LÖSNINGAR 20170110

1.

a)

$$E(\xi) = \int_{9,5}^{10,5} x \cdot 6(x-9,5)(10,5-x)\,dx = \ldots = \boxed{10}$$

$$E(\xi^2) = \int_{9,5}^{10,5} x^2\, 6(x-9,5)(10,5-x)\,dx = \ldots = 100{,}05$$

$$\Rightarrow Var(\xi) = E(\xi^2) - (E(\xi))^2 = 100{,}05 - 10^2 = 0{,}05$$

$$\Rightarrow S(\xi) = \sqrt{Var(\xi)} = \sqrt{0{,}05} = \boxed{0{,}224}$$

b) $P(9{,}8 \le \xi \le 10{,}1 \mid 9{,}9 \le \xi \le 10{,}2)$

$$= \frac{P(\{9{,}8 \le \xi \le 10{,}1\} \cap \{9{,}9 \le \xi \le 10{,}2\})}{P(9{,}9 \le \xi \le 10{,}2)} = \frac{P(9{,}9 \le \xi \le 10{,}1)}{P(9{,}9 \le \xi \le 10{,}2)}$$

$$= \frac{\int_{9,9}^{10,1} 6(x-9,5)(10,5-x)\,dx}{\int_{9,9}^{10,2} 6(x-9,5)(10,5-x)\,dx} = \ldots \approx \boxed{0{,}685}$$

c) $$P(\xi \le 10{,}1) = \int_{9,5}^{10,1} 6(x-9,5)(10,5-x)\,dx = 0{,}648$$

$\Rightarrow$ η är $Bin(n=60,\ p=0{,}648)$

$np(1-p) = 60 \cdot 0{,}648 \cdot (1-0{,}648) = 13{,}686 > 10$

$\Longrightarrow$ η är approx $N(np, \sqrt{np(1-p)}) = N(38{,}88,\ 3{,}699)$

$$\Rightarrow P(\eta \le 43) = P\left(\frac{\eta - 38{,}88}{3{,}699} \le \frac{43-38{,}88}{3{,}699}\right) \approx \Phi(1{,}11) \approx \boxed{0{,}87}$$

2.1

Skatta först σ med s

$$s = \sqrt{\frac{1}{6-1}\sum_{i=1}^{6}(x_i-\bar{x})^2} \quad \text{där} \quad \bar{x} = \frac{1}{6}(250{,}1+\dots+251{,}4)$$
$$= 249{,}967$$

$$\Rightarrow s = \sqrt{\frac{1}{5}\left((250{,}1-249{,}967)^2+\dots+(251{,}4-249{,}967)^2\right)} \approx 1{,}42501$$

$$\chi^2_{0,95} = 1{,}145$$

$\Rightarrow$ Intervallet blir $\left[0, \sqrt{\frac{5\cdot 1{,}42501^2}{1{,}145}}\right] = \boxed{[0,\ 2{,}978]}$

Vi ser att 5 ligger utanför intervallets gränser. Det verkar alltså som att maskinen inte behöver undersökas.

3. Låt T = totala vikten.

$$T = \sum_{i=1}^{860} \xi_i \quad \text{där } \xi_i = \text{vikten för brev nr. } i.$$

$$\mu = E(\xi_i) = 25\cdot 0{,}1 + 50\cdot 0{,}5 + 75\cdot 0{,}3 + 100\cdot 0{,}1 = 60 \text{ gram}$$

$$E(\xi_i^2) = 25^2\cdot 0{,}1 + 50^2\cdot 0{,}5 + 75^2\cdot 0{,}3 + 100^2\cdot 0{,}1 = 4000$$

$$\Rightarrow \sigma^2 = E(\xi_i^2) - (E(\xi_i))^2 = 4000 - 60^2 = 400$$

$$\Rightarrow \sigma = \sqrt{400} = 20 \text{ gram}$$

Centrala Gränsvärdes Satsen $\Rightarrow$ T är approx $N(860\cdot 60, \sqrt{860}\cdot 20)$
$= N(51600, 586.5)$

$$P(\text{säcken går sönder}) = P(T > 52500) =$$

$$= 1 - P(T \leq 52500) = 1 - P\Big(\underbrace{\frac{T - 51600}{586.5}}_{=Z} \leq \frac{52500 - 51600}{586.5}\Big)$$

$$= 1 - P(Z \leq 1{,}53)$$

$$\approx 1 - \Phi(1{,}53) = 1 - 0{,}937 = \boxed{0{,}063}$$

Svar

6.)

