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KRZYSZTOF PODGÓRSKI
IGOR RYCHLIK

Department of Mathematical Sciences
Division of Mathematics
CHALMERS UNIVERSITY OF TECHNOLOGY
UNIVERSITY OF GOTHENBURG
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Krzysztof Podgórski and Igor Rychlik

Department of Mathematical Sciences
Division of Mathematics
Chalmers University of Technology and University of Gothenburg
SE-412 96 Gothenburg, Sweden
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A spatial-temporal model of significant wave height fields with application to reliability and damage assessment of a ship

KRZYSZTOF PODGÓRSKI* AND IGOR RYCHLIK**
Adresses:
* Statistics, Lund University, 220 07 Lund, Sweden
krzysztof.podgorski@stat.lu.se
** Mathematical Sciences, Chalmers University of Technology, SE-412 96 Göteborg, Sweden (Corresponding author)
rychlik@chalmers.se

Abstract

Significant wave heights are modeled by means of a spatial-temporal random Gaussian field. Its dependence structure can be localized by introduction of time and space dependent parameters in the spectrum. The model has the advantage of having a relatively small number of parameters. These parameters have natural physical interpretation and are statistically fitted to represent variability of observed significant wave heights records. The fitted spatial-temporal significant wave field allows for prediction of fatigue accumulation in ship details and of extreme responses encountered. The method is exemplified by analyzing a container ship data relevant for North Atlantic trade and the results show a high agreement with actual on-board measurements.

Keywords: Significant wave heights, hundred years wave, fatigue damage, narrow band approximation, rainflow cycles, level crossings.

1 Introduction

In this paper one gives means to describe variability of encountered sea states by a vessel. The methods are applicable to assessment of the expected damage accumulated in a vessel details, the variability of the damage and prediction of the extreme seas that could be encountered.

The sea load environment is often described by a wave spectrum depending on few parameters. In the simplest case of long-crested sea having Pierson-Moskowitz spectrum these parameters are the significant wave height and the average wave period. The sea surface is then modeled as a sum of non interacting cosine waves leading to a Gaussian random surface that moves in time. A sea state rests for about half an hour period during which the waves acting on ship or offshore structure result in variable stresses causing fatigue damage of structure components. Although the accumulated damage during a sea state is a random quantity, its variability can be assumed negligible in comparison to variation of the sea states and hence it is well approximated by its expected value. For ships the expected damage rate can be then accurately approximated if the significant wave height, vessels speed and heading angle are known. In the following the heading angle at a position on the route will be approximated by a non-random function which depends only on the average velocity of storms and the direction of the rout. For longer time periods the sea conditions change which results in new values of significant wave heights, heading angles and other parameters. Thus the damage accumulation rate is also changing and becomes a function of encountered significant wave heights. This highlights importance of accurate description of the variability of significant wave heights and heading angles along a route for prediction of the severity of encountered loads for a ship detail.

The methodological problems of this sort have been previously worked out to evaluate the expected damage of an offshore structure component, see [19] and [13]. However the distribution of sea state parameters in that context are easier to find since for an offshore structure measurements of sea state are often available and thus standard statistical procedures of fitting parameters to the data can be used. A ship, on the other hand, can take different routes, or alter these dependently on the sea conditions, thus finding distributions of encountered significant wave heights becomes much more challenging. For example some difficulties arise from comparable speed with which ships and sea storms moves across the sea. In this paper we propose a way to address theses challenges and present an approach to computing the expected damage accumulated during a voyage given that the distribution of encountered significant wave heights by a vessel along the route is known. A simple model for significant wave heights variability is proposed. The discussed model features a relatively small number of parameters that have very straightforward physical interpretation. For the applications that are tackled in this work it is sufficient to account for the mean and standard deviation of the significant wave height field logarithms, the average duration of a storm related to the time correlation length (here a storm is defined as an excursion of the significant wave height above the median), the velocity with which storm is moving, and the heading angle of the ship in relation to the propagation of the storm.

Estimation of extreme wave height across the oceans is important for marine safety and design, but it is hampered by a lack of data. Buoy and platform data are geographically limited, and though satellite observations offer global coverage, they suffer from temporal sparsity and intermittency, making application of standard methods of extreme value estimation problematical. A possible strategy in the face of this difficulty is to use extra model assumptions to compensate for the lack of data. In this paper we shall use a spatio-temporal model proposed in [4]. The model is estimated using several types of data, buoys, satellites, hind-cast. Sometimes the significant wave height is measured on board of a ship. However the measurements are often of poor quality and have to be re-calibrated. Numerous missing values constitute an additional problem especially that they are over represented during the storms. All these deficiencies of the data were featured in the set of measurements taken on board of 2800 TEU container ship during several months of a full scale measurement campaign. These data stand behind all our examples.

The risk of meeting extremely high significant wave heights is measured by means of the so-called 100 years significant wave height defined as a level that is exceeded during one year with the probability of $1/100$. Means to estimate the 100 years H_s that can be encountered by a vessel will be presented in Section 4. The computation will require estimates of average sizes of encountered storms as well as significant wave height distributions. These distributions in practice can be distorted since captains usually try to avoid the most severe storms which in turn affects the encountered wave heights. However in our computations captains decisions are neglected and hence the 100 years significant wave height will be overestimated.

In order to evaluate the uncertainty in fatigue damage prediction one can simulate the stochastic sequences of sea condition that could happen along the rout. For the simulated sequence of significant wave heights the accumulated damage can be computed and an estimate of the damage distribution derived. This will be discussed in Section 5.

2 Significant wave height field H_s

In the literature typically the H_s distribution is understood as the long-term distribution of the significant wave height at some location or region. The distribution can be interpreted as variability of H_s at a randomly taken time during a year. Limiting time span to, for example, January month affects the H_s distribution simply because, as it is the case for many geophysical quantities, the variability of H_s depends on season. To avoid ambiguity when discussing the distribution of H_s, time span and region over which the observations of H_s are gathered need to be clearly specify. By shrinking the time span to a single moment t and geographical region to a location $\mathbf{p}$ one obtains (in the limit) the distribution of $H_s(\mathbf{p}, t)$ and this will be the meaning of distribution of H_s as used in this paper.

In order to identify the distributions at all positions $\mathbf{p}$ and times t vast amount of data are needed. Observations of H_s are available at some locations where buoys or platforms are placed. Since the resulting data are usually in the form of regular time series of several observations per day, standard methods of estimation of H_s distribution and intensities of rare extreme events (storms) may be used. Such data are, however, limited both in number and geographical extent. Alternatives offering global spatial coverage are observations of H_s from satellites. Those however suffer from restricted temporal coverage as the satellite returns to the same location only at time intervals of the order of days, and the returns are not equally spaced (see [1]). Thus the satelite observed values of H_s at the location are made at times that are sparse and irregular, posing difficulties for application of standard statistical methods – extra information or stronger model assumptions are needed to compensate for the data limitations. Eventually the two types of data: high time resolution with a limited space coverage (buoys and platforms) and space broadly spread but scarcely sampled in time (satelite data) have to be combined together to reconstruct the significant wave height fields. One can also utilize reconstruction of H_s from numerical ocean-atmosphere models based on large-scale meteorological data called also hindcast. While a hindcast does not represent actual measurements of quantities but extrapolations to the grid locations based on simulations from complex dynamical models, it is defined on regular grids in time and space and hence convenient to use.

In this paper the field $W(t, \mathbf{p})$ will be identified with logarithm of significant wave field $\ln H_s(t, \mathbf{p})$, where $H_s(\mathbf{p}, t)$ is defined as four time standard deviation of sea surface observed in vicinity of a ship, buoy or offshore structure located at a position $\mathbf{p}$. One can think that at time t an image of a sea surface at this location was taken and a standard deviation was estimated from that image.

In several investigations, see [3],[5], [8], it was confirmed that at many locations at the world oceans the distribution of $W(t, \mathbf{p})$ can be defined by seasonally variable mean $m(t, \mathbf{p})$ and variance $\sigma^2(t, \mathbf{p})$. In Figure 3 examples of maps of the estimated parameters m and σ^2 are presented. As expected waves in the Northern Atlantic are higher in winter season, while, surprisingly, the variance seems to be constant over time, i.e. $\sigma^2(t, \mathbf{p}) = \sigma^2(\mathbf{p})$, which was noticed in several works, see [1].

By assuming that $\ln H(t, \mathbf{p}) = W(t, \mathbf{p})$ is Gaussian, the model is completely specified if additionally to $m(t, \mathbf{p})$ and $\sigma^2(\mathbf{p})$, the autocorrelation function $\rho(t, t', \mathbf{p}, \mathbf{p}')$ is given. For this purpose the variability of significant wave height along satellite lines has been studied by several authors, see e.g. [12] and [1] which contains further references. In [12] one has observed that H_s is built up of variability in two scales, a long one on the scale of about two to four degrees and a short one on the scale approximately between half and one degree. Typically a noise is added to the model to account for measurement uncertainty (observational noise) leading to the following general model

$$\ln H_s(t, \mathbf{p}) = m(t, \mathbf{p}) + W_s(t, \mathbf{p}) + W_f(t, \mathbf{p}) + \epsilon(t, \mathbf{p}), \tag{1}$$

where W_s, W_f, and ϵ are independent Gaussian fields corresponding to the smooth component, the fast component and the measurement noise, respectively. Additionally, the model frequently assumes that the covariance is locally isotropic, i.e. variability of H_s along any straight line in space is the same in some small neighborhood of the location.

The spectra locally estimated from the records resembled the Gauss curve. Thus it is reasonable to consider the correlation model that was proposed before in [8] and [4], that is valid for a fixed t and locally in some small area, e.g. four by four degree

$$\rho(\mathbf{p}, \mathbf{p}') = qpe^{-\frac{|\mathbf{p}-\mathbf{p}'|^2}{2L^2}} + q(1-p)e^{-\frac{|\mathbf{p}-\mathbf{p}'|^2}{2L_f^2}} + (1-q)e^{-\frac{|\mathbf{p}-\mathbf{p}'|^2}{2L_e^2}}, \tag{2}$$

where p, q are between zero and one, L is the memory length of the "smooth component" (about two to four degrees) L_f is the memory length of fast component (approx. between half and one degree) while L_e is memory of colored noise. Most often p is close to one and hence for computations of 100 years H_s one can simplify the model and let $p = q = 1$.

