EXAM: Matematisk statistik och diskret matematik (MVE050/MVE051/MSG810). Statistik för fysiker (MSG820).
Time and place: Thursday 24 April 2014, 8:30-12:30, Väg och vatten.
Jour: Anton Muratov, 072-027-1500.
Allowed help: Chalmers-approved calculator, Swedish-English dictionary and Beta handbook.
Grades: Chalmers: 3: 12 points, 4: 18 points, 5: 24 points. GU: G: 12 points, VG: 21 points. Maximal amount of points is 30.

Good luck!

1. (3p) Anna and Beth are going to the party. Anna chooses the color of a dress to wear by tossing a fair coin. "Heads" mean Anna wears black, "tails" mean Anna wears white. Beth chooses the color by tossing a different fair coin. "Heads" mean Beth wears the same color as Anna, "tails" mean Beth wears the opposite color. Introduce the two events $A = \{\text{Anna wears black}\}$ and $B = \{\text{Beth wears black}\}$.

   a) Find the probabilities $\mathbf{P}(A), \mathbf{P}(B), \mathbf{P}(B|A)$. Are the events A and B independent?
   b) Now assume that Anna's coin was fair, but Beth's coin was biased, so that the probability of getting "heads" was $p = 0.6$ for Beth. Find the probabilities $\mathbf{P}(A), \mathbf{P}(B), \mathbf{P}(B|A)$. Are the events A and B independent?

2. (3p) A waiting time X for the bus 16 is distributed uniformly on the interval $(0, 10)$, with a pdf given by

$$f_X(x) = \begin{cases} 0.1, & 0 < x < 10, \\ 0, & \text{otherwise.} \end{cases}$$

   a) Find $\mathbf{P}(X = 1.5)$.
   b) Find $\mathbf{P}(4 < X \leq 8)$.
   c) Find $\mathbf{E}[(X + 2)/10]$.

3. (4p) Rating is a proportion of people supporting the mayor. A newspaper claims that the mayor's rating increased over the last three months by 2% to become 53%. A study involved a survey among $n = 200$ people, with 106 of them supporting the mayor.

   a) Build the null and the alternative hypothesis. Find the p-value of the study findings.
   b) How many people does the study have to involve for the p-value of the same findings to be less than 0.05?

4. (4p) In the game of Blackjack, the aim is to get the score as close to 21 as possible, but not higher. Aces are worth 11 points, Jacks, Queens and Kings are worth 10, and the number cards are worth the same amount of points as the card's value. If a player has exactly 21 point in his hand at some point, he wins immediately. If a player has more than 21 points in his hand at some point, he loses immediately. Assume that a game is played with one standard 52-card deck.

   a) First, player is dealt two cards. What is the probability for those two cards to worth exactly 21 point in total?
   b) What is the probability for the two cards to be worth more than 21 point?

c) Now assume that instead of using a standard 52-cards deck, the cards were taken from ten 52-cards decks shuffled together. What is the probability for the first two cards to be worth more than 21 points together now?

5. (4p) In a lottery, 2 million tickets are sold, each costs 10 SEK. Afterwards, 100000 tickets win 15 SEK, 2000 tickets win 500 SEK, 1000 tickets win 1000 SEK, 100 tickets win 20000 SEK, 10 win 100000 SEK, and the jackpot awarded to one person is two million SEK. Let X be the total gain of one lottery ticket: the winning minus the cost.

   a) What is the probability of winning at least something, if you only have one ticket, i.e. what is $\mathbf{P}(X > 0)$?
   b) What is the expected yield from one lottery ticket, i.e. what is $\mathbf{E}[X]$? What would the expected yield be if there was no jackpot (and the rest of the numbers were the same)?
   c) Find the variance of the number of winning tickets among $n = 100$ tickets (you can think of that number as Binomially distributed).

6. (4p) Find a generating function $A(x)$ of the sequence $\{a_n\}_{n\geq 0}$ defined by the following recursion:

$$\begin{cases} a_0 = 0, \\ a_1 = 1, \\ a_n = 2a_{n-1} + 3a_{n-2} - 2, \quad n = 2, 3, \dots \end{cases}$$

7. (3p) A car driver wants to build a confidence interval for his car's average weekly fuel consumption. He gathers the following data:

   52.5 54.0 47.5 51.0 57.0 52.0 51.0 55.0 51.0 49.5 51.5 [liters/week]

   Build a 99% confidence interval for the mean weekly fuel consumption. What kind of statistical assumptions do you need to make?

8. (5p) Erik is going to a different city for the first time in his life. Each day, from 9 AM to 9 PM his mom calls him according to a Poisson process of intensity 0.5 calls/hour. If he is not answering to more than one call in a row, mom is getting worried. Normally Erik always picks up, but now he wants to go to the movies, where he will turn the phone off. There is two options: watch the whole movie of 2 hours length, or watch the two short ones, one hour each, with a 40 minutes break in between during which he will turn the phone on (but won't see any missed calls).

   a) Is there a difference in terms of chances of worrying mom? In which scenario is Erik more likely to miss two or more calls in a row, and why?
   b) What is the probability of missing two or more calls in a row if Erik decides to go for a two hours movie?
   c) What is the probability of missing two or more calls in a row if Erik instead goes for two shorter movies? Assume that Erik can answer the phone normally during the break.