

EXAM: Matematisk statistik och diskret matematik D (MVE055/MSG810)

Time and place: Wednesday, August 28, 2013, V.

Telefonvakt: Alexey Lindo, tel. 772 82 94

Aids: Chalmers approved calculator and at most one (double-sided) A4 page of own notes. Tables of appropriate statistical distributions are provided.

Grades: Maximal points: 10. You must score at least 3 points on this exam. For the final grade your score here will be combined with scores from the VLE tests on scale 3: 12 points, 4: 18 points, 5: 24 points.

Motivations: All answers/solutions must be motivated. **Language:** You may write your answers in either English or Swedish.

1. (3p) Each of the random variables U and V takes the values ± 1 . Their joint distribution is given by

$$\begin{aligned}\mathbb{P}(U = 1) &= \mathbb{P}(U = -1) = \frac{1}{2} \\ \mathbb{P}(V = 1|U = 1) &= \mathbb{P}(V = -1|U = -1) = \frac{1}{3} \\ \mathbb{P}(V = -1|U = 1) &= \mathbb{P}(V = 1|U = -1) = \frac{2}{3}.\end{aligned}$$

- a) Find the probability that $x^2 + Ux + V = 0$ has at least one real root.
b) Find the expected value of the larger root of $x^2 + Ux + V = 0$ given that there is at least one real root.
c) Find the probability that $x^2 + (U + V)x + U + V = 0$ has at least one real root.

Hint: A formula for finding roots of $ax^2 + bx + c = 0$:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. (2p)
- a) How large a random sample should be taken from a distribution in order for the probability to be at least 0.99 that the sample mean will be within two standard deviations of the mean of the distribution? Use Chebyshev's inequality to determine a sample size that will be sufficient, whatever the distribution.
b) How large a random sample should be taken from a normal distribution in order for the probability to be at least 0.99 that the sample mean will be within one standard deviation of the mean of the distribution?
3. (2p) Three girls A , B and C are playing table tennis. In each game, two of the girls play against each other and the third girl does not play. The winner of any given game n plays in game $n + 1$. The probability that girl x will beat girl y in any game that they play against each other is $\frac{s_x}{(s_x + s_y)}$ for $x, y \in \{A, B, C\}, x \neq y$, where s_A, s_B, s_C represent the playing strengths of the three girls.
- a) Represent this process as a discrete-time Markov chain by defining the possible states and constructing the transition matrix.
b) Determine the probability that the two girls who play each other in the first game will play each other again in the fourth game. Show that this probability does not depend on which two girls play in the first game.

4. (3p)

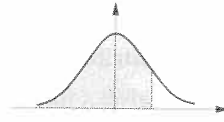
- a) Over a representative mix of client requests, a database server exhibits random response times. The response time T , measured in seconds, is known to have a normal distribution with variance 0.004. Using a sample of size 10, estimate the mean of the distribution and find a 90% confidence interval for the mean. Suppose that the sample contains the following observations.

i	1	2	3	4	5	6	7	8	9	10
T_i	1.72	1.88	1.84	1.81	1.75	1.92	1.80	1.68	1.80	1.81

- b) Use the same context and data as in part a), but now assume a known mean. Specifically, the response time T of a database server, measured in seconds, is known to have a normal distribution with mean 1.80. Estimate the variance and report a 90% confidence interval for the variance.

Lycka till! Good luck!

Normal distribution
Distribution function



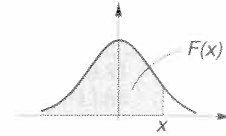
The table gives $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt$. For $x < 0$ values of $\Phi(x)$ can be obtained from $\Phi(-x) = 1 - \Phi(x)$.

x	0	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9 ³ 03	0.9 ³ 06	0.9 ³ 10	0.9 ³ 13	0.9 ³ 16	0.9 ³ 18	0.9 ³ 21	0.9 ³ 24	0.9 ³ 26	0.9 ³ 29
3.2	0.9 ³ 31	0.9 ³ 34	0.9 ³ 36	0.9 ³ 38	0.9 ³ 40	0.9 ³ 42	0.9 ³ 44	0.9 ³ 46	0.9 ³ 48	0.9 ³ 50
3.3	0.9 ³ 52	0.9 ³ 53	0.9 ³ 55	0.9 ³ 57	0.9 ³ 58	0.9 ³ 60	0.9 ³ 61	0.9 ³ 62	0.9 ³ 64	0.9 ³ 65
3.4	0.9 ³ 66	0.9 ³ 68	0.9 ³ 69	0.9 ³ 70	0.9 ³ 71	0.9 ³ 72	0.9 ³ 73	0.9 ³ 74	0.9 ³ 75	0.9 ³ 76