The covariance in (2), is not accounting for any temporal variability and the simplest way to introduce it is through the autoregressive time dependence that leads to the Ornstein-Uhlenbeck type temporal field

$$\tilde{\rho}(\mathbf{p}, \mathbf{p}'; t, t') = \rho(\mathbf{p} - \mathbf{p}' - \mathbf{v}(t - t')) \cdot e^{-\lambda|t-t'|}, \tag{3}$$

where $\mathbf{v}$ is locally constant velocity and λ is accounting for the 'memory' of the significant wave height field. This type of spatial-temporal covariance has been originally used in [14] and [11] for modeling rainfall.

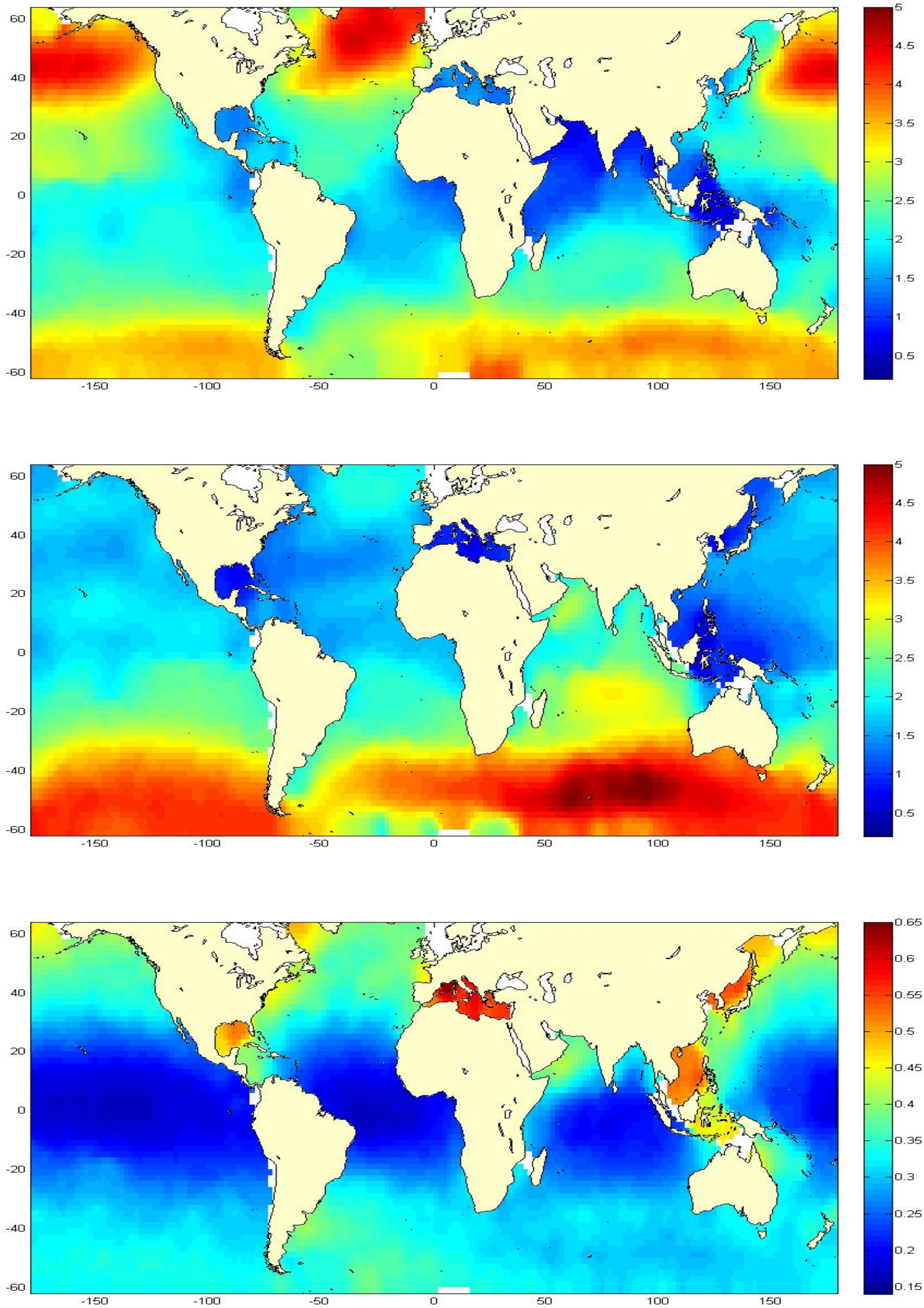

Figure 1: Estimates of median significant wave height in February (top plot) and August (middle plot). In the bottom plot, estimates of variance of $\ln H_s$, as noted the variance is independent of a season.

All the parameters in the above model, including the velocities, are local both in space and time thus, in general, they can be made space and time dependent. For example in Figure 3 *(top)* and *(middle)*, the global map of estimates velocities is presented for two months: February and August. The velocity $\mathbf{v}(t, \mathbf{p})$ is not directly observed in the record and we will later discuss how it can be estimated from the observed motion of the surface.

Remark 1 *Using the autoregressive or, in other words, Ornstein-Uhlenbeck type dependence in (3) exhibited in the factor* $e^{-\lambda|t-t'|}$ *make the covariance non-differentiable along the* t*-axis and the process* $W(t, \mathbf{p})$ *is not smooth in the way it develops in time. This non-differentiability has to be address later when the velocities on a random surface are defined. Essentially, one can consider a smoothed field obtained by convoluting the field with some smoothing kernel, see also [23] for more discussion.*

3 Fatigue

Let $s(t)$, $0 \le t \le T$, be measured, or simulated, stress. In this note the fatigue damage accumulated in material caused by s is measured by means of the rainflow damage rate $D_\beta(s)$ with the damage exponent β, which is computed in the following two steps. First rainflow cycles h_i^{rfc} in $s(t)$ are found, then the rainflow damage is computed according to Palmgren-Miner rule [20], [18], viz.

$$D_\beta(s) = \sum_i a^{-1}(h_i^{rfc})^\beta,$$

where a and β are material dependent constants defining the S-N curve, see [29] for more details on S-N curvws and [26], [10] on rainflow amplitudes. In practice due to uncertainties, stress s is random and so is $D_\beta(s)$. For this reason, one is interested in the average value represented by the expected damage rate $\mathcal{D}_\beta = \mathsf{E}\left[D_\beta(s)\right]$ and in the variability of $D_\beta(s)$ as represented, for example, by its variance.

At the design stage often $\mathcal{D}_\beta$ and the so called S-N method are used to estimate the fatigue life time. The method predicts failure before T, where T is the total exposure time to stress s, if $\mathcal{D}_\beta \ge 1$. Often a safety factor is included and the criterion altered to $\mathcal{D}_\beta \ge 0.5$ or anoter constant smaller than one. Consequently, evaluating $\mathcal{D}_\beta$ during a design life T becomes a central issue in the reliability analysis. In this section we present means to evaluate $\mathcal{D}_\beta$ for a ship. The method is based on representation of the encountered loads as a sequence of stationary sea states during which the expected damage increases at a rate depending on shipping conditions and sea states parameters. The speed of damage increase $d(t)$ per time unit at time t is called the damage rate.

While the presented approach is in principle applicable to any type of ship, we illustrate it when applied to reliability analysis for a container ship traveling regularly between Europe and North America. In [16], [17] approximation of the fatigue damage rate $d(t) = d$ in a container ship was given, viz.

$$d = \frac{0.47\, C^\beta}{a}\left(\frac{H_s^{\beta-1/2}}{3.75} - \frac{2\pi}{g}\frac{H_s^{\beta-1}}{3.75^2}\|\mathbf{v}_{sh}\|\cos\alpha\right) = d(H_s, \mathbf{v}_{sh}, \alpha), \tag{4}$$

where H_s is significant wave height encountered by the ship, $\mathbf{v}_{sh}$ is her velocity, α is the heading angle as given in Figure 2 or by using the velocity of $\mathbf{v}$ of H_s field through

$$\cos\alpha = \frac{\langle \mathbf{v}_{sh}, \mathbf{v}\rangle}{||\mathbf{v_{sh}}|| \cdot ||\mathbf{v}||}.$$

Here $\langle \cdot, \cdot \rangle$ is the inner product between vectors. For compactness of the formulation, the dependence on time t and a position of the ship $\mathbf{p}(t)$ (a route of the ship) is not explicitly shown, i.e. $d = d(t)$, $\mathbf{v}_{sh} = \mathbf{v}_{sh}(t)$, $\mathbf{v} = \mathbf{v}(t, \mathbf{p}(t))$, $H_s = H_s(t, \mathbf{p}(t))$. In the formula a is fatigue strength of material taken from S-N curve. For the particular example considered in this work $a = 10^{12.76}$ while $\beta = 3$. Further $g = 9.81$ $[m/s^2]$ is the gravity acceleration. The constant C depends on the type of a ship and the geometry of ship detail; ship speed, heading angle. It can be evaluated using linear strip code, beam theory and stress concentration factor, here $C = 20$. It is also assumed that actual encountered waves travel in the same direction as that the field H_s moves in.

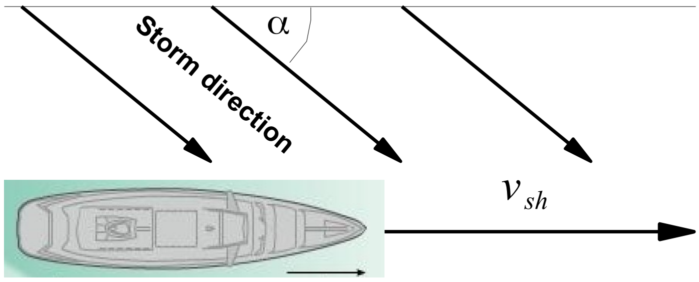

Figure 2: Heading angle α of a ship.

The formula (4) has been derived using the narrow band approximation for the damage rate proposed first in [9] under the assumption that stresses are Gaussian. Note that under this assumption the narrow band method overestimate the damage rate, see [27] for the proof. It is well known that container vessels responses are non-Gaussian in harsh seas (high H_s values) due to high frequency vibrations involving transients caused by impulse forces, e.g. slams. The high frequency responses have small energy often below 5% of the total energy (variance) but causes increase of damage rate by about 20%-30%. Hence one could question the use Gaussian processes to model stresses. However in the cases studied the narrow band approximation, based on the Gaussianity assumption, overestimates the damage rate by about 30% which is close to the contribution to fatigue due to high frequency vibrations. Consequently the simple formula (4) predicts fairly well the damage accumulation in ship details.

