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1. a) $E(\xi) = \int_0^{3/2} x \frac{4}{9}(3x - x^2)\,dx = \frac{4}{9}\int_0^{3/2} (3x^2 - x^3)\,dx =$

$= \frac{4}{9}\left[3x^3/3 - x^4/4\right]_0^{3/2} = \frac{4}{9}\left(\frac{81}{8\cdot 3} - \frac{81}{16\cdot 4}\right) = \ldots = \frac{15}{16} \approx \boxed{0{,}9375}$

$E(\xi^2) = \int_0^{3/2} x^2 \frac{4}{9}(3x - x^2)\,dx = \frac{4}{9}\left[\frac{3x^4}{4} - \frac{x^5}{5}\right]_0^{3/2} = \frac{4}{9}\left(\frac{3\cdot 81}{4\cdot 16} - \frac{243}{5\cdot 32}\right)$

$= \ldots = \frac{81}{80} \approx 1{,}0125$

$\Rightarrow \mathrm{Var}(\xi) = E(\xi^2) - (E(\xi))^2 \approx 1{,}0125 - 0{,}9375^2 \approx \boxed{0{,}1336}$

b) $P(\xi \leq 1) = \int_0^1 \frac{4}{9}(3x - x^2)\,dx = \frac{4}{9}\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{4}{9}\left(\frac{3}{2} - \frac{1}{3}\right) =$

$= \frac{4}{9}\cdot\left(\frac{9}{6} - \frac{2}{6}\right) = \frac{4}{9}\cdot\frac{7}{6} = \frac{28}{54} = \frac{14}{27} \approx \boxed{0{,}5185}$

$P(\xi > 0{,}5) = \int_{0.5}^{1.5} \frac{4}{9}(3x - x^2)\,dx = \frac{4}{9}\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_{1/2}^{3/2} =$

$= \frac{4}{9}\left(\frac{27}{8} - \frac{27}{24} - \frac{3}{8} + \frac{1}{24}\right) = \frac{4}{9}\left(\frac{24}{8} - \frac{26}{24}\right) = \ldots = \frac{23}{27} \approx \boxed{0{,}8519}$

c) $P(\xi \leq 1 \mid \xi > 0{,}5) = \frac{P(0{,}5 < \xi \leq 1)}{P(\xi > 0{,}5)} = \frac{P(\xi \leq 1) - P(\xi \leq 0{,}5)}{P(\xi > 0{,}5)} =$

$= \frac{P(\xi \leq 1) - (1 - P(\xi > 0{,}5))}{P(\xi > 0{,}5)} \underset{b}{=} \frac{0{,}5185 - (1 - 0{,}8519)}{0{,}8519} \approx \boxed{0{,}4348}$

2. Låt ξ_i = antal paket dag i $\quad (i = 1, \ldots, 365)$

$$\mu = E(\xi_i) = 2 \cdot 0{,}9 + 1 \cdot 0{,}1 = 1{,}9$$

$$E(\xi_i^2) = 4 \cdot 0{,}9 + 1 \cdot 0{,}1 = 3{,}6 + 0{,}1 = 3{,}7$$

$$\Rightarrow Var(\xi_i) = E(\xi_i^2) - (E(\xi_i))^2 = 3{,}7 - 1{,}9^2 = 0{,}09$$

$$\Rightarrow \sigma = \sqrt{0{,}09} \approx 0{,}3$$

CGS $\Rightarrow T = \sum_{i=1}^{365} \xi_i$ approximativt $N(365 \cdot 1{,}9, \sqrt{365} \cdot 0{,}3)$

antalet paket under 1 år. $\qquad = N(693{,}5,\ 5{,}73)$

$$P(T \geq 712) = P\left(\frac{T - 693{,}5}{5{,}73} \geq \frac{712 - 693{,}5}{5{,}73} \right)$$

$$\approx 1 - \Phi(3{,}23) = 1 - 0{,}9994 = \boxed{0{,}0006}$$

3a) $\bar{x} = \dfrac{0{,}56 + \ldots + 0{,}62}{5} = 0{,}6$. $\qquad \alpha = 0{,}05$

$$s^2 = \frac{1}{5-1} \cdot \left((0{,}56 - 0{,}6)^2 + \ldots + (0{,}62 - 0{,}6)^2 \right) = 0{,}00075$$

$s = \sqrt{0{,}00075} = 0{,}0274$. $\quad$ Frihetsgrader $= 5 - 1 = 4$.

