EXAM: Matematisk statistik och diskret matematik (MVE051/MSG810). Statistik för fysiker (MSG820).
Time and place: Thursday 19 December 2013, 14:00–18:00, Väg och vatten.
Jour: Malin Palö, 031-772 5379.
Allowed help: Chalmers-approved calculator, Swedish-English dictionary and Beta handbook.
Grades: Chalmers: 3: 12 points, 4: 18 points, 5: 24 points. GU: G: 12 points, VG: 21 points. Maximal amount of points is 30.

Good luck!

1. (2p) Assume that X and Y are independent random variables and that $\mathbf{E}[X] = \mathbf{E}[Y] = \mathbf{Var}[X] = \mathbf{Var}[Y] = 0.5$. Choose the correct statement, and motivate your choice:
   (a) $\mathbf{E}[2 * X + 2 * Y] = 2, \mathbf{Var}[2 * X - 2 * Y] = 0$,
   (b) $\mathbf{E}[2 * X - 2 * Y] = 0, \mathbf{Var}[2 * X + 2 * Y] = 2$,
   (c) $\mathbf{E}[2 * X + 2 * Y] = 2, \mathbf{Var}[2 * X - 2 * Y] = 4$,
   (d) none of the above is correct.

2. (2p) Assume that 100 people have answered Problem 1 independently, each choosing one of the four possible options uniformly at random, so that every option has the same probability to be chosen. What can you say about the distribution of X, the total number of people, who guessed the correct answer? Choose the two correct statements (there are exactly two):
   (a) X is Binomially distributed with parameters $n = 100$, $p = 0.25$.
   (b) X is Binomially distributed with parameters $n = 4$, $p = 0.5$.
   (c) X is Binomially distributed with parameters $n = 100$, $p = 0.5$
   (d) X is Binomially distributed with some other parameters.
   (e) The distribution of X can be approximated by a Normal distribution with parameters $\mu = 100$, $\sigma = 0.25$
   (f) The distribution of X can be approximated by a Normal distribution with parameters $\mu = 25$, $\sigma^2 = 18.75$
   (g) The distribution of X can be approximated by a Normal distribution with parameters $\mu = 50$, $\sigma^2 = 25$
   (h) The distribution of X can be approximated by a Normal distribution with some other parameters.

3. (2p) Find the probability of guessing a correct answer for a Problem 2, if one chooses the answer uniformly at random among all possible combinations of two options. (Hint: use the classical definition of probability)

4. (4p) Assume that 100 students answer the Problem 1 independently of each other, and that each has the same probability of getting a correct answer. Denote that probability by p. Out of curiosity, Anton wants to test a hypothesis $H_0 : p = 0.25$, corresponding to the situation where everybody attempts to guess the answer, choosing one of the oprions uniformly at random, against the alternative $H_1 : p > 0.25$.

a) Anton uses the test statistic $\hat{p}$. Find the critical region for it on the significance levels $\alpha = 0.05$ and $\alpha = 0.01$.
b) Assume that out of 100 students, 34 have answered correctly. Find the results of the hypothesis test on $\alpha = 0.05$ and $\alpha = 0.01$. What is the p-value of the corresponding tests?

5. (3p) Assume the total proportion of the students that fail the exam is p. Anton starts grading the exams one by one, works until he grades the first failed exam, then goes for a coffee.

a) What is the distribution of the number of works Anton is going to grade before his first cup of coffee? (you can assume that different students' results are independent of each other, and that the total ammount of exams to grade is absurdly immense, or even infinite)
b) Find the probability (write a formula) that Anton will grade at least 3 works before having a coffee.

6. (5p) Assume that the time Y needed to finish all the 9 questions of the exam is distributed normally, with parameters $\mu = 3.5$ hr, $\sigma = 1$ hr.

a) If 4 hours are given to complete the exam, what is the probability to be on time?
b) Again, assume $n = 100$ people are taking the exam. Given the probability p from part 'a)', what is the exact distribution of X, the number of students who finish on time? (Hint: let 'finishing on time' correspond to the 'success' in the series of 100 independent experiments)
c) If X denotes the number of people who finish on time, find $\mathbf{P}(X > 50)$. (Hint: use the Normal approximation)

7. (4p) Beth is not sure about the answer for the Problem 1. Her mind goes wondering according to a Markov chain (X_n), starting in $X_0 =$ 'a)', governed by the following transition matrix:

$$A = \begin{array}{c} \\ \text{a)} \\ \text{b)} \\ \text{c)} \\ \text{d)} \end{array} \begin{array}{c} \begin{array}{cccc} \text{a)} & \text{b)} & \text{c)} & \text{d)} \end{array} \\ \begin{pmatrix} 0.5 & 0.25 & 0 & 0.25 \\ 0 & 0.5 & 0.25 & 0.25 \\ 0.25 & 0 & 0.5 & 0.25 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

Beth's mind wonders for a little while (2 steps of the Markov chain), and the answer she chooses is then given by X_2, the state of the Markov chain after 2 steps.

a) Sketch a state diagram of the corresponding Markov chain.
b) Which answer will Beth pick with the maximal probability? (Hint: find the distribution vector of a MC's state after two steps, pick the state with the maximal probability)
c) Which answer would Beth pick eventually, if her mind was given an infinite amount of time to wonder (that is, if the corresponding Markov chain made a very large number of steps), and why?

