LÖSNINGAR 20170110

1.

a)

$$E(\xi) = \int_{9,5}^{10,5} x \cdot 6(x-9,5)(10,5-x)\,dx = \ldots = \boxed{10}$$

$$E(\xi^2) = \int_{9,5}^{10,5} x^2\, 6(x-9,5)(10,5-x)\,dx = \ldots = 100,05$$

$$\Rightarrow Var(\xi) = E(\xi^2) - (E(\xi))^2 = 100,05 - 10^2 = 0,05$$

$$\Rightarrow S(\xi) = \sqrt{Var(\xi)} = \sqrt{0,05} = \boxed{0,224}$$

b) $P(9,8 \leq \xi \leq 10,1 \mid 9,9 \leq \xi \leq 10,2)$

$$= \frac{P(\{9,8 \leq \xi \leq 10,1\} \cap \{9,9 \leq \xi \leq 10,2\})}{P(9,9 \leq \xi \leq 10,2)} = \frac{P(9,9 \leq \xi \leq 10,1)}{P(9,9 \leq \xi \leq 10,2)}$$

$$= \frac{\int_{9,9}^{10,1} 6(x-9,5)(10,5-x)\,dx}{\int_{9,9}^{10,2} 6(x-9,5)(10,5-x)\,dx} = \ldots \approx \boxed{0,685}$$

c) $P(\xi \leq 10,1) = \int_{9,5}^{10,1} 6(x-9,5)(10,5-x)\,dx = 0,648$

$\Rightarrow \eta$ är $Bin(n=60,\ p=0,648)$

$np(1-p) = 60 \cdot 0,648 \cdot (1-0,648) = 13,686 > 10$

$\Rightarrow \eta$ är approx $N(np, \sqrt{np(1-p)}) = N(38,88,\ 3,699)$

$$\Rightarrow P(\eta \leq 43) = P\left(\frac{\eta - 38,88}{3,699} \leq \frac{43-38,88}{3,699}\right) \approx \Phi(1,11) \approx \boxed{0,87}$$

2.1

Skatta först σ med s

$$s = \sqrt{\frac{1}{6-1}\sum_{i=1}^{6}(x_i - \bar{x})^2} \quad \text{där} \quad \bar{x} = \frac{1}{6}(250{,}1 + \dots + 251{,}4)$$

$$= 249{,}967$$

$$\Rightarrow s = \sqrt{\frac{1}{5}\left((250{,}1 - 249{,}967)^2 + \dots + (251{,}4 - 249{,}967)^2\right)} \approx 1{,}42501$$

$$\chi^2_{0,95} = 1{.}145$$

$$\Rightarrow \text{Intervallet blir } \left[0, \sqrt{\frac{5 \cdot 1{.}42501^2}{1{.}145}}\right] = \boxed{[0,\ 2{,}978]}$$

Vi ser att 5 ligger utanför intervallets gränser. Det verkar alltså som att maskinen inte behöver undersökas.

3. Låt T = totala vikten.

$$T = \sum_{i=1}^{860} \xi_i \quad \text{där } \xi_i = \text{vikten för brev nr. } i.$$

$$\mu = E(\xi_i) = 25 \cdot 0{,}1 + 50 \cdot 0{,}5 + 75 \cdot 0{,}3 + 100 \cdot 0{,}1 = 60 \text{ gram}$$

$$E(\xi_i^2) = 25^2 \cdot 0{,}1 + 50^2 \cdot 0{,}5 + 75^2 \cdot 0{,}3 + 100^2 \cdot 0{,}1 = 4000$$

$$\Rightarrow \sigma^2 = E(\xi_i^2) - \left(E(\xi_i)\right)^2 = 4000 - 60^2 = 400$$

$$\Rightarrow \sigma = \sqrt{400} = 20 \text{ gram}$$

Centrala Gränsvärdes Satsen $\Rightarrow$ T är approx $N(860 \cdot 60, \sqrt{860} \cdot 20)$
$= N(51600, \ 586{,}5)$

$$P(\text{säcken går sönder}) = P(T > 52500) =$$

$$= 1 - P(T \leq 52500) = 1 - P\Bigg(\underbrace{\frac{T - 51600}{586{,}5}}_{=Z} \leq \frac{52500 - 51600}{586{,}5}\Bigg)$$

$$= 1 - P(Z \leq 1{,}53)$$

$$= 1 - \Phi(1{,}53) = 1 - 0{,}937 = \boxed{0{,}063}$$

Svar

4.

