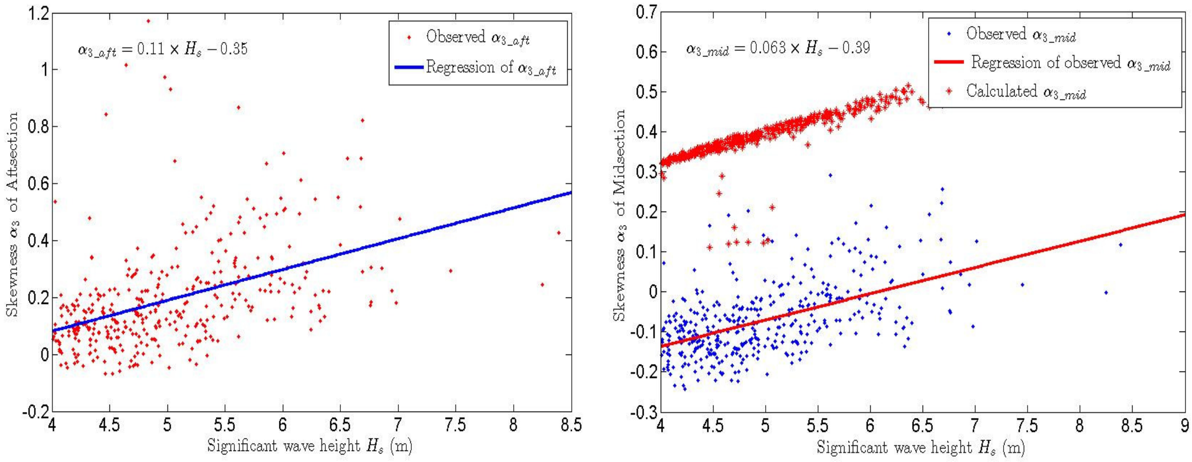


Fig. 8: Linear Regression of Skewness as a function of significant wave heights. Left plot: Results for After-section; Right plot: Results for Mid-section, also including the computed skewness by the closed expression in Jensen and Mansour [21].

4.2 Encountered waves

For the computation of upcrossings, besides the relation between the stochastic parameters Θ and wave environments W (mainly characterized by significant wave height H_s), one also needs to know the expected number of encountered sea states K and the long term distribution of H_s. The first quantity is related to the expected sailing time while the second depends on the shipping. Since the available stress measurements are from North Atlantic, this region will be considered in what follows.

The variability of sea environments, mainly H_s, has been extensively studied and many databases are available. In the following, the measurements of H_s from the onboard radar installed on the above 2800TEU container ship are used for the extreme estimation. Further, the distribution of H_s recommended by DNV [22] is also used as an input of Method 2, and compared with the onboard measurements.

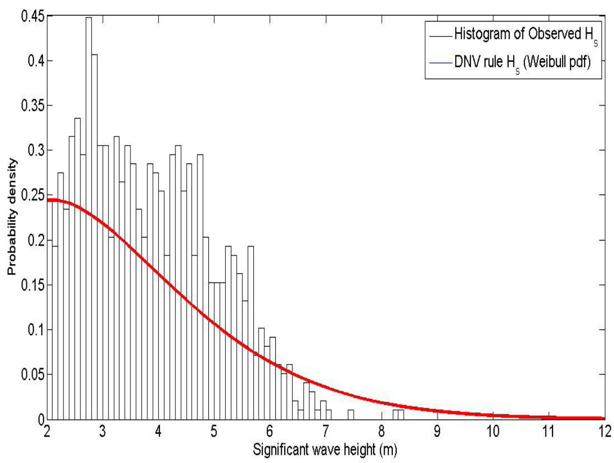


Fig. 9: Empirical distribution of H_s measured by onboard radar, and long term distribution of H_s from DNV Rule for the North Atlantic operation. Note that here the Weibull distribution for H_s is different from the one used to fit the long term cdf of response crest. The Weibull distribution here can be directly obtained from DNV [22] for different ocean zones.

The distribution of H_s measured onboard contains much more moderate seas than DNV [22] recommended, see Fig. 9. For fatigue estimation, the moderate seas are the most important conditions. Ship fatigue design based on DNV recommended will underestimate 50% of the fatigue damage in comparison with the observed wave environments. For extreme analysis, large H_s are of more interest to estimate the extreme values. Fig. 9 tells us that the probability of high H_s from DNV [22] is larger than that from the onboard observations. Hence, we expect that using DNV [22] recommended H_s will give larger values of extreme responses than that using observed H_s.