The damage rate d is the expected damage increase per time unit for a known sea state parameter H_s and $\|\mathbf{v}_{sh}\| \cos\alpha$. However both quantities vary in an unpredictable way along ship's routes. This variability is often model by probability distributions also called long-term distributions. The distributions are used at the design stage of a ship detail to estimate the expected damage for possibly encountered H_s, $\mathbf{v}_{sh}, \alpha$ for a planed shipping. In this sense the expected damage rate d is a random variable itself and the expected damage during the design life $\mathsf{E}[\mathcal{D}_{life}]$ is given by

$$\mathsf{E}[\mathcal{D}_{life}] = \mathsf{E}[d(H_s, \mathbf{v}_{sh}, \alpha)] \cdot \mathsf{E}[N]\,\Delta t \tag{5}$$

where $\mathsf{E}[\cdot]$ now stands for averaging over the long-term distributions of the sea states, $\mathsf{E}[N]$ is the expected number of encountered sea states all lasting for a period Δt, e.g. half of an hour. Obviously the long-term distributions of $H_s, \mathbf{v}_{sh}, \mathbf{v}$ and N, are not easy to determine as it requires a specification of shipping and knowledge of $H_s(t, \mathbf{p})$ distributions over oceans during seasons. This work aims at presenting an organized way of approaching to this challenging problem.

The method is constructive and requires a detailed shipping plan which consists of voyages between harbors. A single voyage is then represented by a route i.e. a sequence of times t_i (in hours) of the (approximately) constant sea states and corresponding positions in space $\mathbf{p}_i = \mathbf{p}(t_i)$ (in degrees). In our examples we considered the constant time sea state as lasting half an hour, so that the constant increment of t_i's is $\Delta t = 0.5$. Since the sea climate depends on season one requires also information on a season a voyage is undertaken, i.e. a month of a year. In Figure 3 *Bottom* a route that will be considered in examples is presented as a black line. The route started close to England and took 408 hours to sail (time in an American harbor included).

Now for a given route which sails in T hours the accumulated damage is

$$\mathcal{D} = \int_0^T d(t)\,dt \approx \sum d(t_i)\Delta t. \tag{6}$$

As mentioned before the expected damage

$$\mathsf{E}[\mathcal{D}] \approx \sum \mathsf{E}[d(t_i)]\,\Delta t,$$

is used to estimate risk for cracking of ships details. Here expectation $\mathsf{E}[d(t_i)]$ averages the variability of significant wave heights, heading angle, and the ship velocity at time t_i when ship is at position

$\mathbf{p}(t_i)$. The variability of significant wave height is described by its probability distribution at time t_i and the position p_i. In order to find the distribution for any route one needs atlases of distributions of $H_s(\mathbf{p})$ at any position and season (here represented by a month). Several such atlases exist and an example of such maps is presented in Figure 1 one estimated from H_s recorded by satellites.

The second problem is description of variability of the ship velocity $\mathbf{v}_{sh}$ and heading angle α. Here the ships velocity and heading along the route are approximated by their average values that are derived as follows. The speed is taken as the "design speed" and the direction is evaluated as the tangent to the rout. The heading is taken as the angle between the average ship velocity and the average velocity of H_s at position $\mathbf{p}$ and time t. The average velocity of H_s is discussed next.

3.1 Heading angle - Velocity fields

In order to evaluate $\|\mathbf{v}_{sh}\| \cos\alpha$ – the quantity that is needed in (4) – ships velocity $\mathbf{v}_{sh}$ and the velocity of wave field along entire route are needed. For any position on the route both velocities are uncertain and difficult to measure and could be modeled as random vectors. In the presented approach to evaluation of the damage rate, the velocities will be replaced by their expected value. For this, the average ships velocity is estimated using the rout. The positions are estimated for half hour periods and by assuming that the ship is moving along straight lines between the positions. Analysis of H_s dynamics is a much more challenging problem, even on the stage of definition of the velocity of a sea state movement.

In pioneering work [15] a concept of velocity was introduced studied for random, moving surfaces. This concept of velocity has been further investigated in [22] and called the velocity of apparent waves and further extended in [6] to describe dynamics of contour levels in a moving random field. Several possible definitions of velocities were studied. Here the following definition for the velocity of movement of a field $W(t, x, y)$ at position $\mathbf{p} = (x, y)$ is used

$$\mathbf{V}(\mathbf{p}, t) = \left(-\frac{W_t}{W_x}, -\frac{W_t}{W_y}\right).$$

The contour line of H_s field may describe the edges of a storm and the velocities defined on that contour line $W(t, \mathbf{p}) = u$ describe movements of a storm. The velocity $\mathbf{V}$ observed on the contour is random and for a stationary and homogeneous Gaussian fields has mean

$$\mathbf{v} = \left(-\frac{\mathbb{C}ov(W_t, W_x)}{\mathbb{V}ar(W_x)}, -\frac{\mathbb{C}ov(W_t, W_y)}{\mathbb{V}ar(W_y)}\right). \tag{7}$$

We use here the same notation for the velocity as that in (3) and it is not obvious that the mean velocity on the contours coincide with the velocity driving dynamics in the model but it was shown in [7] and [23] that these two velocities coincide in a wide range of models. Note that the mean velocity is independent of the level u but here, for the sake of interpretation, we choose the contour level u to be zero, i.e. median of H_s field.

Remark 2 *It is worth to notice that W_t for the model given in (3) with the autoregressive temporal dependence is not well defined. However for any smoothing of the data either by filtering or discretization, the resulting models will have random velocities on the contours that are well defined. It has been shown in [23] that the median value of these distributions is always located at $\mathbf{v}$, used in (3), independently of the choice of the smoothing. Therefore despite the problem with existence of W_t, the median value of the velocities for such fields is still well defined and can be obtained from a smoothed version of the field.*

Let us also remark on this occasion that the usage of the median in [23] was dictated by the fact that velocities observed at an arbitrary point have a Cauchy distribution that does not have the mean value. However when the velocities are observed only at the constant level contours the mean of this biased sampling distribution is well defined and coincide with the median due to the symmetry of distributions, see [6] for detailed discussion of this problem.

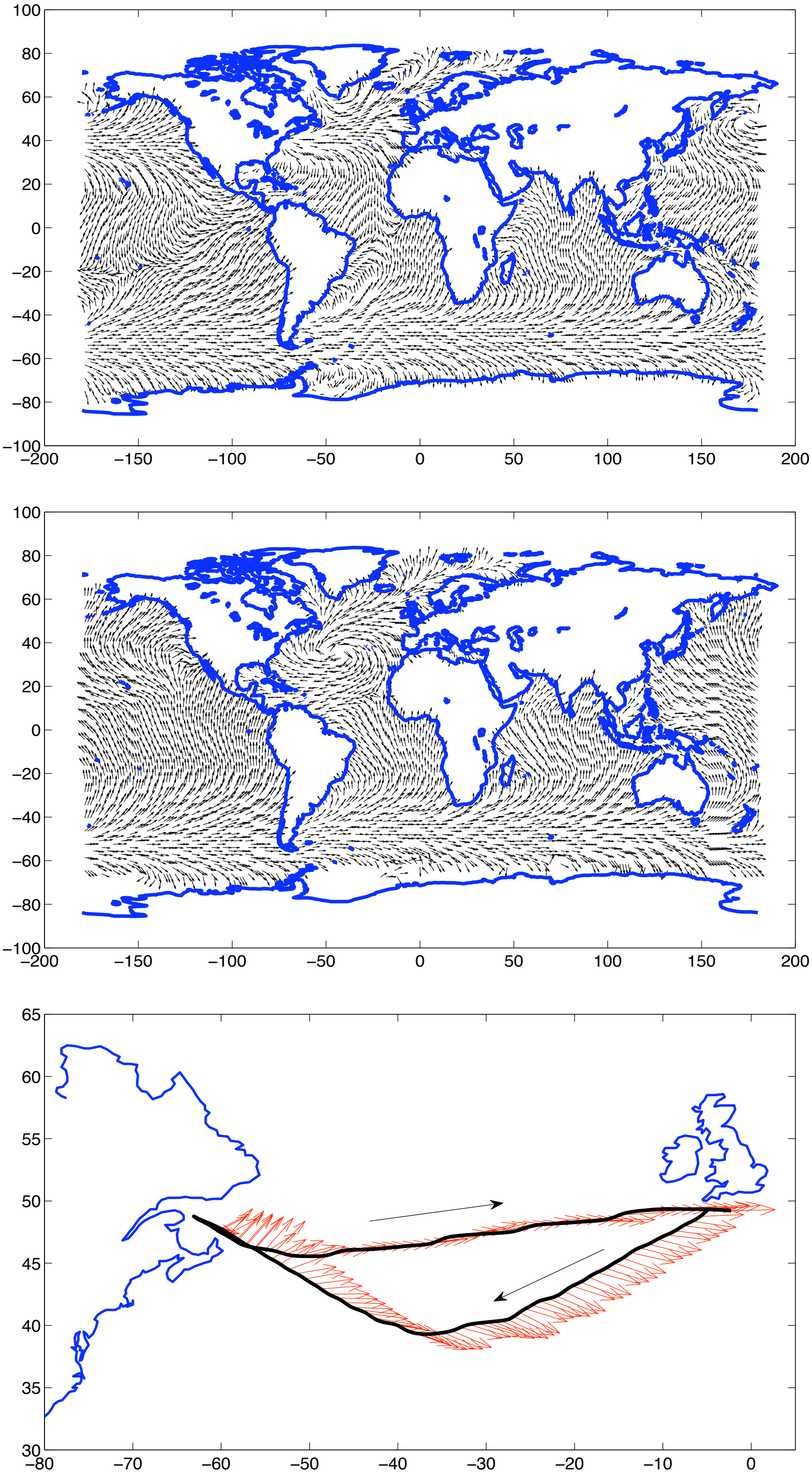

Figure 3: . Estimates of the median H_s-velocity $\mathbf{v}$ for the wave fields in February *(top)* and August *(middle)*. The lengths of vectors are not up to scale. *Bottom:* Estimates of the median velocity $\mathbf{v}$ along the route in January.