a) $P(E) = 1 - P(\overbrace{\text{partiet accepteras}}^{=E^c})$

$P(\text{partiet acc.}) = P(d_1=0) + P(d_1=1)P(d_2=0) + P(d_1=1)P(d_2=1) + P(d_1=2)P(d_2=0)$

$= \binom{25}{0} 0{,}03^0 0{,}97^{25} + \binom{25}{1} 0{,}03^1 0{,}97^{24} \binom{50}{0} 0{,}03^0 0{,}97^{50}$

$+ \binom{25}{1} 0{,}03^1 0{,}97^{24} \binom{50}{1} 0{,}03^1 0{,}97^{49} + \binom{25}{2} 0{,}03^2 0{,}97^{23} \binom{50}{0} 0{,}03^0 0{,}97^{50}$

$= 0{,}697 \Rightarrow P(E) = 1 - 0{,}697 = \boxed{0{,}303}$ (Approx ok eftersom $\frac{75}{10000} < 0{,}1$)

b) $P(D|E) = \frac{P(D \cap E)}{P(E)} = \frac{P(E) - P(D \cap E^c)}{P(E)} =$

$= \frac{0{,}303 - (P(d_1=1)P(d_2=0) + P(d_1=1)P(d_2=1))}{0{,}303}$

$= \ldots = \boxed{0{,}338}$

c) $P(\xi = 25) = P(\text{beslut efter urval } 1) =$

$= P(d_1=0) + P(d_1 \geq 3) = P(d_1=0) + 1 - P(d_1=1) - P(d_1=2)$

$= \binom{25}{0} 0{,}03^0 0{,}97^{25} + 1 - \binom{25}{1} 0{,}03 \cdot 0{,}97^{24} - \binom{25}{2} 0{,}03^2 0{,}97^{23}$

$= 0{,}972$

$P(\xi = 75) = 1 - 0{,}972 = 0{,}028$

$\Rightarrow AJN = 25 \cdot 0{,}972 + 75 \cdot 0{,}028 = \boxed{26{,}4}$ $(= E(\xi))$

$E(\xi^2) = 25^2 \cdot 0{,}972 + 75^2 \cdot 0{,}028 = 765$

$\Rightarrow Var(\xi) = E(\xi^2) - (E(\xi))^2 = 765 - 26{,}4^2 = \boxed{68{,}04}$

5.)

ξ_1 är $N(100, 10)$, ξ_2 är $N(100, 10)$

ξ_1 & ξ_2 är oberoende.

$\eta_1 = 100\xi_1$ $\qquad \eta_2 = 50\xi_1 + 50\xi_2$.

a) $P(\xi_1 \leq 95) = P\left(\frac{\xi_1 - 100}{10} \leq \frac{95-100}{10}\right) = \Phi\left(-\frac{1}{2}\right) =$

$= 1 - \Phi(1/2) = 1 - 0{,}6915 = \boxed{0{,}3085}$

b) $P(\eta_1 \geq 12000) = 1 - P(\eta_1 \leq 12000) = 1 - P(\xi_1 \leq 120)$

$= 1 - P\left(\frac{\xi_1 - 100}{10} \leq \frac{120-100}{10}\right) = 1 - \Phi(2) = 1 - 0{,}9772$

$= \boxed{0{,}0228}$

c) $P(\eta_2 \geq 12000) = P(50(\xi_1 + \xi_2) \geq 12000)$

$= P(\xi_1 + \xi_2 \geq 240) = 1 - P(\xi_1 + \xi_2 \leq 240)$

$= \left\{ \xi_1 + \xi_2 \text{ är } N(200, \sqrt{200}) \right\} = 1 - P\left(\frac{\xi_1 + \xi_2 - 200}{\sqrt{200}} \leq \frac{240-200}{\sqrt{200}}\right)$

$= 1 - \Phi\left(\frac{20}{\sqrt{200}}\right) = 1 - \Phi\left(\frac{20}{\sqrt{2}\cdot 10}\right) = 1 - \Phi\left(\frac{2}{\sqrt{2}}\right) = 1 - \Phi(\sqrt{2})$