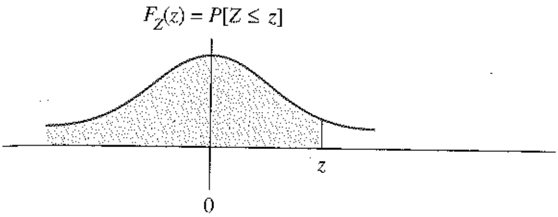
8. (4p) Assume that p is the total proportion of statistical problems that Beth can solve.

   a) How many questions does the final exam have to contain in order to build a 95% confidence interval for that proportion of length at most 0.2? You can assume that Beth solves different questions independently of each other, each with probability p.
   b) Would that number increase or decrease if we had the prior estimate of p?

9. (4p) Alice, Bob, Claire and Dean decide to cooperate, dividing the 9 questions between themselves. Alice wants to get between 3 and 9 questions, Bob wants to get between 0 and 4, and Clair and Dean have no preferences whatsoever.

   a) Denote by y_1, y_2, y_3, y_4 the respective number of questions each of the four students gets, and write the corresponding diophantine equation with the constraints.
   b) Write the generating function for that combinatorical problem.
   c) How many ways is there to make a division so that everyone is happy?

TABLE V
Cumulative distribution: Standard normal



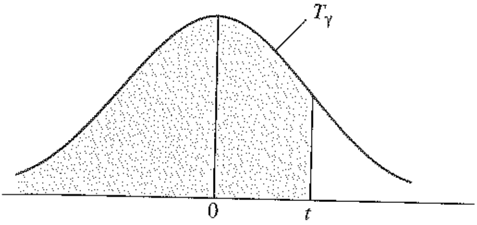
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
	$F_Z(z) = P[Z \le z]$									
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1921	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

TABLE V

Cumulative distribution: Standard normal *(concluded)*

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

TABLE VI
T distribution



Column heading = cumulative probability
Row heading = degrees of freedom
Row ∞ = standard normal values

$P[T_\gamma \le t]$

γ	.6	.75	.9	.95	.975	.99	.995	.999	.9995
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	318.317	636.607
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	22.327	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.611	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.385	3.646
31	0.256	0.682	1.309	1.696	2.040	2.453	2.744	3.375	3.633
32	0.255	0.682	1.309	1.694	2.037	2.449	2.738	3.365	3.622
33	0.255	0.682	1.308	1.692	2.035	2.445	2.733	3.356	3.611
34	0.255	0.682	1.307	1.691	2.032	2.441	2.728	3.348	3.601
35	0.255	0.682	1.306	1.690	2.030	2.438	2.724	3.340	3.591
36	0.255	0.681	1.306	1.688	2.028	2.434	2.719	3.333	3.582
37	0.255	0.681	1.305	1.687	2.026	2.431	2.715	3.326	3.574
38	0.255	0.681	1.304	1.686	2.024	2.429	2.712	3.319	3.566
39	0.255	0.681	1.304	1.685	2.023	2.426	2.708	3.313	3.558
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	3.307	3.551
41	0.255	0.681	1.303	1.683	2.020	2.421	2.701	3.301	3.544
42	0.255	0.680	1.302	1.682	2.018	2.418	2.698	3.296	3.538
43	0.255	0.680	1.302	1.681	2.017	2.416	2.695	3.291	3.532
44	0.255	0.680	1.301	1.680	2.015	2.414	2.692	3.286	3.526

TABLE VI
T* distribution *(concluded)