It is assumed that locally the logarithm of H_s is well approximated by a stationary and homogeneous Gaussian field and the average velocities can be computed using (7). In the larger scale of couple of degrees and weeks the average velocity may change. This time-space dependence will be made explicit in the notation by writing $\mathbf{v}(t, \mathbf{p})$ if needed. For the field that is (locally) stationary and homogeneous a power spectral density $S(\kappa_x, \kappa_y, \omega)$ can be defined and the mean velocity $\mathbf{v}$ can be conveniently computed using the spectral moments of S, viz.

$$\mathbf{v}(t, \mathbf{p}) = \left(-\frac{\lambda_{101}}{\lambda_{200}}, -\frac{\lambda_{011}}{\lambda_{020}} \right), \tag{8}$$

where

$$\lambda_{ijk} = \int \kappa_x^i \kappa_y^j \omega^k \, S(\kappa_x, \kappa_y, \omega) \, d\kappa_x \, d\kappa_y \, d\omega).$$

In Figure 3 examples of maps of the average velocities $\mathbf{v}(\mathbf{p}, t)$ are presented for month February and November. The velocities have been derived in [4] using (8) and spectral moments estimated from ERA 40 data.

3.2 Expected fatigue rates along a route – an example

Here we shall illustrate the presented methods by estimating the expected damage accumulation rate along the route presented in Figure 3 *(Bottom)* that took 17 days to sail in January month. The damage rate is given by (4) with ship velocity and H_s field velocity replaced by their expected values, as represented in (8). The average H_s fields velocities along the route are available and presented in Figure 3. What remains is computations of expectations of H_s^2 and $H_s^{2.5}$. For log normally distributed H_s defined by $m = \mathsf{E}\,[\ln H_s]$ and $\sigma^2 = \mathbb{V}ar\,(\ln H_s)$ the average value of H_s^r is given by

$$\mathsf{E}\,[H_s^r] = \exp(r\, m + r^2 \sigma^2 / 2).$$

Since the duration of the voyage is less than one month the dependence of distribution of H_s on time can be neglected and hence the damage rate can be computed when the parameters values $m(\mathbf{p})$ and $\sigma^2(\mathbf{p})$ are known for locations $\mathbf{p}$ along the rout. This can be done using the model proposed in [8] and fitted to all locations on oceans in [3]. Typical fits following this approach see Figure 1.

The expected damage rate can be thus evaluated and it is presented in Figure 4-*Right* (dashed line) as a function of time. Since during the voyage the encountered H_s were measured one can also compute the observed damage intensities. The quality of measurements was poor, needed recalibration and there are missing data particularly during the periods when H_s are high. For these reasons the 'observed' damage rate should be treated as an illustration how the damage accumulation could look like and not as precise measurement values. The so-obtained observed damage rates are presented in the figure as a solid (irregular) line.

Let us comment the results:

- Using the evaluated damage rates presented in Figure 4 the accumulated damage for the entire voyage can be computed. The average damage rate (dashed line) results in expectation that 0.99% of the total fatigue life has been "consumed" in this 17 days long voyage. This estimate applies to a midship location in 2800 TEU container ship, see [15], [16] for more details. During six winter months between 6 and 8 such voyages can be undertaken which indicates high risk for cracking in some ship details before the design life of 20 years.

- The observed damage rates, although uncertain due to large estimation errors and missing values, give accumulation of 0.65% of the life during the voyage (when corrected for missing values the damage increases to 0.72%). The difference between observed and the expected damage is caused not only by randomness of the sea conditions but also by captain decisions and routing programs. A wish to avoid predicted large storms resulted in the decision of taking longer route to the south to avoid the worse sea conditions but despite these precautions still three storms have been met en route to America.

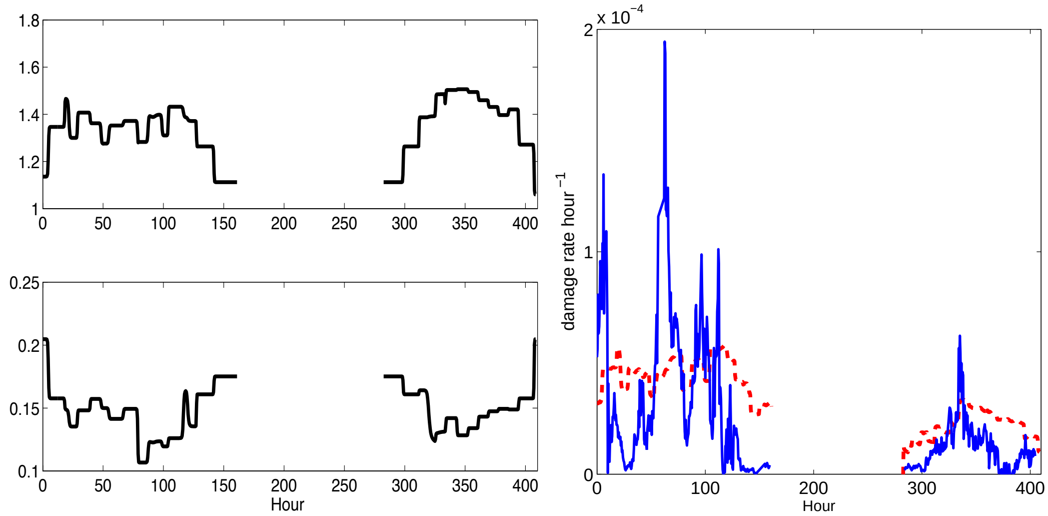

Figure 4: Analyses for the route in Figure 3 *(Bottom)* in dependence on sailing time. *Left:* Mean (top) and variance (bottom) of $\ln H_s(t)$. *Right:* The expected (mean) damage rate (dashed line) vs. the observed damage rate (solid line).

- Since on the route back to Europe one is sailing in similar direction as the motion of storms the damage rate is much smaller. The weather in the middle of this route is harsher than the conditions that can be met on the chosen route to America as seen from the parameters m and σ^2 in Figure 4 *(Left)*. Mathematically, it is the term $\|\mathbf{v}_{sh}\| \cos\alpha$ that makes the damage rate smaller, and as a result 80% damage were accumulated on the way to America and only 20% on the way back. The difference is a well known fact and hence sailing from America to Europe is often taken along the shortest route with a maximum speed.

4 Extreme responses - the 100 years significant wave height

In this paper we classify the sea state at time t as a storm if H_s exceeds the median $\exp(m(t))$. The highest value of H_s is attended at storm crests. Severity of a storm is measured by means of values of its return times, evaluated assuming a stationary climate. For example a 100 years storm is encountered when storm crest exceeds a threshold h_{100}, which is called a hundred years significant wave height. Here, the threshold h_{100} is chosen in such a way that the return period of the so defined 100 years storm is actually 100 years. Most of the structures are designed to withstand waves for a sea state with $H_s = h_{100}$. In this section we shall give means to estimate return values of significant wave height, e.g. 100 years value, which is a measure of harshness of a shipping plan (here identified with a collection of routes). This is a more difficult problem than estimation of the expected damage since it requires extrapolation to very high levels for which not much of relevant data are available – as mentioned before, spatio-temporal data having good coverage in space and sufficiently dense in time are limited. This lack of the data necessarily has to be remedied by more specified model assumptions.

In previous work [28], an approach to estimation of extreme wave heights using crossings methods was discussed based on a assumption that the logarithms of significant wave heights can be accurately represented by a Gaussian model. Here the method is adopted to estimation of the return values of extreme significant wave heights that can be encountered by a vessel.

In this section we consider the logarithm of H_s along a route $t \mapsto (t, \mathbf{p}(t))$, i.e. process $W(t) = W(t, \mathbf{p}(t)) = \ln H_s(t, \mathbf{p}(t))$, where t is measured in years. The 100 years return value h_{100}, say, is defined as a level which can be exceeded during one year with probability $1/100$. More precisely, let $M = \max_{0 \le t \le t_{year}} W(t)$, where t_{year} represents the period of one year expressed in the time unit of the temporal argument t. Then h_{100} is a solution to the following equation

$$P(M > \ln h_{100}) = \frac{1}{100}. \tag{9}$$

Other return periods can be also considered, e.g. 20 years significant wave height h_{20} solves $P(M > \ln h_{20}) = 1/20$.

Our approach is based on the theory of level-crossings of stochastic processes. We have the following obvious relationship between crossings of a level u and the distribution of the maximum over a period of time

$$P(M > u) = P(W(0) > u) + P(N(u) > 0, W(0) \leq u), \tag{10}$$

where $N(u)$ is the number of upcrossings of the level u by $W(t)$ during one year. For Gaussian model of W, as u increases, the probability $P(W(0) > u)$ become negligible, so that the restriction $W(0) \leq u$ in the second probability in (10) becomes less relevant. Thus for large u

$$P(M > u) \approx P(N(u) > 0).$$

Furthermore, we have obvious inequalities

$$\mathsf{E}\left[N(u)\right] - 2\mathsf{E}\left[N(u)(N(u)-1)\right] \leq P(N(u) > 0) \leq \mathsf{E}\left[N(u)\right].$$

When W is satisfying some smoothness assumptions, the term $\mathsf{E}\left[N(N-1)\right]$ tends faster to zero than $\mathsf{E}\left[N\right]$ (see, for example, Chapter 4 in [2]), and so

$$P(M > u) \approx \mathsf{E}\left[N(u)\right] \tag{11}$$

The expectation in (11) can be evaluated by means of Rice's formula – an approach referred to as Rice's method.

We assume that at time t the process $W(t)$ is approximately stationary for about half an hour. In other words we assume that logarithm of H_s measured from a ship is Gaussian has variance $\sigma^2(t)$, mean $m(t) = m(\mathbf{p}(t), t)$ and variance of derivative $\lambda_2(t)$. Our assumptions means in practice that the mean of derivative process $W'(t)$ is negligible ($m'(t) \approx 0$) and that $W(t)$ is independent of $W'(t)$ for any fixed t. Now, if the sea state sequence were known for a long period of time, then $N(u)$ could be estimated and extrapolated to very high levels. The extrapolation could be used to estimate $\mathsf{E}\left[N(\ln h_{100})\right]$ and hence h_{100}. When we do not have measured H_s, for example at the design stage, then we can use the model of Section 2 and the probability $P(M > u)$ could be estimated using (11) and Rice's formula [24], [25], so that h_{100} can be approximated by solving

$$\mathsf{E}\left[N(\ln h_{100})\right] = \frac{1}{2\pi} \int_0^{t_{year}} \sqrt{\frac{\lambda_2(t)}{\sigma^2(t)}} \exp\left(-\frac{(\ln h_{100} - m(t))^2}{2\sigma^2(t)}\right) dt = 1/100. \tag{12}$$

The parameter

$$\tau = \tau(t) = \pi \sqrt{\frac{\sigma^2(t)}{\lambda_2(t)}} \tag{13}$$

needs to be evaluated but first we give its physical interpretation (see for example [21] for a similar discussion).