$\approx 1 - \Phi(1{,}41421) = 1 - 0{,}9207 = \boxed{0{,}0793}$

d) P(Ivars värd mer än 1000kr mer än Ingrids portfölj)

$= P(\eta_1 - \eta_2 \geq 1000) = P(100\xi_1 - 50\xi_1 - 50\xi_2 \geq 1000)$

$= P(50\xi_1 - 50\xi_2 \geq 1000) = P(\xi_1 - \xi_2 \geq 20)$

$= \left\{ \begin{array}{l} E(\xi_1 - \xi_2) = 100 - 100 = 0 \\ Var(\xi_1 - \xi_2) = Var(\xi_1) + (-1)^2 Var(\xi_2) = Var(\xi_1) + Var(\xi_2) = 100 + 100 = 200 \\ \Rightarrow S(\xi_1 - \xi_2) = \sqrt{200} \Rightarrow \xi_1 - \xi_2 \text{ är } N(0, \sqrt{200}) \end{array} \right\}$

Fortsättning →

5 forts.) $= P\left(\frac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{200}} \geq \frac{20}{\sqrt{200}} \right) = 1 - \Phi(\sqrt{2}) =$

$= 1 - 0{,}9207 = \boxed{0{,}0793}$

6.) $P(A \cap C) \underset{\substack{\uparrow \\ \text{oberoende}}}{=} P(A)P(C) = 0{,}01 \cdot 0{,}02 = 0{,}0002$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$
$$= 0{,}01 + 0{,}01 + 0{,}02 - 0{,}003 - 0{,}0002 - 0{,}003 + 0$$
$$= \boxed{0{,}0338}$$

$$P(\xi = 0) = 1 - P(A \cup B \cup C) = 1 - 0{,}0338 = \boxed{0{,}9662}$$
$$P(\xi = 2) = P(A \cap B) + P(B \cap C) + P(A \cap C) =$$
$$= 0{,}003 + 0{,}003 + 0{,}0002 = \boxed{0{,}0062}$$
$$P(\xi = 1) = 1 - P(\xi = 0) - P(\xi = 2) = 1 - 0{,}9662 - 0{,}0062$$
$$= \boxed{0{,}0276}$$

b) $n = 50 \qquad p = 0{,}0338 \quad (= P(A \cup B \cup C))$

Låt d = antalet defekta.

$$P(d \leq 1) = P(d=0) + P(d=1) = \binom{50}{0} 0{,}0338^0 (1 - 0{,}0338)^{50} + \binom{50}{1} \cdot 0{,}0338^1 (1 - 0{,}0338)^{49}$$
$$= (1 - 0{,}0338)^{50} + 50 \cdot 0{,}0338 \cdot (1 - 0{,}0338)^{49}$$
$$\approx 0{,}179206 + 0{,}313452 = \boxed{0{,}492658}$$

$$\Longrightarrow P(\text{partiet avvisas}) = 1 - 0{,}492658 = \boxed{0{,}507342}$$

7)

a) Låt ξ_A = livslängden för komponent A, ξ_B = livsl. för B, ξ_C = livsl för C.

ξ_A är Exp(λ) där $\lambda = \frac{1}{1,5}$

$$P(\xi_A \geq 2) = \int_2^\infty \frac{1}{1,5} e^{-\frac{x}{1,5}} dx = \left[-e^{-\frac{x}{1,5}} \right]_2^\infty = e^{-\frac{2}{1,5}} \approx 0,264$$

Samma sak för ξ_B och ξ_C

P(systemet funkar efter 2 år)

$$= P\left(\{\xi_A \geq 2\} \cup \left(\{\xi_B \geq 2\} \cap \{\xi_C \geq 2\} \right) \right)$$

$$\underset{\substack{\uparrow \\ \text{Additionssatsen}}}{=} P(\xi_A \geq 2) + P(\{\xi_B \geq 2\} \cap \{\xi_C \geq 2\}) - P(\{\xi_A \geq 2\} \cap \{\xi_B \geq 2\} \cap \{\xi_C \geq 2\})$$

$$\underset{\substack{\uparrow \\ \text{oberoende}}}{=} P(\xi_A \geq 2) + P(\xi_B \geq 2) P(\xi_C \geq 2) - P(\xi_A \geq 2) P(\xi_B \geq 2) P(\xi_C \geq 2)$$

$$= 0,264 + 0,264^2 - 0,264^3 = \boxed{0.315}$$

b) Bestäm $P(\eta \leq x)$: (← fördelnings funktionen)