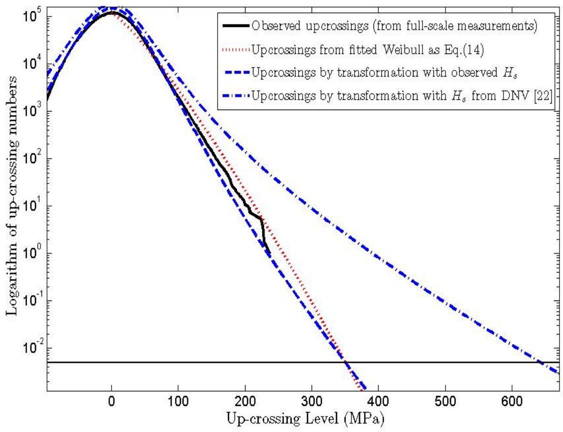
The large difference of H_s obtained from above 2 approaches could be a consequence of the routing plan system installed in the measured ship. However, the difference may be just caused by statistical errors, since the distribution of observed H_s is obtained from only half year's measurements while the DNV [22] recommended H_s is collected over many years.

4.3 Results of extreme values

In the following, the so-called 100-year response x_T ($T=100$ years) will be estimated by both Method 1 and 2 on the basis of the six month ($t = 0.5$) full-scale measurements. According to Eq. (7), the expected number of up-crossings by the level of 100-year response is equal to t/T=0.05. If the onboard observed H_s are used as the input of Method 2, it is denoted as Method 2(a). While if Method 2 uses DNV [22] recommended H_s, it is then denoted by Method 2(b).

Figure 10 presents the expected number of up-crossings and the estimation of x_{100} for both after section and mid section. The expected numbers of up-crossings computed by Method 1 (Eq. (14)) and Method 2(a) are very close to each other and converge fast to the observed up-crossings at high levels. For the after section, the value of x_{100} is about 210Mpa and 230Mpa estimated by Method 1 and Method 2(a), respectively. For the midsection, the two methods give almost identical results of x_{100}, 350Mpa.

The expected number of up-crossings computed by Method 2(b) significantly deviates from the other two methods, in particular for the midsection. Method 2(b) overestimates more than 40% of the 100-year than Method 1 and Method 2(a). It is due to that the distribution of H_s used in Method 2(b) is quite difference from the measurements, see Fig.9. Hence, for extreme prediction during ship's design stage, it is extremely important to describe the encountered waves accurately along its operation period.



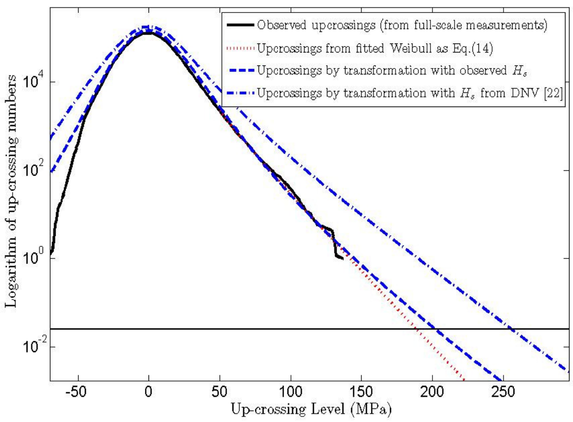


Fig. 10: Results of the estimation of 100-year responses based on the six months' full-scale measurements. The expected numbers of up-crossings are computed by Method 1 as Eq. (14) (dotted lines), and Method 2 with onboard measured H_s (dashed lines) and H_s recommended by DNV [22] (dash-dotted lines). Solid lines represent the observed up-crossings. Horizontal dash-dotted lines represent the expected number of up-crossings related to the 100 year response. **Left**: Results for After-section; **Right**: Results for Mid-section.

5 Conclusions

This paper presented a simple approach for the prediction of extreme response, e.g. 100-year stress x_{100}. In the method, Winterstein's transformed Gaussian approach is used to model the non-Gaussian ship responses. The expected numbers of upcrossings by the real ship responses are then computed by Rice's formula from the transformed Gaussian processes. The computed upcrossings are easily applied to estimate the values of extreme responses. The accuracy of this method is validated by the typical Weibull fitting method, on the basis of full-scale measurements of a 2800TEU container ship.

Parameters of the transformed Gaussian model, i.e., standard deviation and skewness of stationary ship responses, are derived as a function of encountered significant wave height. The relation between skewness and H_s can be directly computed using only wave induced responses. It can be achieved through a simple nonlinear numerical analysis assuming as a rigid ship body.

Finally, the proposed method is conveniently applicable for extreme estimation with limited information, mainly encountered significant wave height H_s. However, due to the strongly relation between the encountered H_s and estimates of extreme responses, a correct distribution of encountered H_s should always be initially determined for applications.

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