ESS011
Mathematical statistics and signal processing

Lecture 1: Introduction

CHALMERS

Practicalities 1/2

- Course website:
  `www.math.chalmers.se/Stat/Grundutb/CTH/ess011/1314`
- $\sim$ 22 lectures
- exam at the end
- homework given each week
- Book: Milton & Arnold, *Introduction to probability and statistics*

Two parts but one course!

A. Probability and statistics by Tuomas Rajala (ca 2 lectures/week)
B. Signal processing by Bill Karlström (ca 1 lecture/week)

One exam, one score. Go to both lectures!

Practicalities 2/2

Exercises:

- Person in charge: Viktor Johnsson
- Two (2) groups per week, Tuesdays and Fridays, 13-15 , Room EL43
- Each student participates in one (1) group per week!

- Homework given ca a week before the exercise session
- In the exercise session, mark the exercises you solved/attempted
- A random sample of students will present their solution each session
- Marking i.e. doing homework adds points to final evaluation (0-10%)
- First week exercises are a guided session with Viktor

Other:

- Due to Easter, course schedule fragmented: Follow the website

Course content

In this course, you will learn

- what the word "probability" means
- how to compute probabilities in simple scenarios
- how to do basic mathematics involving random events
- what kind of models scientists use for random events

and after Easter

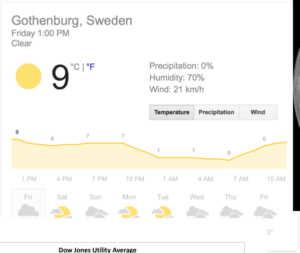
- why probability is important for something called "statistics"
- how to handle uncertainty and unknown errors in data
- how to derive and use simple statistical tools

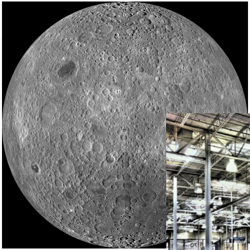
and, hopefully, in the end

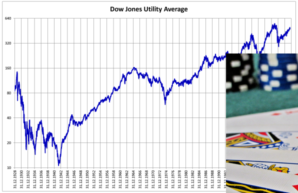
- why the understanding of randomness is important for everyone

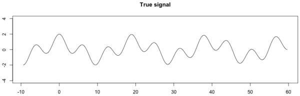
Part 1: Randomness and probabilities

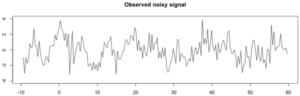
We live in a random and uncertain world.











Part 1: Randomness and probabilities

Different views on randomness:

- Random disturbances: we observe not the true event but a noisy version (e.g. signals)
- Uncertainty of the situation: We don't know/observe/account for everything affecting the system
- Intrinsic variability: The system will always be "random" as too many sources are affecting it for us to measure and account for them all

Often all three are involved when we study physical phenomenon.

Deterministic vs Stochastic system

Deterministic: always the same outcome with the same initial setup

A dropped coin falls down to the floor.

e.g. the models (laws) of classical physics: $F = mg, E = mc^2$, ...

Stochastic: the outcome varies, even with the same initial setup

The coin flip comes up tails, or maybe not.

The child will live to become 80-90 years old, or maybe more, or less.

How does the outcome vary?
Can we predict the event?

Random experiments

To discuss randomness, we need *probability theory*. Start with

Random experiment: An experiment or situation of interest of which outcome is not certain.

For example:

- Choose the cashier queue in a supermarket, and measure the time spent in queue.
- Forecast the weather tomorrow.
- Flip a coin, what comes out.

One execution of the experiment is called a **trial**.

Random experiments

Consider the following experiment:

- Raise your pen, and let go. Does it fall?

Of course it does. This is deterministic as we can trust gravity.

Now repeat the drop some times, and observe:

- Which direction does the end of the pen point?

each trial's outcome varies since

- Your hand position is not the same each time
- The pen bounces a bit differently each time
- etc.

There is always a degree of uncertainty in the outcome.

Q: How to quantify that degree of uncertainty?

Probability

To describe the outcome of a random experiment, we use *probabilities*. Define

Probability: The measure of certainty in an outcome of a random experiment.

Things we all sort of know already:

- Probability close to 1 means almost sure thing
- Probability close to 0 means almost never going to happend
- In coin flip, probability 0.5 means it can go both ways.
- Probability is often also called *chance*, 0.25 chance of rain.
- Sometimes we use 0-100% instead of 0-1, e.g. the 25% chance of rain.

Probability: relative frequency

First idea of how to quantify probability:

Assume the experiment can be repeated many times.

Let's say you toss a coin 100 times, and observe tails 42 times. Then

$$\text{probability of tails} = \frac{42}{100}$$

we *infer* the probability from experience.