Vi har $\{\eta \geq x\} = \{\xi_A \geq x\} \cup (\{\xi_B \geq x\} \cap \{\xi_C \geq x\})$

På samma sätt som i a) fås att

$$P(\eta \geq x) = P(\xi_A \geq x) + P(\xi_B \geq x) P(\xi_C \geq x) - P(\xi_A \geq x) P(\xi_B \geq x) P(\xi_C \geq x)$$

$$= P(\xi_A \geq x) + P(\xi_A \geq x)^2 - P(\xi_A \geq x)^3$$

$$= \left\{ P(\xi_A \geq x) = \int_x^\infty \frac{1}{1,5} e^{-\frac{t}{1,5}} dx = \dots = e^{-\frac{x}{1,5}} \right\}$$

$$= e^{-\frac{x}{1,5}} + e^{-\frac{2x}{1,5}} - e^{-\frac{3x}{1,5}} \Rightarrow$$

Forts →

7.) forts $P(\eta \le x) = 1 - P(\eta \geq x) = \boxed{1 - e^{-\frac{x}{1,5}} - e^{-\frac{2x}{1,5}} + e^{-2x}}$

gäller om $x \geq 0$

Om $x < 0$ är farstis $P(\eta \le x) = 0$

Frekvensfunktionen $f(x) = F'(x)$

$$= \boxed{\frac{e^{-\frac{x}{1,5}}}{1,5} + \frac{2}{1,5} e^{-\frac{2x}{1,5}} - 2e^{-2x} \quad \text{för } x \geq 0}$$

och $f(x) = 0$ om $x < 0$.

8.

a) Teckentabell för CD:

+
+
+
+
−
−
−
−
−
−
−
−
+
+
+
+

$$\Rightarrow l_{CD} = \frac{7{,}0+7{,}1+8{,}2+8{,}3+8{,}4+8{,}3+9{,}7+9{,}8}{8} - \frac{(8{,}6+8{,}4+9{,}6+9{,}8+6{,}8+6{,}5+8{,}0+8{,}3)}{8} = \boxed{0{,}1}$$

b) Välj till exempel

$E = ABC \quad F = BCD \quad G = ACD \quad H = ABCD$

$\Rightarrow I_1 = ABCE \quad I_2 = BCDF \quad I_3 = ACDG \quad I_4 = ABCDH$

$I_5 = I_1 I_2 = A\cancel{BC}E\cancel{BC}DF = AEDF \quad I_6 = I_1 I_3 = \cancel{A}B\cancel{C}E\cancel{AC}DG = BDEG$

$I_7 = I_1 I_4 = \cancel{ABC}E\cancel{ABC}DH = DEH \quad I_8 = I_2 I_3 = B\cancel{CD}F A\cancel{CD}G = ABFG$

$I_9 = I_2 I_4 = \cancel{BCD}F A\cancel{BCD}H = AFH \quad I_{10} = I_3 I_4 = \cancel{ACD}G\cancel{A}B\cancel{CD}H = BGH$

$I_{11} = I_1 I_2 I_3 = \cancel{AB}\cancel{C}E\cancel{BCD}F\cancel{A}C\cancel{D}G = CEFG$

$I_{12} = I_1 I_2 I_4 = \cancel{ABC}E\cancel{BCD}F\cancel{AB}C\cancel{D}H = BCEFH$

$I_{13} = I_1 I_3 I_4 = \cancel{ABC}E\cancel{ACD}G A\cancel{B}C\cancel{D}H = ACEH$

$I_{14} = I_2 I_3 I_4 = \cancel{BCD}F\cancel{ACD}G\cancel{AB}CDH = CDFGH$

$I_{15} = I_1 I_2 I_3 I_4 = \cancel{ABC}E\cancel{BC}DF\cancel{AC}DG AB\cancel{CD}H = ABDEFGH$

Kortaste ordet har längd 3 $\Rightarrow$ upplösningen = III.