$$P(A\cap C) \underset{\substack{\uparrow \\ \text{oberoende}}}{=} P(A)P(C) = 0{,}01 \cdot 0{,}02 = 0{,}0002$$

$$P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)$$
$$= 0{,}01+0{,}01+0{,}02-0{,}003-0{,}0002-0{,}003+0$$
$$= \boxed{0{,}0338}$$

$$P(\xi=0) = 1-P(A\cup B\cup C) = 1-0{,}0338 = \boxed{0{,}9662}$$
$$P(\xi=2) = P(A\cap B)+P(B\cap C)+P(A\cap C) =$$
$$= 0{,}003+0{,}003+0{,}0002 = \boxed{0{,}0062}$$
$$P(\xi=1) = 1-P(\xi=0)-P(\xi=2) = 1-0{,}9662-0{,}0062$$
$$= \boxed{0{,}0276}$$

b)

$n=50 \quad p=0{,}0338 \quad (=P(A\cup B\cup C))$

Låt d = antalet defekta.

$$P(d\le 1) = P(d=0)+P(d=1) = \binom{50}{0}0{,}0338^0(1-0{,}0338)^{50}$$
$$+\binom{50}{1}\cdot 0{,}0338^1(1-0{,}0338)^{49}$$
$$= (1-0{,}0338)^{50} + 50\cdot 0{,}0338\cdot(1-0{,}0338)^{49}$$
$$= 0{,}179206 + 0{,}313452 = \boxed{0{,}492658}$$

$$\Longrightarrow P(\text{partiet avvisas}) = 1-0{,}492658 = \boxed{0{,}507342}$$

7)

a)

Låt ξ_A = livslängden för komponent A, ξ_B = livsl. för B, ξ_C = livsl för C.

ξ_A är $Exp(\lambda)$ där $\lambda = \frac{1}{1,5}$

$$P(\xi_A \geq 2) = \int_2^\infty \frac{1}{1,5} e^{-\frac{x}{1,5}} dx = \left[-e^{-\frac{x}{1,5}} \right]_2^\infty = e^{-\frac{2}{1,5}} \approx 0,264$$

Samma sak för ξ_B och ξ_C

P(systemet funkar efter 2 år)

$$= P\left(\{\xi_A \geq 2\} \cup \left(\{\xi_B \geq 2\} \cap \{\xi_C \geq 2\} \right) \right)$$

$$\underset{\uparrow}{=} P(\xi_A \geq 2) + P(\{\xi_B \geq 2\} \cap \{\xi_C \geq 2\}) - P(\{\xi_A \geq 2\} \cap \{\xi_B \geq 2\} \cap \{\xi_C \geq 2\})$$

Additionssatsen

$$\underset{\hat{A}}{=} P(\xi_A \geq 2) + P(\xi_B \geq 2) P(\xi_C \geq 2) - P(\xi_A \geq 2) P(\xi_B \geq 2) P(\xi_C \geq 2)$$

oberoende

$$= 0,264 + 0,264^2 - 0,264^3 = \boxed{0.315}$$

b) Bestäm $P(\eta \leq x)$: (← fördelningsfunktionen)

Vi har $\{\eta \geq x\} = \{\xi_A \geq x\} \cup \left(\{\xi_B \geq x\} \cap \{\xi_C \geq x\}\right)$

På samma sätt som i a) fås att

$$P(\eta \geq x) = P(\xi_A \geq x) + P(\xi_B \geq x) P(\xi_C \geq x) - P(\xi_A \geq x) P(\xi_B \geq x) P(\xi_C \geq x)$$

$$= P(\xi_A \geq x) + P(\xi_A \geq x)^2 - P(\xi_A \geq x)^3$$

$$= \left\{ P(\xi_A \geq x) = \int_x^\infty \frac{1}{1,5} e^{-\frac{t}{1,5}} dx = \ldots = e^{-\frac{x}{1,5}} \right\}$$

$$= e^{-\frac{x}{1,5}} + e^{-\frac{2x}{1,5}} - e^{-\frac{3x}{1,5}} \Rightarrow$$

Forts ⇒

7.) forts $P(\eta \leq x) = 1 - P(\eta \geq x) = \boxed{1 - e^{-\frac{x}{1,5}} - e^{-\frac{2x}{1,5}} + e^{-2x}}$

gäller om $x \geq 0$

Om $x < 0$ är fastis $P(\eta \leq x) = 0$

Frekvens funktionen $f(x) = F'(x)$

$$= \frac{e^{-\frac{x}{1,5}}}{1,5} + \frac{2}{1,5} e^{-\frac{2x}{1,5}} - 2e^{-2x} \quad \text{för } x \geq 0$$

och $f(x) = 0$ om $x < 0$.