95% konf int $= \bar{x} \pm t_{0{,}025}(4) \cdot \dfrac{s}{\sqrt{n}} = 0{,}6 \pm \dfrac{2{,}78 \cdot 0{,}0274}{\sqrt{5}}$

$= \boxed{0{,}6 \pm 0{,}034}$

b) $s = 0{,}0274$. $\quad \chi^2_{0{,}005} = 14{,}86$

$\alpha = 0{,}01 \quad \chi^2_{0{,}995} = 0{,}2070$

99% konf int: $\left[\sqrt{\dfrac{(n-1)s^2}{\chi^2_{\alpha/2}}}, \sqrt{\dfrac{(n-1)s^2}{\chi^2_{1-\alpha/2}}} \right] = \left[\dfrac{2 \cdot 0{,}0274}{\sqrt{14{,}86}}, \dfrac{2 \cdot 0{,}0274}{\sqrt{0{,}2070}} \right]$

$\approx \boxed{[0{,}014,\ 0{,}120]}$

4.) Låt A_i = bro i hel. $(i = 1, 2, 3, 4)$.

a) $P\left(\begin{matrix}\text{vandraren} \\ \text{kan passera bägge floderna}\end{matrix}\right) = P\left((A_1 \cup A_2) \cap (A_3 \cup A_4)\right)$

$\underset{\text{ober.}}{=} P(A_1 \cup A_2) \cdot P(A_3 \cup A_4) = \left(P(A_1) + P(A_2) - P(A_1 \cap A_2)\right)\left(P(A_3) + P(A_4) - P(A_3 \cap A_4)\right)$

$\underset{\text{ober}}{=} (0{,}9 + 0{,}9 - 0{,}9^2)(0{,}9 + 0{,}9 - 0{,}9^2) = 0{,}99^2 = \boxed{0{,}9801}$

b) Låt $B = \{\overset{\text{man}}{\text{kan}} \text{ passera flod 1}\}$. $\left(\begin{matrix} B = A_1 \cup A_2 \\ D = A_3 \cup A_4 \end{matrix}\right)$

$C = \{\overset{\text{man}}{\text{kan}} \text{ passera bägge}\}$. $D = \{\text{kan passera flod 2}\}$

Söker $P(B \mid C^c)$.

Vi har $P(B \mid C^c) = \dfrac{P(B \cap C^c)}{P(C^c)}$.

$P(B \cap C^c) = P(B) - P(B \cap C) \underset{P(B) = 0{,}99 \text{ enligt a)}}{=} 0{,}99 - P(B \cap C) \ldots$

$\underset{B \cap C = B \cap D}{=} 0{,}99 - P(B \cap D) \underset{\text{oberoende}}{=} 0{,}99 - P(B)P(D) = 0{,}99 - 0{,}99^2 = 0{,}0099$

$P(C^c) = 1 - P(C) \underset{\text{se a}}{=} 1 - 0{,}9801 = 0{,}0199$

$\Longrightarrow P(B \mid C^c) = \dfrac{0{,}0099}{0{,}0199} \approx \boxed{0{,}4975}$

5.) Låt ξ_i = fågel i:s avstånd till S_1 $(i=1,2,3)$

a) $P(\xi_1 \leq 3) = \int_0^3 \frac{1}{20}\,dx = \boxed{\frac{3}{20}}$

b) $P(\xi_1 \leq 3,\ \xi_2 \leq 3,\ \xi_3 > 17) = P(\xi_1 \leq 3)P(\xi_2 \leq 3)P(\xi_3 > 17)$

$= \left(\frac{3}{20}\right)^2 \cdot \int_{17}^{20} \frac{1}{20}\,dx = \left(\frac{3}{20}\right)^2 \cdot \left(\frac{20-17}{20}\right) = \left(\frac{3}{20}\right)^3 = \boxed{0{,}003375}$

c) P(avståndet mellan S_1 och F_1 åtminstone dubbelt så stort som avståndet mellan S_2 och F_1)

$= P\left(\xi_1 \geq \frac{2}{3}\cdot 20\right) = P\left(\xi_1 \geq \frac{40}{3}\right) = 1 - P\left(\xi_1 < \frac{40}{3}\right)$

$= 1 - \frac{40}{3\cdot 20} = 1 - \frac{2}{3} = \boxed{\frac{1}{3}}$

d) η = antal fåglar på avstånd ≤ 3 från S_1.

