EXAM: Matematisk statistik och diskret matematik (MVE050/MVE055/MSG810). Statistik för fysiker (MSG820).
Time and place: Saturday 15 December 2012, 08.30–12.30, Väg och vatten.
Jour: Anton Muratov, tel. 072 027 1500.
Allowed help: Chalmers-approved calculator, Swedish-English dictionary and Beta handbook.
Grades: Chalmers: 3: 12 points, 4: 18 points, 5: 24 points. GU: G: 12 points, VG: 21 points. Maximal amount of points is 30.

Good luck!

1. (3p) The joint distribution of the discrete random variables X and Y is given by the density table:

$x\backslash y$	0	1
1	0.4	0.2
2	0.1	0.3

   a) Find $\mathbf{E}[X]$, $\mathbf{E}[Y]$, $\mathbf{E}[XY]$
   b) Find $\mathbf{Cov}(X, Y)$
   c) Find $\mathbf{E}[5X - Y + 17]$

2. (3p) Draw a card at random from a standard 52-card deck. Denote $A = \{\text{draw a nine}\}$, $B = \{\text{draw a diamond}\}$, $C = \{\text{draw a number}\}$.

   a) Are the events B and C independent? A and C? Why?
   b) Find $\mathbf{P}(A \cap B|C)$.
   c) Remove one card (the Ace of spades) from the deck. Are A and B independent now? Why?

3. (2p) The continuous random variable X has a density $f_X(x)$:

$$f_X(x) = \begin{cases} \frac{x}{a}, & 20 < x < 40, \\ 0, & \text{otherwise.} \end{cases}$$

   a) Find the value of a so that $f_X(x)$ becomes a proper density function.
   b) Using the obtained value of a, find the probability $\mathbf{P}(X < 25)$.

4. (4p) Assume the total proportion p of red-haired people in Sweden is 0.015. The dentist examines $n = 10$ people every day, which you can assume to be independent of each other.

   a) What is the probability for the dentist to meet at least 2 red-haired individuums during the same day?
   b) What is the probability to examine at least 1 red-haired person every day during the whole working week (5 days)?
   c) What is the expected number of days a new dentist has to work before meeting the first red-haired person?

5. (3p) Bad thoughts are popping up in Peter's head according to a Poisson process with intensity $\lambda = 2$ [tph] (thoughts per hour). When bad thoughts are there, Peter becomes uneasy and can not eat.

   a) It takes Peter 40 minutes to eat his lunch. What's the probability for him to eat the lunch in peace, without any bad thoughts?
   b) If Peter wakes up 6:30 and has a breakfast at 7, what's the probability for him to get at least 3 bad thoughts before breakfast?

6. (3p) Find a generating function $A(x)$ of the sequence (a_n), defined by the following recursion:

$$\begin{cases} a_0 = 2, \\ a_1 = 2, \\ a_n = a_{n-1} + a_{n-2} - 1, \quad n = 2, 3, \ldots \end{cases}$$

   Note: you don't have to find the (a_n) itself, just the generating function.

7. (4p) Stefan is on a quest to find out the average IQ of a seagull. Omitting details about how he measures the IQ of his involuntary subjects, we present his data:

$$7.5 \quad 7.0 \quad 8.5 \quad 4.0 \quad 6.0 \quad 7.0 \quad 5.5$$

   a) Help Stefan construct the 95% confidence interval for μ – the actual average IQ of a seagull.
   b) What's the length of your interval? Based on your estimate for the actual variance, how many seagulls does Stefan have to catch for his 95% confidence interval to be of length <1?

8. (5p) Tim started working as a ticket controller for a bus company in the beginning of November. Before this job, he had a feeling that around 20% of people under 21 years old usually did not have tickets. During first month of his job, out of $n = 132$ such young people he met, 17 were without tickets. Is there a statistically significant reason to update his old beliefs? Test the hypothesis $H_0 : p = 0.2$ against the appropriate alternative. Decide if you need a two- or one-sided test. What's the p-value for the test? Can you reject H_0 on a significance level $\alpha = 0.01$?

9. (3p) Barbara is playing roulette in a casino, always putting in the minimal bet of 2 cents. Let us denote by X_i the change in her capital after i-th game. One can assume all of the games to be independent of each other, played by the same rules (thus X_i's are distributed identically to some random variable X). The expected gain after one game is negative: $\mu_X = -0.06$ cents, the standard deviation is $\sigma_X = 2$ cents. Barbara has 2 dollars, her plan is to play $n = 100$ games and then quit, no matter what.

   a) Find the expected value of Barbara's total gain $S_{100} = X_1 + X_2 + \ldots + X_{100}$.
   b) Find an approximate probability for her total gain S_{100} to be positive. (Hint: use the Central Limit Theorem)