γ	.6	.75	.9	.95	.975	.99	.995	.999	.9995
45	0.255	0.680	1.301	1.679	2.014	2.412	2.690	3.281	3.520
46	0.255	0.680	1.300	1.679	2.013	2.410	2.687	3.277	3.515
47	0.255	0.680	1.300	1.678	2.012	2.408	2.685	3.273	3.510
48	0.255	0.680	1.299	1.677	2.011	2.407	2.682	3.269	3.505
49	0.255	0.680	1.299	1.677	2.010	2.405	2.680	3.265	3.500
50	0.255	0.679	1.299	1.676	2.009	2.403	2.678	3.261	3.496
51	0.255	0.679	1.298	1.675	2.008	2.402	2.676	3.258	3.492
52	0.255	0.679	1.298	1.675	2.007	2.400	2.674	3.255	3.488
53	0.255	0.679	1.298	1.674	2.006	2.399	2.672	3.251	3.484
54	0.255	0.679	1.297	1.674	2.005	2.397	2.670	3.248	3.480
55	0.255	0.679	1.297	1.673	2.004	2.396	2.668	3.245	3.476
56	0.255	0.679	1.297	1.673	2.003	2.395	2.667	3.242	3.473
57	0.255	0.679	1.297	1.672	2.002	2.394	2.665	3.239	3.470
58	0.255	0.679	1.296	1.672	2.002	2.392	2.663	3.237	3.466
59	0.254	0.679	1.296	1.671	2.001	2.391	2.662	3.234	3.463
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	3.232	3.460
61	0.254	0.679	1.296	1.670	2.000	2.389	2.659	3.229	3.457
62	0.254	0.678	1.295	1.670	1.999	2.388	2.658	3.227	3.455
63	0.254	0.678	1.295	1.669	1.998	2.387	2.656	3.225	3.452
64	0.254	0.678	1.295	1.669	1.998	2.386	2.655	3.223	3.449
65	0.254	0.678	1.295	1.669	1.997	2.385	2.654	3.221	3.447
66	0.254	0.678	1.295	1.668	1.997	2.384	2.652	3.218	3.444
67	0.254	0.678	1.294	1.668	1.996	2.383	2.651	3.217	3.442
68	0.254	0.678	1.294	1.668	1.995	2.382	2.650	3.215	3.440
69	0.254	0.678	1.294	1.667	1.995	2.382	2.649	3.213	3.437
70	0.254	0.678	1.294	1.667	1.994	2.381	2.648	3.211	3.435
71	0.254	0.678	1.294	1.667	1.994	2.380	2.647	3.209	3.433
72	0.254	0.678	1.293	1.666	1.993	2.379	2.646	3.207	3.431
73	0.254	0.678	1.293	1.666	1.993	2.379	2.645	3.206	3.429
74	0.254	0.678	1.293	1.666	1.993	2.378	2.644	3.204	3.427
75	0.254	0.678	1.293	1.665	1.992	2.377	2.643	3.203	3.425
76	0.254	0.678	1.293	1.665	1.992	2.376	2.642	3.201	3.423
77	0.254	0.678	1.293	1.665	1.991	2.376	2.641	3.200	3.422
78	0.254	0.678	1.292	1.665	1.991	2.375	2.640	3.198	3.420
79	0.254	0.678	1.292	1.664	1.990	2.375	2.640	3.197	3.418
80	0.254	0.678	1.292	1.664	1.990	2.374	2.639	3.195	3.416
81	0.254	0.678	1.292	1.664	1.990	2.373	2.638	3.194	3.415
82	0.254	0.677	1.292	1.664	1.989	2.373	2.637	3.193	3.413
83	0.254	0.677	1.292	1.663	1.989	2.372	2.636	3.191	3.412
84	0.254	0.677	1.292	1.663	1.989	2.372	2.636	3.190	3.410
85	0.254	0.677	1.292	1.663	1.988	2.371	2.635	3.189	3.409
86	0.254	0.677	1.291	1.663	1.988	2.371	2.634	3.188	3.407
87	0.254	0.677	1.291	1.663	1.988	2.370	2.634	3.187	3.406
88	0.254	0.677	1.291	1.662	1.987	2.369	2.633	3.186	3.405
89	0.254	0.677	1.291	1.662	1.987	2.369	2.632	3.184	3.403
90	0.254	0.677	1.291	1.662	1.987	2.369	2.632	3.183	3.402
91	0.254	0.677	1.291	1.662	1.986	2.368	2.631	3.182	3.401
92	0.254	0.677	1.291	1.662	1.986	2.368	2.630	3.181	3.400
93	0.254	0.677	1.291	1.661	1.986	2.367	2.630	3.180	3.398
94	0.254	0.677	1.291	1.661	1.986	2.367	2.629	3.179	3.397
95	0.254	0.677	1.291	1.661	1.985	2.366	2.629	3.178	3.396
96	0.254	0.677	1.290	1.661	1.985	2.366	2.628	3.177	3.395
97	0.254	0.677	1.290	1.661	1.985	2.365	2.627	3.176	3.394
98	0.254	0.677	1.290	1.661	1.984	2.365	2.627	3.176	3.393
99	0.254	0.677	1.290	1.660	1.984	2.365	2.626	3.175	3.392
100	0.254	0.677	1.290	1.660	1.984	2.364	2.626	3.174	3.391
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	3.090	3.291