η är $\mathrm{Bin}\left(n=3,\ p=\frac{3}{20}\right)$

$P(\eta=2) = \binom{3}{2}\cdot\left(\frac{3}{20}\right)^2\left(1-\frac{3}{20}\right) = \frac{3!}{2!\cdot 1!}\cdot\frac{9\cdot 17}{20^3} =$

$= \frac{3\cdot 19\cdot 17}{20^3} \approx \boxed{0{,}057375}$

6. a)

$$l_A = \frac{(55+75+53+79)}{4} - \frac{(53+77+54+70)}{4} = \frac{8}{4} = \boxed{2}$$

$$l_B = \frac{(77+75+70+79)}{4} - \frac{(53+55+54+53)}{4} = \frac{86}{4} = \boxed{21.5}$$

$$l_C = \frac{54+53+70+79}{4} - \frac{(53+55+77+75)}{4} = \boxed{-1}$$

Teckenkolumner för AC, AB och ABC:

	AC	AB	ABC
1	+	+	−
2	−	−	+
3	+	−	+
4	−	+	−
5	−	+	+
6	+	−	−
7	−	−	−
8	+	+	+

$$l_{AC} = \frac{53+77+53+79}{4} - \frac{(55+75+54+70)}{4} = \frac{8}{4} = \boxed{2}$$

$$l_{AB} = \frac{53+75+54+79}{4} - \frac{(55+77+53+70)}{4} = \frac{6}{4} = \boxed{1.5}$$

$$l_{ABC} = \frac{55+77+54+79}{4} - \frac{(53+75+53+70)}{4} = \frac{14}{4} = \boxed{3.5}$$

b) $D = ABC \quad E = BC \quad F = AC$

$I_1 = DABC = ABCD$

$I_2 = EBC = BCE$

$I_3 = FAC = ACF$

$I_4 = I_1 I_2 = A\cancel{B}\cancel{C}D\cancel{B}\cancel{C}E = ADE$

$I_5 = I_2 I_3 = B\cancel{C}EA\cancel{C}F = ABEF$

$I_6 = I_1 I_3 = \cancel{A}B\cancel{C}D\cancel{A}\cancel{C}F = BDF$

$I_7 = I_1 I_2 I_3 = \cancel{A}\cancel{B}\cancel{C}D\cancel{B}\cancel{C}E\cancel{A}CF = CDEF$

Längden på kortaste ordet $= 3 \Longrightarrow$
upplösningen $= \boxed{3}$

Alias till A:
$AI_1 = \cancel{A}\cancel{A}BCD = \boxed{BCD}$

$AI_2 = \boxed{ABCE}$

$AI_3 = AACF = \boxed{CF}$

$AI_4 = AADE = \boxed{DE}$

$AI_5 = AABEF = \boxed{BEF}$

$AI_6 = \boxed{ABDF}$

$AI_7 = \boxed{ACDEF}$

7.) a) Grupp 19: $\bar{x} = \frac{6,5+6,3+6,1}{3} = 6,3$

$R = 6,5 - 6,1 = 0,4$

Grupp 20: $\bar{x} = \frac{7,0+7,2+6,8}{3} = 7,0$

$R = 7,2 - 6,8 = 0,4$

b) $\bar{\bar{x}} = \frac{1}{20}(7,0+6,9+\ldots+7,0) \approx 6,75$

Om man räknar med det som stod i texten. Om man istället tar $\bar{x} = 6.9$ i grupp 18 blir det $\bar{\bar{x}} = 6,755$

$\bar{R} = \frac{1}{20}(0,3+0,5+\ldots+0,4) \approx 0,345$

$\bar{X}$-diagram: $K_n = \bar{\bar{x}} - A_2\bar{R} = 6,75 - 1,023 \cdot 0,345 = 6,39707$

$K_ö = \bar{\bar{x}} + A_2\bar{R} = 6,75 + 1,023 \cdot 0,345 = 7,10294$

Vi ser att exempelvis provgrupp 7 har $\bar{x} = 6,3$ som är lägre än K_n. Alltså är processen ej i statistisk kontroll. Samma slutsats blir det ifall vi räknar med $\bar{x} = 6,9$ i provgrupp 18.