Let us consider a small time interval like, say, half an hour chosen so that the dependence of the parameters on time can be neglected. By employing Rice's formula one can see that the average number of upcrossings of the level $m(t)$ per time unit is equal to $\mu = \sqrt{\lambda_2(t)/\sigma^2(t)}/(2\pi)$. If we define a storm as an excursion of H_s above its median, then the number of storms per time unit is equal to the number of upcrossings, and thus μ represents intensity of the storms. By our definition of a storm, the average time between storms is the same as the average duration of a storm, so that $0.5\mu^{-1}$ interprets as the average duration of a storm, i.e. τ is the average duration of a storm encountered by a vessel. Note that τ would be the average if $W(t)$ were stationary in time, i.e. H_s homogeneous and stationary and ship were sailing along straight line. In this application $\tau = \tau(t)$ is a local parameter which changes along the route so it depends on a position on the route and a season.

4.1 Estimation of $\tau(t)$

Denote by $r(s) = \sigma^2\rho(s)$ the covariance between $W(t)$ and $W(t+s)$, while σ^2 and $\rho(s)$ are variance and correlation function, respectively. By assumed local stationarity of the encountered H_s process one has that

$$\tau = \pi\sqrt{\sigma^2/\lambda_2} = \pi\sqrt{\sigma^2/-r''(0)} = \frac{\pi}{\sqrt{-\rho''(0)}}. \tag{14}$$

The problem of estimation of τ given in (13) reduces to estimation of $\rho(s)$. The most straightforward approach would be to statistically estimate the autocorrelation for observed H_s on board of a ship. However such measurements are: seldom available, often unreliable, have incomplete coverage, and could be biased because of the way data are collected. Hence other means of computing the parameter are of interest.

Here we shall presented such a method and consider two special cases for which extensive data sets are available: firstly, H_s observed from a satellite, corresponding to encountered significant waves by a vessel moving with speed 5.8 km/s, i.e. practically with "infinite" speed as compared with the H_s dynamics scale; secondly, H_s measured by a buoy or on an offshore structure, i.e. encountered H_s by vessel moving with a zero speed – this case was discussed in [28] where the total variation of logarithms of the significant wave height were used to estimate $\sqrt{\lambda_2(t)}$. Then, under some additional model assumptions, the in-between case of a moving vessel is treated for which τ (equivalently λ_2 or $\rho''(0)$) is obtained by combining the methods for the two extreme cases above.

4.1.1 H_s encountered by a satellite

The TOPEX-POSEIDON satellite is moving with the ground speed of 5.8 km/s. Because of its high speed temporal variability of H_s can be neglected and it is natural to consider the spatial autocorrelation $\tilde{\rho}$, where $\rho(t) = \tilde{\rho}(tv)$ and $d = tv$ is distance in degree and $v = 0.0523$ degree per second.

The log-normal model is used as defined before in (1). The field W is normal and when considered for fixed t, while $\mathbf{p}$ are positions in some small area, e.g. four by four degree, it has the covariance as defined in (2). Most often p and q are close to one and hence for computations of 100 years H_s one can simplify the model by setting $p = 1$ and $q = 1$. We have $\rho''(0) = v^2\tilde{\rho}''(0)$ and since $\tilde{\rho}''(0) = -(L)^{-2}$

$$\tau = \frac{\pi L}{v}, \tag{15}$$

giving τ about four minutes.

We note the interpretation of spatial covariance parameter L. Namely, from (15) we obtain that πL is the average length (along a line) of a storm in degrees. This interpretation would be exact if $W(\mathbf{p})$ were homogeneous. In general, L is a local parameter which changes in space and with season as can be seen in Figure 5 were maps of πL are shown for February and August.

4.1.2 Estimation of $\tau(t)$ at a fixed position.

It is tempting but wrong to use the formula (15) to estimate τ for moving vessels. Since ship moves about 500 times slower than the satellite does thus τ for a ship would be around 33 hours which is too short when sailing in storm direction, see Figure 7, and in the case of buoy τ would be infinity which is obviously wrong. The reason for it is that the previous derivation work by assuming that satellite measures "frozen" H_s field. The assumption was reasonable since satellite moves so fast relative to H_s dynamics that no movement of H_s field is observed. From satellite measurements one can not observe that waves travels (as swell) over oceans and in general H_s fields moves. However as illustrated in Figure 3, where maps of average H_s velocities were shown, the storms are moving and this movement has to be accounted for in the cases when the vessel velocity is comparable to the movements of wave systems and in particular if one considers a fixed location.

Simplistically, limiting the window to four by four degree in space and 10 hours in time one may assume that approximately in that window the velocity of significant wave heights $\mathbf{v}$ is non-random and constant. This leads to a "frozen field" moving with a constant velocity in time. Further assuming that movements of fast components and noise is zero ($p = 1$, $q = 1$), then the observed correlation in time at fixed position $\mathbf{p}_0$ would be

$$\rho(t; \mathbf{p}_0) = e^{-\frac{|\mathbf{v}t|^2}{2L^2}}. \tag{16}$$

The true local dynamics of H_s field is not likely to be accurately modeled by such a movement of a "frozen" surface. Hence a more careful analysis of buoy data need to be performed by adding truly temporal stochastic dependence. This is often done by a simple autocorrelation function $\rho(t) = \exp(-\lambda|t|)$ to model dynamics of logarithms of H_s. This is a very convenient model since sampling H_s regularly in time will form an AR(1) process. For example, the AR(1) model was investigated in [1] to model logarithms of H_s measured by Buoy 46005. It was concluded that "residuals are not fully described by an AR(1) process". Despite this, the authors still recommended the AR(1) structure to be included in the model and the following shows how a combination of AR(1) and (16) can be introduced.

Having in mind the formula (16) and using AR(1) modeling presented in [4], the correlation of $W(t)$ takes form

$$\rho(t) = qe^{-\frac{t^2}{2T^2}}e^{-\lambda|t|} + (1-q)\,1_0(t), \quad 0 \le t \le 10, \tag{17}$$

where t has units hours and $1_0(t) = 1$ if $t = 0$ and zero otherwise. The $1-q$-term is the iid noise term, typically introduced to account for the so-called nugget effect observed in estimated correlation, and the factor $e^{-\lambda|t|}$ is the correlation of Ornstein-Uhlenbeck process which when sampled on a regular grid becomes AR(1). In [4], the model was successfully fitted to twenty buoys from NDBC-NOAA and the tables with parameters were given, proving that this extension of AR(1) model can be useful for the waves models.

It should be noted that the Ornstein-Uhlenbeck term $e^{-\lambda|t|}$ causes that the $\lambda_2 = -r''(0)$ is not well defined even if the noise would be neglected, i.e. $q = 1$. Consequently smoothing of the signal or other approximations are needed to evaluate the parameter τ needed in estimation of 100 years encountered significant wave heights by means of Rice's method.

It should be emphasized, that a direct approach to evaluation of τ by using the records to estimate $\rho''(0)$ is problematic not only because of the model assumptions as in the argument above. There is a general difficulty in estimation of τ by means of the second order derivative of covariance function at zero due to sampling frequency of the data. For example, H_s are reported every hour for an buoy and every six hours for hind-cast data, e.g. ERA 40. Consequently, estimation of the derivative using finite differences can be quite inaccurate. One way to go around the problem would be to estimate τ by taking averages of time periods that H_s spends above/ below its median value, i.e. directly using the physical interpretation of τ as the average duration of storms. The approach would require more studies and, in particular, accounting for the dependence of τ on season. Investigations of this issues are planed in future. Meanwhile the parametric model for covariance (17) is utilized providing a way of fitting τ based on the estimation of λ and T, that is done by fitting (17) to empirical covariances.

Once λ and T are fitted, then τ can be found some 'smoothed' versions of the relation between τ on one hand and T and λ on the other hand. Is the past research, see [4], a smooth simplified method has been developed based on the time that the autocorrelation of $\ln H_s$ drops to 0.6. Namely, we observe that if the actual correlation would not have the Ornstein-Uhlenbeck part ($\lambda = 0$), then τ obtained from the second derivative of such simplified covariance would be equal to πT. The value of the simplified correlation at $t = \tau/\pi = T$ is approximately equal to 0.6. Thus a rough estimate of τ in the presence of λ is obtained by considering the time T_0 at which value of the complete correlation drops to 0.6 and then taking $\tau = \pi T_0$. Consequently τ could be directly from the fitted $r(s)$ to the data by solving equation $\rho(\tau/\pi) = 0.6$ or by evaluating it for the $\rho(s)$ given in (17), which through the second order Taylor expansion results in

$$\tau = \pi\,(-\lambda\,T^2 + \sqrt{\lambda^2 T^4 + T^2}). \tag{18}$$

Example 1 (Hundred years H_s at location of Buoy 46005) *In [1] data H_s measured 1978-1999, from Buoy 46005, have been throughfully analyzed. The proposed model for $m(s)$ was a seasonal component with linear trend, viz.*

$$m(s) = 0.7697 + 0.1072\sin(2\pi\,s) + 0.4375\cos(2\pi\,s) + 2.6626\,10^{-5} \cdot 365.25(s - 1978), \tag{19}$$