For large values of x the following approximation can be used.

$$\frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2} \cdot \left(\frac{1}{x} - \frac{1}{x^3} \right) < 1 - \Phi(x) < \frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2} \cdot \frac{1}{x}$$

$\Phi(4) = 0.9468329$ $\Phi(5) = 0.967133$

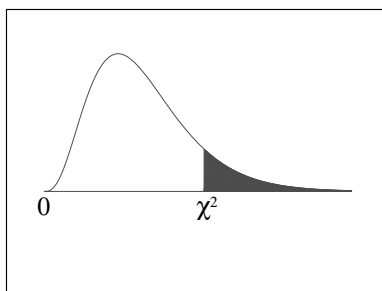


The t -distribution

The table gives x for given values of the distribution function $F(x)$ for a t -distribution with r degrees of freedom. For $x < 0$ values of $F(x)$ can be obtained from $F(-x) = 1 - F(x)$.

$F(x)$	0.75	0.90	0.95	0.975	0.990	0.995	0.9975	0.9995
$r = 1$	1.000	3.078	6.314	12.71	31.82	63.66	127.3	636.6
2	0.8165	1.886	2.920	4.303	6.965	9.925	14.09	31.60
3	0.7649	1.638	2.353	3.182	4.541	5.841	7.453	12.92
4	0.7407	1.533	2.132	2.776	3.747	4.604	5.598	8.610
5	0.7267	1.476	2.015	2.571	3.365	4.032	4.773	6.869
6	0.7176	1.440	1.943	2.447	3.143	3.707	4.317	5.959
7	0.7111	1.415	1.895	2.365	2.998	3.499	4.029	5.408
8	0.7064	1.397	1.860	2.306	2.896	3.355	3.832	5.041
9	0.7027	1.383	1.833	2.262	2.821	3.250	3.690	4.781
10	0.6998	1.372	1.812	2.228	2.764	3.169	3.581	4.587
11	0.6974	1.363	1.796	2.201	2.718	3.106	3.497	4.437
12	0.6955	1.356	1.782	2.179	2.681	3.055	3.428	4.318
13	0.6938	1.350	1.771	2.160	2.650	3.012	3.372	4.221
14	0.6924	1.345	1.761	2.145	2.624	2.977	3.326	4.140
15	0.6912	1.341	1.753	2.131	2.602	2.947	3.286	4.073
16	0.6901	1.337	1.746	2.120	2.583	2.921	3.252	4.015
17	0.6892	1.333	1.740	2.110	2.567	2.898	3.222	3.965
18	0.6884	1.330	1.734	2.101	2.552	2.878	3.197	3.922
19	0.6876	1.328	1.729	2.093	2.539	2.861	3.174	3.883
20	0.6870	1.325	1.725	2.086	2.528	2.845	3.153	3.850
21	0.6864	1.323	1.721	2.080	2.518	2.831	3.135	3.819
22	0.6858	1.321	1.717	2.074	2.508	2.819	3.119	3.792
23	0.6853	1.319	1.714	2.069	2.500	2.807	3.104	3.767
24	0.6848	1.318	1.711	2.064	2.492	2.797	3.090	3.745
25	0.6844	1.316	1.708	2.060	2.485	2.787	3.078	3.725
26	0.6840	1.315	1.706	2.056	2.479	2.779	3.069	3.707
27	0.6837	1.314	1.703	2.052	2.473	2.771	3.056	3.690
28	0.6834	1.313	1.701	2.048	2.467	2.763	3.047	3.674
29	0.6830	1.311	1.699	2.045	2.462	2.756	3.038	3.659
30	0.6828	1.310	1.697	2.042	2.457	2.750	3.030	3.646
34	0.6818	1.307	1.691	2.032	2.441	2.728	3.002	3.601
39	0.6808	1.304	1.685	2.023	2.426	2.708	2.976	3.559
44	0.6801	1.301	1.680	2.015	2.414	2.692	2.956	3.526
49	0.6795	1.299	1.677	2.010	2.405	2.680	2.940	3.501
59	0.6787	1.296	1.671	2.001	2.391	2.662	2.916	3.464
69	0.6781	1.294	1.667	1.995	2.382	2.649	2.900	3.438
79	0.6776	1.292	1.664	1.990	2.374	2.640	2.888	3.418
89	0.6773	1.291	1.662	1.987	2.369	2.632	2.879	3.404
99	0.6770	1.290	1.660	1.984	2.365	2.626	2.871	3.392
∞	0.6745	1.282	1.645	1.960	2.326	2.576	2.807	3.291

Chi-Square Distribution Table



The shaded area is equal to α for $\chi^2 = \chi^2_{\alpha}$.

<i>df</i>	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169