R-diagrammet: $K_n = D_3\bar{R} = 0 \cdot 0,345 = 0$

$K_ö = D_4\bar{R} = 2,575 \cdot 0,345 = 0,88838$

8.) Låt d_1 = antalet defekter i urval 1

d_2 = ——— " ——— 2

Accepterar efter urval 1 om $d_1 = 0$ eller $d_1 = 1$

$P(d_1 = 0 \text{ eller } d_1 = 1) = P(d_1 = 0) + P(d_1 = 1) =$

$\approx \binom{20}{0} \cdot 0{,}01^0 \cdot 0{,}99^{20} + \binom{20}{1} \cdot 0{,}01^1 \cdot 0{,}99^{19}$

använd bin approx

$= 0{,}99^{20} + 20 \cdot 0{,}01 \cdot 0{,}99^{19} \approx 0{,}818 + 0{,}165 \approx 0{,}983$

Dvs $A_1 \approx 0{,}983$

Accepterar efter urval 2 om $d_1 = 2$ och $d_2 = 0$ eller

$d_1 = 2$ och $d_2 = 1$ eller

$d_1 = 3$ och $d_2 = 0$

$P(d_1 = 2 \text{ och } d_2 = 0) \approx P(d_1 = 2) \cdot P(d_2 = 0) \approx$

approx. ober.

$\binom{20}{2} \cdot 0{,}01^2 \cdot 0{,}99^{18} \cdot \binom{40}{0} \cdot 0{,}01^0 \cdot 0{,}99^{40} \approx \ldots \approx 0{,}0106$

$P(d_1 = 2 \text{ och } d_2 = 1) = P(d_1 = 2)\, P(d_2 = 1) \approx$

$\binom{20}{2} \cdot 0{,}01^2 \cdot 0{,}99^{18} \cdot \binom{40}{1} \cdot 0{,}01^1 \cdot 0{,}99^{39} \approx 0{,}0043$

$P(d_1 = 3 \text{ och } d_2 = 0) \approx P(d_1 = 3)\, P(d_2 = 0)$

$= \binom{20}{3} 0{,}01^3 \cdot 0{,}99^{17} \cdot \binom{40}{0} \cdot 0{,}01^0 \cdot 0{,}99^{40} \approx 0{,}0006$

Dvs $A_2 \approx 0{,}0106 + 0{,}0043 + 0{,}0006 \approx 0{,}0155$

$ATI(0{,}01) = 20 \cdot 0{,}983 + 60 \cdot 0{,}0155 + 1000(1 - 0{,}983 - 0{,}0155) \approx 22{,}09$ →

9.) Eftersom $P(-1 \leq \xi \leq 1) = 1$ gäller att

$$P(0 \leq \eta \leq 1) = 1.$$

Alltså är $P(\eta \leq x) = 0$ om $x < 0$ och

$$P(\eta \leq x) = 1 \quad \text{om} \quad x > 1.$$

Om $0 \leq x \leq 1$ fås

$$P(\eta \leq x) = P(\xi^2 \leq x) = P(|\xi| \leq \sqrt{x}) =$$

$$= P(-\sqrt{x} \leq \xi \leq \sqrt{x}) = P(\xi \leq \sqrt{x}) - P(\xi \leq -\sqrt{x})$$

$$= \frac{1+\sqrt{x}}{2} - \frac{(1-\sqrt{x})}{2} = \frac{1}{2} + \frac{\sqrt{x}}{2} - \frac{1}{2} + \frac{\sqrt{x}}{2} = \sqrt{x}$$

Svar:
$$\boxed{P(\eta \leq x) = \begin{cases} 0 & \text{då } x < 0 \\ \sqrt{x} & \text{då } 0 \leq x \leq 1 \\ 1 & \text{då } x > 1 \end{cases}}$$