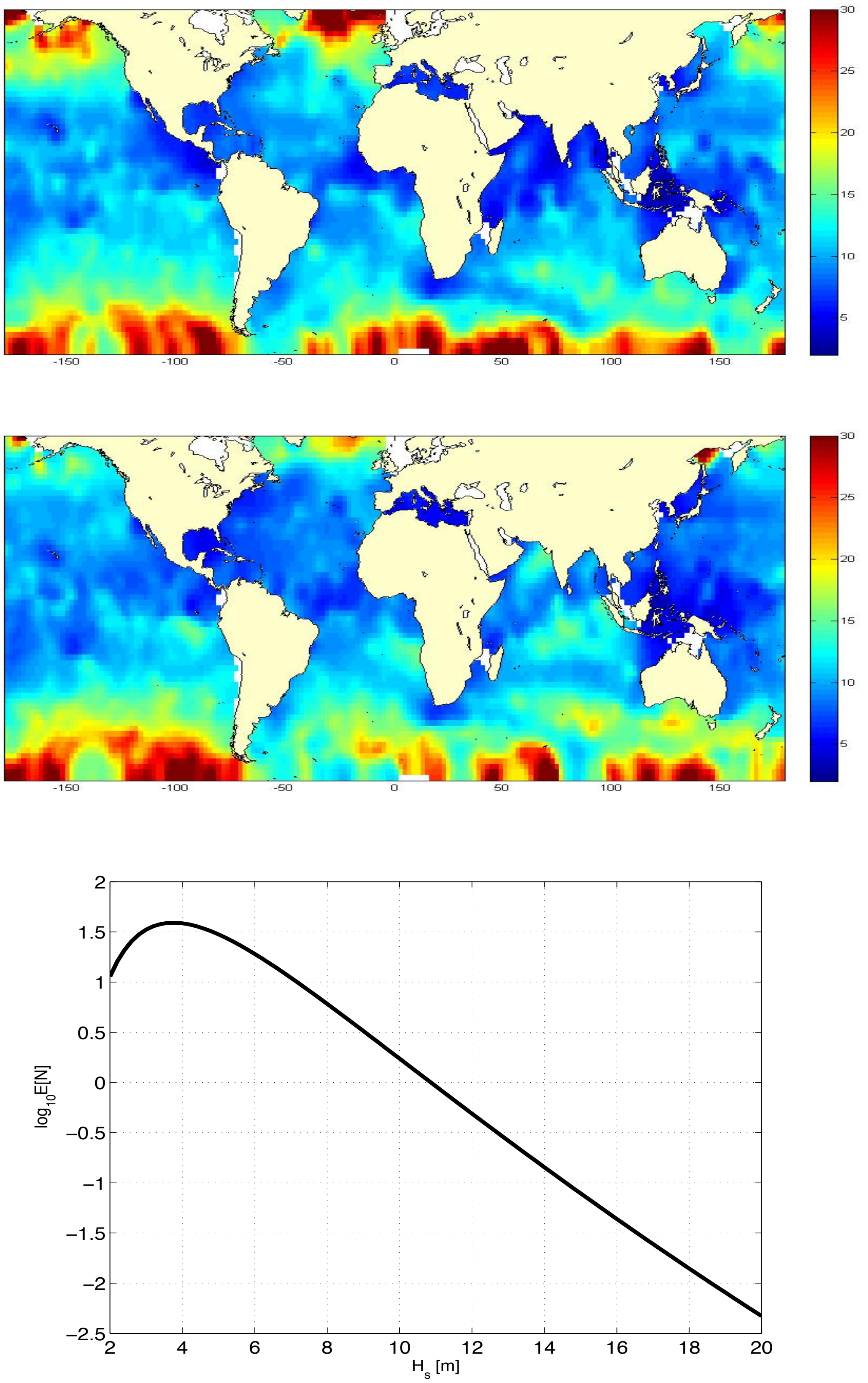

Figure 5: Estimates of storm size in degrees, i.e. average excursion length above median significant wave height $\pi\, L$, in February *(top)* and August *(middle)*. *Bottom:* Expected number of crossings of level u by $H_s(t)$ during a year of shiping in the North Atlantic.

while

$$\sigma(s) = 0.432 - 1.143\,10^{-5} \cdot 365.25(s - 1978), \tag{20}$$

where s has units years. In the Table 2.4 page 21 of that reference, the long-term correlation function $\rho(t)$ is given. To the function $\rho(t)$, $1 \le t \le 10$ the correlation function (17) is fitted and the fitted parameters are $T = 23.6$ and $\lambda = 0.009$ hours^{-1}. Using (18) the parameter $\tau = 60$ hours.

The estimate of 100 years H_s using Rice's method and parameters based on analysis of [1], e.g. the fitted correlation to data from the table and regressions (19-20), is 15.4 meters, see Figure 6 solid line ($s = 1999$) and 17.6 meters, dashed doted line $s = 1979$. Finally as reported in [28] the 100 years significant wave hight estimated using yearly maximums and the Gumbel fit was 16.8 meters while the generalized extreme distribution fit gave estimate 13.7 meters, see the dot and the star in Figure 6, respectively.

4.1.3 H_s measured on board of a ship

Similar reasoning that was motivating formula (17) can be applied to a vessel moving with velocity $\mathbf{v}_{shp}$. The local covariance between logarithms of encountered H_s is of the following form

$$\rho(t) = p e^{-\frac{|\mathbf{v}_{shp} - \mathbf{v}|^2\, t^2}{2\,L^2}} e^{-\lambda |t|} + (1-p) 1_0(t),$$

In order to find the average duration of storm when on board of such a vessel, one can use the relation (18) that leads to

$$\tau = \pi \left(-\lambda\, T_v^2 + \sqrt{\lambda^2 T_v^4 + T_v^2} \right), \qquad T_v = L / ||\mathbf{v}_{shp} - \mathbf{v}||). \tag{21}$$

In general, τ depends on route (season and position of vessel and speed computed from the rout) which is function of time t hence, given yearly routing, the expected number of times the encountered significant wave heights by a vessel can be predicted by

$$\mathsf{E}\left[N(u)\right] = \frac{365.2 \cdot 24}{2} \int_0^1 \frac{1}{\tau(s)} \exp\left(\frac{-(\ln u - m(s))^2}{2\sigma^2(s)} \right) ds, \tag{22}$$

Here s has units years while τ is in hours leading to the factor 4382.4 in (22).

Example 2 (100 years H_s observed from a vessel) *Here yearly shipping is equivalent to seven passages over Atlantic. We can use (21), (22) and (12), to evaluate h_{100}. However the problem is that at present we have no estimates of λ for the globe but only for the 20 buoys locations reported in [4]). Instead, we arbitrarily choose λ to be 0.011 hours^{-1}. The remaining parameters are evaluated using the estimates reported in [3] and shown in Figures 1 and 3 (bottom). The functions $m(s)$, $\sigma^2(s)$ are presented in Figure 4 (left) while $\tau(s)$ in Figure 7 (left). The estimated encountered 100 years H_s is about 18.5 meters, see Figure 5 (bottom).*

Example 3 (100 years H_s at location of Buoy 46005, using the model) *We consider a buoy as a ship moving with speed zero and use (21), (22) and (12), to evaluate h_{100}. We choose the same λ as in Example 1, viz. 0.011 hours^{-1}. The remaining parameters are evaluated using the estimates reported in [4]) and extrapolated to the position of the buoy, giving $\sigma(s) = 0.3523$,*

$$m(s) = 0.9493 + 0.3452 \cos(2\pi\, s) + 0.2669 \sin(2\pi\, s).$$

The parameters L and T_v vary with season and are given in the following table

L in degrees:	*5.1*	*9.1*	*5.6*	*6.0*	*4.5*	*3.9*	*3.6*	*3.5*	*3.6*	*3.8*	*6.1*	*4.7*
T_v in hours:	*24.5*	*43.9*	*27.2*	*29.1*	*21.6*	*18.9*	*17.5*	*16.9*	*17.4*	*18.5*	*29.6*	*22.7*

The speed was taken constant $||\mathbf{v}|| = 0.207$ degree per hour. The direction of the velocity $\mathbf{v}$ varies with

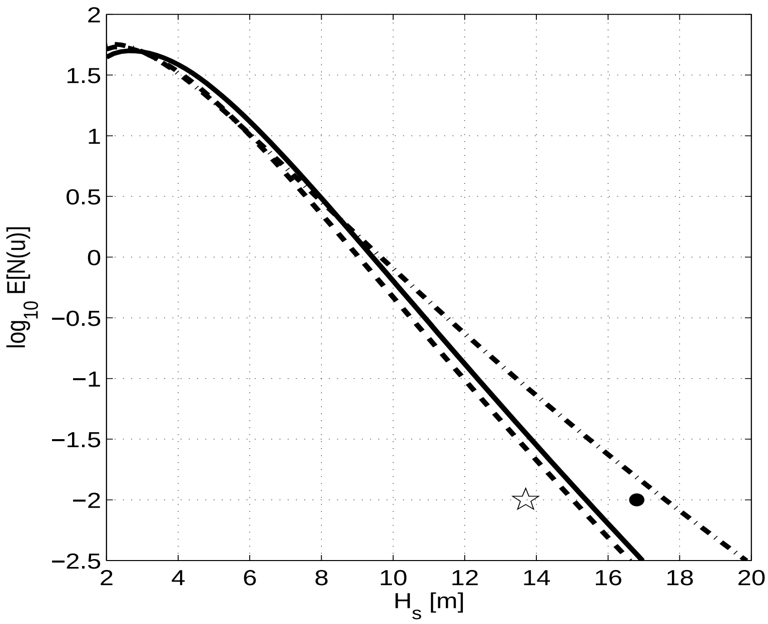

Figure 6: Expected number of crossings of level u by $H_s(t)$ during a year, estimated for Buoy 46005, using estimate given in [1] the solid line for year 1999 and the dashed dotted line for year 1979. The dashed line is the expected number using the spatio-temporal model. The dot and star are estimates of h_{100} ussing 23 yearly maximums, the Gumbel fit (black dot) and the generalized extreme distribution (GEV) fit (star).

time. Since L *are estimated for each month separately one can see considerable estimation errors. The encountered 100 years* H_s *estimated in this example is about 15 meters, see Figure 6 the dashed line.*

We conclude that because of a trends (increasing mean and decreasing standard deviation the Rice's estimate of 100 years significant wave height h_{100} *changes from 17.6 meters in 1979 to 15.4 meters in 1999. More standard statistical methods employing observed yearly maxims estimate* h_{100} *to be 13.7 meters if GEV model is used and 16.8 meters for Gumbel model. (One need to to note that about 20% of buoy observations are missing.) The Rice's estimate employing spatio-temporal model gave 15 meters estimate of* h_{100}*. The values are quite similar although show some spread due to uncertainties when estimating such extremely seldom events as an occurrence of 100 years storm.*

4.2 Hierarchical model for parameters

In the above approach sea conditions encountered by a vessel during a year are described by a deterministic functions $(m(t), \sigma^2(t), \tau(t))$ sampled at frequency 2 per hour. The integral (12) is basically a sum of a long sequence of encountered crossing intensities. The result is a function of about 50 thousand values of parameters. Often a more convenient approach is to compute the integral by means of the so called long-term distribution of parameters. Denote by $f(m, \sigma^2, \tau)$ the pdf of the distribution which could be approximated by the normalized histogram of encountered values of parameters along the route. The long term distributions describes variability of parameters values encountered on the route at time t taken at random.