8. a) Teckentabell för CD:

+
+
+
+
−
−
−
−
−
−
−
−
+
+
+
+

$$\Rightarrow l_{CD} = \frac{7{,}0+7{,}1+8{,}2+8{,}3+8{,}4+8{,}3+9{,}7+9{,}8}{8} - \frac{(8{,}6+8{,}4+9{,}6+9{,}8+6{,}8+6{,}5+8{,}0+8{,}3)}{8} = \boxed{0{,}1}$$

b) Välj till exempel

$E = ABC \quad F = BCD \quad G = ACD \quad H = ABCD$

$\Rightarrow I_1 = ABCE \quad I_2 = BCDF \quad I_3 = ACDG \quad I_4 = ABCDH$

$I_5 = I_1 I_2 = A\cancel{BC}E\cancel{BC}DF = AEDF \quad I_6 = I_1 I_3 = \cancel{A}B\cancel{C}E\cancel{AC}DG = BDEG$

$I_7 = I_1 I_4 = \cancel{ABC}E\cancel{ABC}DH = DEH \quad I_8 = I_2 I_3 = B\cancel{CD}FA\cancel{CD}G = ABFG$

$I_9 = I_2 I_4 = \cancel{BCD}FA\cancel{BCD}H = AFH \quad I_{10} = I_3 I_4 = \cancel{ACD}G\cancel{A}B\cancel{CD}H = BGH$

$I_{11} = I_1 I_2 I_3 = \cancel{AB}C\cancel{E}\ \cancel{BC}\cancel{D}F\cancel{A}C\cancel{D}G = CEFG$

$I_{12} = I_1 I_2 I_4 = \cancel{AB}CE\cancel{BCD}F\cancel{AB}C\cancel{D}H = BCEFH$

$I_{13} = I_1 I_3 I_4 = \cancel{AB}CE\cancel{ACD}GA\cancel{B}C\cancel{D}H = ACEH$

$I_{14} = I_2 I_3 I_4 = \cancel{BC}\cancel{D}F\cancel{AC}DG\cancel{AB}CDH = CDFGH$

$I_{15} = I_1 I_2 I_3 I_4 = \cancel{ABC}E\cancel{BC}DF\cancel{AC}DG\,AB\cancel{CD}H = ABDEFGH$

Kortaste ordet har längd 3 $\Rightarrow$ upplösningen = III.

Fråga 4

a)

$$\mathbb{P}(E) = 1 - \mathbb{P}(E^C)$$

$$\mathbb{P}(E^C) = \mathbb{P}(\xi_1 = 0) + \sum_{i=1}^{2} \mathbb{P}(\xi_1 = i)\mathbb{P}(\xi_2 \leq 2 - i|\xi_1 = i) = \binom{25}{0} 0.03^0 \cdot 0.97^{25}$$

$$+ \binom{25}{1} 0.03^1 \cdot 0.97^{24} \cdot \binom{50}{0} 0.03^0 \cdot 0.97^{50} + \binom{25}{1} 0.03^1 \cdot 0.97^{24} \cdot \binom{50}{1} 0.03^1 \cdot 0.97^{49}$$

$$+ \binom{25}{2} 0.03^2 \cdot 0.97^{23} \cdot \binom{50}{0} 0.03^0 \cdot 0.97^{50}$$

$$= 46.70\% + 7.87\% + 12.18\% + 2.92\% = 69.67\%$$

$$\mathbb{P}(E) = 1 - \mathbb{P}(E^C) = 1 - 69.67\% = 30.33\%$$

b)

$$\mathbb{P}(D|E) = \frac{\mathbb{P}(E \cap D)}{\mathbb{P}(E)} = \frac{\mathbb{P}(E|D)\mathbb{P}(D)}{\mathbb{P}(E)} = \frac{\mathbb{P}(\text{avvisa}|\xi_1 = 1)\mathbb{P}(\xi_1 = 1)}{0.3033}$$

