

**EXAM:** Matematisk statistik och diskret matematik (MVE050/MSG810). Statistik för fysiker (MSG820).

**Time and place:** Wednesday 28 August 2013, 14:00–18:00, Väg och vatten.

**Jour:** Alexey Lindo, tel. 031 772 82 94.

**Allowed help:** Chalmers-approved calculator, Swedish-English dictionary and Beta handbook.

**Grades:** Chalmers: 3: 12 points, 4: 18 points, 5: 24 points. GU: G: 12 points, VG: 21 points. Maximal amount of points is 30.

Good luck!

1. (3p) Assume you throw a fair coin ten times (each time, independently of everything else, you get either ‘heads’ or ‘tails’, with equal probabilities).
  - a) What is the probability to obtain not less than 3 and not more than 7 ‘heads’?
  - b) What is the probability to obtain not less than 3 ‘heads’ or not more than 7 ‘tails’?
2. (3p) During the first half of the lunch hour (12:00 – 12:30) the students arrive to the end of the canteen queue according to a Poisson process with intensity  $\lambda = 6$  ppl/minute, and leave according to a Poisson process with intensity  $\lambda = 3$  ppl/minute. Assume that there is no queue at 12:00. What is the expected number of people in the queue at 12:19? (find it as the difference between the expectations of the number of people coming in and the number of people leaving)
3. (3p) Adam has a radio which has a function of autofind that finds one of the two radio-stations A and B, with probabilities 0.3 and 0.7, correspondently. On the radiostation A they play advertisements 15% of time (that is, if you turn on station A at any given moment, probability to hear an advertisement is 0.15). On radiostation B, they play advertisements 5% of time.
  - a) Adam turns on the radio and hears a news report. What is the probability that it is station A turned on?
  - b) Next morning, Adam turns on the radio and hears a detergent advertisement. What is the probability that it is station B?
4. (2p) Bob tries to get a ‘pass’ on the statistics course. Given his current knowledge, the probability to pass is about 0.15. Assume his level of knowledge doesn’t change between the re-exams, and that different trials are independent. What is the average amount of trials he needs to pass the course?
5. (3p) How many integer solutions does the following system have:

$$\begin{cases} x_1 + x_2 + x_3 = 15, \\ x_1 \geq 5, \\ x_2 \geq 4, \\ x_3 \geq 0. \end{cases}$$

6. (6p) Charlie claims the proportion of winning lottery tickets is less than 12%. To verify his hypothesis, he buys 50 lottery tickets, out of which 5 are lucky.

- a) Formulate the appropriate null and alternative hypotheses, choose a test statistic and conduct the corresponding statistical test on 95% significance level. What is the critical value of a test statistic you have chosen?
- b) What is the  $\alpha$ -value for your test?
- c) Assume that the true proportion of winning tickets is 7%. What is the  $\beta$ -value for the test?
7. (5p) Dean wants to estimate the average weight  $\mu$  of contents of a 100 gram nuts pack. He buys 10 packs and gets the following data (in grams):

107, 98, 97, 103, 102, 105, 107, 105, 103, 101

- a) Give a point estimate for  $\mu$ .
- b) Assume Dean doesn't have a prior knowledge of the standard deviation. Find a 95% two-sided confidence interval for  $\mu$ .
- c) Assume that the real value of  $\sigma$  is equal to the sample standard deviation  $s$  you found in part b). Find a 95% confidence interval for  $\mu$ , using the real standard deviation.
8. (5p) Edward wants to open an ice-cream stand, but he wants to do it the smart way, so he decides to study the market first. Edward wants to know the amount of the linear dependence between the air temperature and ice-cream consumption. He goes to Brunnsparcken on days with different temperature, and counts the number of people eating ice-cream. He gets the following data:

$x$ , degrees	10	15	20	25	30	35
$y$ , number	2	7	8	19	22	24

Leaving aside the questionable design of Edward's experiment, provide him with the linear regression analysis: find the least-squares estimates for  $\beta_0$ ,  $\beta_1$ , write out the regression equation and plot the regression line.