By using the long-term pdf we can write the expectation in (12) as

$$\mathsf{E}\left[N(\ln h_T)\right] = \frac{1}{2}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\frac{1}{\tau}\exp\left(\frac{-(\ln h_T - m)^2}{2\sigma^2}\right) f(m, \sigma^2, \tau)\, dm\, d\sigma\, d\tau \qquad (23)$$

The formula (23) can be further simplified by introducing the biased sampling pdf of the parameters

$$\tilde{f}(m, \sigma^2) = c^{-1}\int_0^\infty \frac{1}{\tau} f(m, \sigma^2, \tau)\, d\tau, \qquad (24)$$

where c is the expected frequency of storms encountered by a ship during a year. This leads to

$$\mathsf{E}\left[N(\ln h_T)\right] = c\int_{-\infty}^{\infty}\int_{0}^{\infty}\exp\left(\frac{-(\ln h_T - m)^2}{2\sigma^2}\right)\tilde{f}(m,\sigma^2)\,dm\,d\sigma^2. \tag{25}$$

Note that factor c is equal to the expected number of encountered storms, defined as an excursion of H_s above its median, during a year.

Remark 3 *In previous sections we have presented maps for* $\exp(m)$ *and* σ^2*. Such maps could be used to compute the long term distribution of* m *and* σ^2 *whenever a detailed plan of shipping, i.e. positions* $\mathbf{p}$ *and seasons for planned routes, is chosen. Basically one can estimate the long term pdf* $f(m,\sigma^2)$*. However* $\tilde{f}(m,\sigma^2)$ *is in general not equal to the long term pdf* $f(m,\sigma^2)$ *and hence the joint distribution of* (m,σ^2,τ) *needs to be estimated as well.*

5 Simulation of encountered H_s along a route.

In the previous sections means to estimate expected fatigue damage and risk for encountering an extreme sea state (100 years significant wave height) for a specific shipping plan were presented. The computations involved long-term distributions of sea climate parameters; median and coefficient of variation H_s; heading angle; and τ (parameter measuring average duration of encountered storms). These parameters describe local properties of the encountered sea states along the planned routes. The long term distribution of the parameters were defined as the values of parameters at a randomly chosen point along a rout.

The expected damage and hundreds years encountered significant wave heights represent quantities (averages) that are useful at the design stage of a vessel, a choice of ship routes or for planing of maintainance schedules. However the accumulated damage may differ from the expected one and there can be a need to quantify the variability of the accumulated damage by means of its variance or even distribution. For such purposes one needs to simulate random sequences of encountered sea states and evaluate the accumulated fatigue damages, which become random itself. Their variability can be describe by means of a single parameter, e.g. variation coefficient, or more completely by a fitted probability distribution. There are also other applications of explicit random model for the sequence of encountered sea states. They can be used for prediction of missing values, future sea states to be encountered based on the available information, duration and severity of storms, etc.

In the following a simple random model for a sequence of encountered sea states will be described. The model is uniquely defined by spatial and time distributions of the previously mentioned local parameters; median and coefficient of variation of H_s; average duration of storms τ; and average velocity $\mathbf{v}$ of storms (level contours of H_s). The parameters have clear physical meaning and their evolution in time could be described by means of trends derived from climate change models. This would give a possibility to study distributions of more complex events (like sizes of storms) at a future seas.

5.1 Non-stationary log-normal model for H_s encountered along a route.

We continue to assume that the logarithms of encountered significant wave heights $W(t) = \ln H_s(t,\mathbf{p}(t))$ are normally distributed with means $m(t)$ and variances $\sigma^2(t)$, $0 \le t \le T$, where t is in hours. It was also assumed that $W(t)$ is locally stationary, the notion which is not precisely defined but it essentially means that mean $m(t)$ and variance $\sigma^2(t)$ change slowly and that for points t_1, t_2 such that $|t_i - t| < 10$, say, the correlation ρ between $W(t_1)$ and $W(t_2)$ is a function of the difference $t_2 - t_1$. The parameter τ , the average duration of a storm, is proportional to the second derivative of ρ, viz. $\tau = \pi/\sqrt{-\rho''(0)}$. Hence a very crude approximation of the autocorrelation is derived by means of Gauss function, viz.

$$\rho(t_1,t_2) \approx e^{-\frac{\pi^2(t_2-t_1)^2}{2\tau(t)^2}}. \tag{26}$$

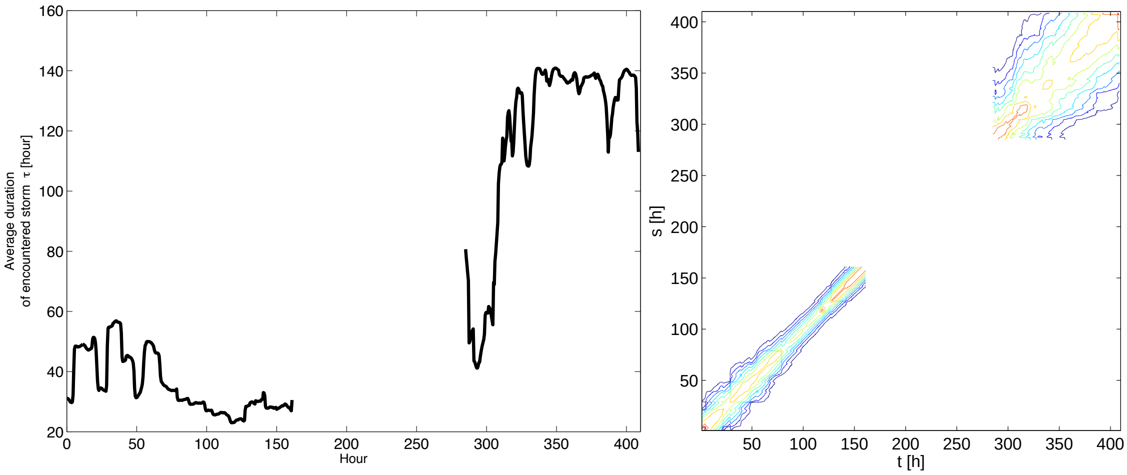

Figure 7: Parameters of $W(t) = \ln H_s(t, \mathbf{p}(t))$ for the route presented in Figure 3 *(bottom)*. *Left:* The expected duration τ of encountered storm. *Right:* Covariance (31) between $W(t)$, $W(s)$. The covariance is zero for $160 < t < 270$ or $160 < s < 270$ since the vessel is close to or in the harbor.

The approximation does not define ρ for arbitrary t_1, t_2 in the interval $[0, T]$ but only only in vicinity of the time t. Before we propose a formula for correlation ρ for any pair of times t_1, t_2, we present how to simulate stationary $W(s)$ with correlation (26) through spectral representation of $W(s)$.

Stationary Gaussian process $W(s)$ with correlation (26) has a power spectral density

$$S_t(\omega) = \frac{\tau(t)}{\sqrt{2\pi}} \, e^{-\omega^2\, \tau(t)^2/2\pi^2} \tag{27}$$

and can be evaluated by the following integral

$$W(s) = m(t) + \sigma(t) \int \exp(-is\,\omega) \sqrt{S_t(\omega)} \, dB(\omega), \tag{28}$$

where $B(\omega)$ is a Brownian motion.

A non stationary process can be derived by exploiting that the spectrum S_t in (28) depends on time and let $t = s$, viz.

$$W(s) = m(s) + \sigma(s) \int \exp(-is\,\omega) \sqrt{S_s(\omega)} \, dB(\omega). \tag{29}$$

If the spectrum changes slowly $W(s)$ in time defined in (29) satisfies our loose requirements on a locally stationary Gaussian process. In order to simulate the process one could approximate the integral in (29) or evaluate a covariance between $W(s)$ and $W(t)$ for process defined in (29) for any times s, t. This approach will be used here.

Some simple analysis calculations shows that

$$Cov(W(t), W(s)) = \int \exp(-i(s-t)\,\omega) \sqrt{S_s(\omega) S_t(\omega)} \, d\omega = r(t,s), \tag{30}$$

say, where S_{t_i} are defined in (27). Assuming that $\sigma(t)$ and $\tau(t)$ are known and S_t is given by (27) the integral in (30) can be computed to yield

$$r(t,s) = \sigma(t)\sigma(s) \sqrt{\frac{2\tau(t)\tau(s)}{\tau(t)^2 + \tau(s)^2}} \, e^{-\pi^2 (t-s)^2/(\tau(s)^2 + \tau(t)^2)}. \tag{31}$$

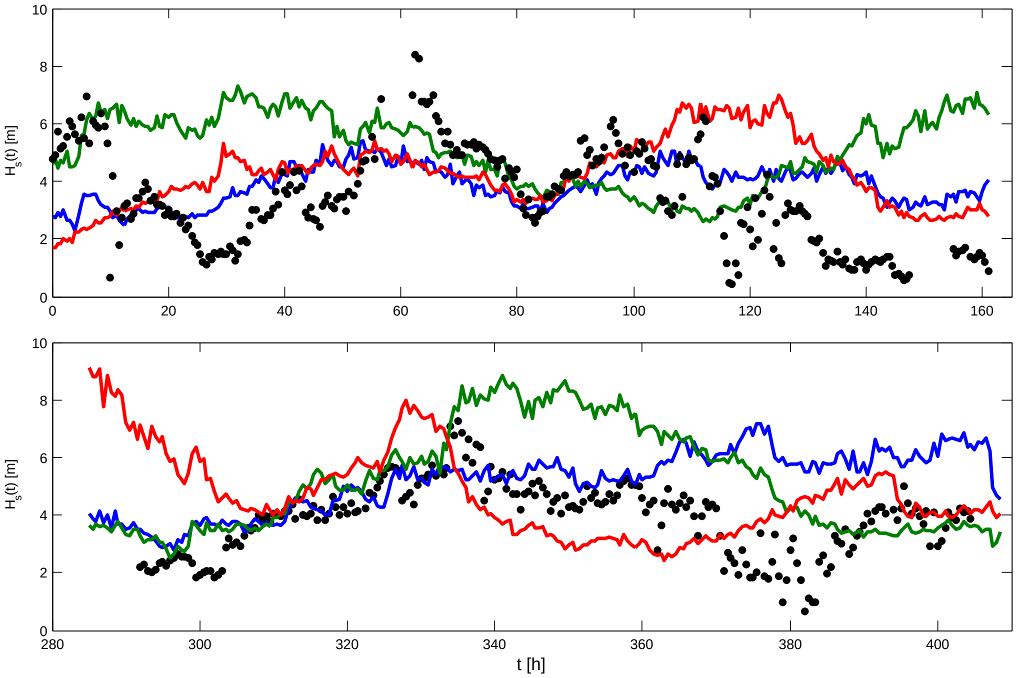

Figure 8: Three simulated H_s encountered along the route presented in Figure 3 – they are exhibited in solid lines while dots represent measured H_s on board of a vessel sailing the same route.