Relative frequency: The probability of an outcome is the proportion it has occured of all our executions of the experiment.

Probability: relative frequency

Relative frequency is an *approximation*: You toss another 100 coins, and observe 65 tails. Then

$$\text{probability of tails} = \frac{65}{100} \neq \frac{42}{100}$$

This "error" is the bread of statistics and we will master it after Easter.

What you would "expect" from a coin toss is $\frac{1}{2}$. Let's model this.

Probability: classical model

The coin can land in one of two states: heads or tails.

Assuming each is equally likely, our *model* for the coin toss is

$$\text{probability of tails} = \frac{n(\{\text{tails}\})}{n(\{\text{heads, tails}\})} = \frac{1}{2}$$

Another example: Throw a die. It can land in one of six sides. What is the probability it will be 1 or 4?

$$\text{probability of 1 or 4} = \frac{n(\{1,\ 4\})}{n(\{1,2,3,4,5,6\})} = \frac{2}{6}$$

Classical model of probability: The probability of a set of **equally likely** outcomes is their proportion out of all possible outcomes.

* * *

Mathematical foundation: Sample space and events

Let's begin the mathetical construction of probability theory.

Define

- **Sample space** S: all possible outcomes that can occur in the experiment.

coin $S = \{\text{heads, tails}\}$, die cast $S = \{1, 2, 3, 4, 5, 6\}$, pen angle $S = [0, 2\pi)$

Note:

- Every physical outcome of the experiment corresponds to *exactly* one element in S.
- The elements in S are called *sample points*.

(16th century, Cardano)

Mathematical foundation: Sample space and events

Then, define

- **Event** A: a subset of sample points, i.e. $A \subseteq S$.

e.g. roll 1 or 4: $A = \{1, 4\} \subset S$

The classical formula for probability with equally likely sample points:

$$P(A) := \frac{n(A)}{n(S)}$$

e.g. die cast $P(\{1, 4\}) = \dfrac{2}{6}$, coin toss $P(\{\text{tails}\}) = \dfrac{1}{2}$.

The two extremes:

- The *certain event* $A = S$: $P(S) = 1$
- The *impossible event* $A = \emptyset$ (empty set): $P(\emptyset) = 0$

Set theory: Or = union

Now let's consider the die cast with events

$$\begin{aligned} A_o &:= \{\text{odd number}\} = \{1,3,5\} \\ A_e &:= \{\text{even number}\} = \{2,4,6\} \\ A_a &:= \{\text{above 3}\} = \{4,5,6\} \end{aligned}$$

What about: $B := \{\text{odd number } \textbf{or} \text{ above 3}\}$?

We can use *set union* for the word **or**, and get

$$B = A_o \cup A_a = \{1,3,4,5,6\}$$

Notice that $P(B) \geq P(A_o)$ and $P(B) \geq P(A_a)$.

What is $P(A_e \cup A_o)$?

Set theory: And = intersection

What about $C := \{$even number **and** above 3$\}$?

As we want both conditions to be true, we use *set intersection*:

$$C = A_e \cap A_a = \{4, 6\}$$

Notice that $P(C) \leq P(A_e)$ and $P(C) \leq P(A_a)$.

What is $P(A_e \cap A_o)$?

Set theory: Not = complement

What about $D := \{\textbf{not} \text{ above } 3\}$?

This is set of sample points of S **not** in A_a, so we can use the *set complement*:

$$D = A_a' = S \setminus A_a = \{1, 2, 3\}$$

The complement of A is often denoted by A^c or A' or $\bar{A}$, and sometimes written as $A' = S - A$

Now we can compute more complicated ideas:

- Above three and not an odd number $= A_a \cap A_o' = \{4, 6\}$
- Even or odd, and not above three $= (A_e \cup A_o) \cap A_a' = \{1, 2, 3\}$.

Mutually exclusive sets

Often we are interested in events that can not occur at the same time.

For example, for coin $A_o = \{odd\}$ and $A_e = \{even\}$ can not happend at the same time.

Then, the **and** rule implies $A_o \cap A_e = \emptyset$.

Mutually exclusive sets: Two events A_1 and A_2 are *mutually exclusive* if and only if ("iff") $A_1 \cap A_2 = \emptyset$. Furthermore, Events $A_1, A_2, A_3, ...$ are mutually exclusive iff for all pairs i and j with $i \neq j$ we have $A_i \cap A_j = \emptyset$.

Summary of lecture 1

- Probability: measure of certainty on the outcome of a random experiment
- Incredients: Sample space, sample points, events.
- How to translate "and", "or" and "not" into set operations on events: $A \cup B$, $A \cap B$, A'
- Mutually exclusive events have an empty intersection

Tomorrow we will get to 17th century and start computing probabilities that involve e.g. several dice.

Remember to make a decision about the exercise group!