$$= \frac{\mathbb{P}(\xi_1 + \xi_2 \geq 3|\xi_1 = 1)\mathbb{P}(\xi_1 = 1)}{0.3033} = \frac{\mathbb{P}(\xi_2 \geq 2)\binom{25}{1}0.03^1 \cdot 0.97^{24}}{0.3033}$$

$$= (1 - \mathbb{P}(\xi_2 \leq 1))\frac{0.3611}{0.3033} = \left(1 - \binom{50}{0} 0.03^0 \cdot 0.97^{50} - \binom{50}{1} 0.03^1 \cdot 0.97^{49}\right) \frac{0.3611}{0.3033}$$

$$= 0.4447 \frac{0.3611}{0.3033} = 52.95\%$$

c)

$\mathbb{E}[\xi] = 25 \cdot \mathbb{P}(\text{bestämmer oss i urval 1}) + 75 \cdot \mathbb{P}(\text{bestämmer oss i urval 2})$
$\mathbb{P}(\text{bestämmer oss i urval 1}) = \mathbb{P}(\xi_1 = 0 \cup \xi_1 \geq 3) = \mathbb{P}(\xi_1 = 0) + \mathbb{P}(\xi_1 \geq 3)$

$$= \mathbb{P}(\xi_1 = 0) + 1 - \mathbb{P}(\xi_1 \leq 2) = \mathbb{P}(\xi_1 = 0) + 1 - \sum_{i=0}^{2} \mathbb{P}(\xi_1 = i) = 1 - \mathbb{P}(\xi_1 = 1) - \mathbb{P}(\xi_1 = 2)$$

$$= 1 - \binom{25}{1} 0.03^1 \cdot 0.97^{24} - \binom{25}{2} 0.03^2 \cdot 0.97^{23} = 100\% - 36.11\% - 13.40\% = 50.49\%$$

$\mathbb{P}(\text{bestämmer oss i urval 2}) = 1 - 50.49\% = 49.51\%$
$\Rightarrow \mathbb{E}[\xi] = ASN(3\%) = 25 \cdot 0.5049 + 75 \cdot 0.4951 = 49.755$
$Var[\xi] = \mathbb{E}[\xi^2] - \mathbb{E}[\xi]^2 = 25^2 \cdot 0.5049 + 75^2 \cdot 0.4951 - 49.755^2 = 624.94$
$\Rightarrow \sqrt{Var[\xi]} = 24.99$

Fråga 5

a)

$$\mathbb{P}(\xi_1 \leq 95) = \{\xi \sim \mathbb{N}(\mu = 100, \sigma = 10)\} = F_z\left(\frac{95 - 100}{10}\right) = F_z(-0.5) = 30.85\%$$

b)

$\eta_1 = 100\xi_1 \sim \mathbb{N}(\mu, \sigma), \quad \mu = 100 \cdot 100 = 10^4, \quad \sigma = \sqrt{100^2 \cdot 100} = 10^3$
$$\mathbb{P}(\eta_1 \geq 12000) = 1 - F_z\left(\frac{12000 - 10000}{1000}\right) = 1 - F_z(2) = 1 - 0.9773 = 2.28\%$$

c)

$$\eta_2 = 50\xi_1 + 50\xi_2 \sim \mathbb{N}(\mu, \sigma), \quad \mu = 50 \cdot 100 + 50 \cdot 100 = 10^4, \quad \sigma = \sqrt{50^2 \cdot 100 + 50^2 \cdot 100}$$

$= 50\sqrt{200} \approx 707.11$
$$\mathbb{P}(\eta_2 \geq 12000) = 1 - F_z\left(\frac{12000 - 10000}{707.11}\right) = 1 - F_z(2.828) = 1 - 0.9977 = 0.23\%$$

d)

$$\eta_3 = \eta_1 - \eta_2 = 50\xi_1 - 50\xi_2 \sim \mathbb{N}(\mu, \sigma), \quad \mu = 50 \cdot 100 - 50 \cdot 100 = 0, \quad \sigma = \sqrt{50^2 \cdot 100 + 50^2 \cdot 100}$$

$$= 50\sqrt{200} \approx 707.11$$

$$\mathbb{P}(\eta_3 > 1000) = 1 - F_z\left(\frac{1000}{707.11}\right) = 1 - F_z\,(1.414) = 1 - 0.9213 = 7.87\%$$