5.1.1 Example: Simulation of H_s along a route.

Here we demonstrate the use of the proposed model to simulate the significant wave heights encountered along the route presented in the Figure 3. The mean value $m(t)$ and $\sigma^2(t)$ are shown in Figure 4. The average duration of encountered storm (in hours) $\tau(t)$ is given in Figure 7. One can see that the time duration of storms/calm periods on the route from Europa to America is about 40 hours while the sailing time is about 160 hours. Thus one can expect to meet three storms on this route. On the other hand, the route from America to Europe took about 125 hours, the average storm period is 130 hours and hence one can expect to meet one storm.

We turn now to more detailed description of the simulation algorithm. The inputs are functions $m(t)$, $\sigma^2(t)$ and $\tau(t)$ presented in Figures 4 and 3. The functions are sampled with time step 0.5 hour (the same as frequency of on board measured H_s). In the following, we denote by $\mathbf{t}$, $\mathbf{s}$ vectors of time points sampled with frequency two per hour along the rout. Further the significant wave height observed at points $\mathbf{t}$ is denoted by $H_s(\mathbf{t})$, while their logarithms by $W(\mathbf{t})$ and the vector of their means by $m(\mathbf{t})$. The covariance matrix between $W(\mathbf{t})$, $W(\mathbf{s})$, with entries $r(t_i, s_j)$ computed using (31), is denoted by $\Sigma(\mathbf{t}, \mathbf{s})$. Using the introduced notation the samples of $H_s(\mathbf{t})$ can be simulated as follows

$$H_s(\mathbf{t}) = \exp\left(m(\mathbf{t}) + \sqrt{\Sigma(\mathbf{t},\mathbf{t})}\mathbf{Z}\right),$$

where $\mathbf{Z}$ is a vector of independent standard normal variables and $\sqrt{\Sigma}$ is any matrix such that $\Sigma = \sqrt{\Sigma}\sqrt{\Sigma}^T$.

Example 4 *For the voyage presented in Figure 3 and times t_i sampled with frequency 2 per hour the covariance $\Sigma(\mathbf{t}, \mathbf{t})$ is shown in Figure 7 (right). One can see that correlation between W_i decreases fast for t_i on the route to America, since one is sailing "against" storms while $W(t_i)$ are strongly correlated on the route from America to Europe since one is traveling with storms. Finally, since the stay in the harbor is much longer than the correlation length of $W(t)$, the covariance between $W(t)$ and $W(s)$ for t when ship is on the route to America while s is on the route to Europe is zero.*

In Figure 8 three samples of simulated significant wave heights are compared with the measured ones (dots). One can see that there are missing values in the measured signals and that during certain periods the measurements have quite high volatility.

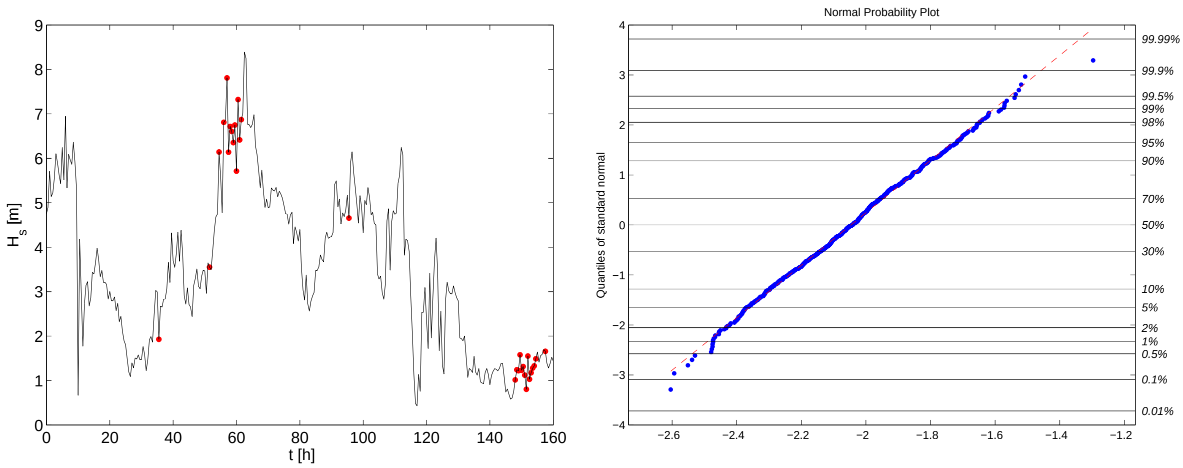

Figure 9: Further analysis of the route presented on Figure 3. *Left:* Measured H_s along the route from Europe to America. The dots mark reconstructed values that has been missing in the record. *Right:* One thousand simulated values of $\log_{10}(D)$, accumulated for the ship sailing the route, plotted on a normal probability paper. The logarithm of the observed damage $\log_{10}(D^{obs})$ is -2.18.

5.1.2 Extrapolation of missing values

As one can see in Figure 8 there are some times when some H_s measurements are missing. Since those are in the stormy period it can lead to underestimation of the observed damage. Here we give formulas to simulate the missing values.

Let $\mathbf{s}$ be a vector of times when measurements are missing while $\mathbf{t}$ be the remaining times. Denote the available measurements by $h_s(\mathbf{t})$ and let $\tilde{m}(\mathbf{s})$ and $\tilde{\Sigma}(\mathbf{s}, \mathbf{s})$ be the mean and covariance of $W(\mathbf{s}))$ conditioned that $W(\mathbf{t})) = \ln h_s(\mathbf{t})$, respectively. Then the missing values $H_s(\mathbf{s})$ can be simulated from the conditional distribution by

$$H_s(\mathbf{s}) = \exp(\tilde{m}(\mathbf{s}) + \sqrt{\tilde{\Sigma}(\mathbf{s}, \mathbf{s})}\mathbf{Z}).$$

Finally the conditional mean is given by

$$\tilde{m}(\mathbf{s}) = m(\mathbf{s}) + \Sigma(\mathbf{s}, \mathbf{t})\Sigma(\mathbf{t}, \mathbf{t})^{-1}(\ln h_s(\mathbf{t}) - m(\mathbf{t}))^T$$

while the conditional covariance is

$$\tilde{\Sigma}(\mathbf{s}, \mathbf{s}) = \Sigma(\mathbf{s}, \mathbf{s}) - \Sigma(\mathbf{s}, \mathbf{t})\Sigma(\mathbf{t}, \mathbf{t})^{-1}\Sigma(\mathbf{t}, \mathbf{s}).$$

In Figure 9 *(left)* some reconstruction is presented. In order to make the reconstruction more similar to the rest of signal we have assumed that there is a measurement error of $\ln H_s$ which is normally distributed with zero mean and variance 0.02. More precisely one have added to $\Sigma(\mathbf{t}, \mathbf{t})$ a matrix $0.02\,\mathbf{I}$, where $\mathbf{I}$ is the identity matrix.

5.1.3 Estimating distribution of the accumulated damage on a route

The route used here for illustration of the described results was selected from 6 month long full scale measuring campaign on board of a 2800 TEU container ship. Container ships are long, slender, thin-walled structures with open cross-sections experiencing large oscillations of wave induced stresses with superimposed vibrations caused by resonances and slamming. Such variability of stresses makes fatigue life prediction very uncertain, see Mao et al. (2010). Although the fatigue design life of a container ship operating in the North Atlantic Ocean is usually more than 20 years cracks in ships details are found earlier than expected. For example it was reported in [30] that fatigue cracks were found in a container vessel after less than eight years of service. To identify source of such early damage vibrations of container ships were studied both theoretically, in wave tanks on ship models, and in full scale measuring campaigns. The measured H_s used in the paper were taken from one of such full scale measurements on a 2800 TEU container ship mentioned above undertaken to study the variability of stresses at some location of the ship were cracks were observed.

Here we had considered a trip to America and back to Europe undertaken in January month. The observed damage was 0.66% of the expected total life, which increases to 0.75% after prediction of missing H_s measurements. The median damage computed using the log-normal model for the encountered H_s is 0.99%. The observed damage is slightly below the theoretical median as can be seen in Figure 9 where logarithms of 1000 simulated damages are plotted on normal paper. This bias is not entirely surprising since captains are using routing programs to avoid the most sever storms. For eight undertaken voyages during the measuring campaign only once the observed damage exceeded the median computed using the model presented in this paper.

The presented model could be used to detect problems in the design of the vessel. A very crude analyzes could be performed in the following way. One may assume that for the North Atlantic trade the yearly accumulated damage is equivalent to 8 presented voyages during one year. These would lead to the median damage during 20 years of service of about 160%, i.e. the predicted life is about 12 years. Since predictions of fatigue life are uncertain one often conservatively predicts high risks for fatigue problems at a ship detail when the damage exceeds 50% level, which in our case would be 6 years.

6 Conclusions

This work demonstrates that a relatively simple model for the global significant wave field can be fitted from a variety data and records: satellite data, buoy and platform data, hindcast. Despite its simplicity, the model allows quite accurately analyze the fatigue damage and reliability of a ship traveling along known routes. The same model can be used to provide assessment of the risk associated with the 100 year significant wave height. We obtain different values of the 100 year significant wave heights for a stationary location (buoy or platform) and for a vessel traveling along a route on North Atlantic. Finally, the simulated data from the models allowed for a study of variability of the damage accumulation – an important issue at the design stage of the ship operation.

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