

EXAM: Matematisk statistik och diskret matematik D (MVE055/MSG810)

Time and place: Wednesday 15 January 2014, em, V.

Jour: Alexey Lindo, tel. 772 82 94

Aids: Chalmers approved calculator and at most one (double-sided) A4 page of own notes.

Grades: Maximal points : 10. You must score at least 3 points on this exam. For the final grade your score here will be combined with scores from the VLE tests on scale 3: 12 points, 4: 18 points, 5: 24 points.

Motivations: All answers/solutions must be motivated.

Language: You may write your answers in either english or swedish.

1. (1p) Show that if $\mathbb{P}(A) > 0$, then $\mathbb{P}(AB|A) \geq \mathbb{P}(AB|A \cup B)$.
2. (1.5p) Let X be a Poisson random variable with parameter λ . Show that $\mathbb{P}(X = i)$ increases monotonically and then decreases monotonically as i increases, reaching its maximum when i is the largest integer not exceeding λ .
Hint: Consider $\frac{\mathbb{P}(X=i)}{\mathbb{P}(X=i-1)}$.
3. (1.5p) Let X be such that $\mathbb{P}(X = 1) = p = 1 - \mathbb{P}(X = -1)$. Find $c \neq 1$ such that $\mathbb{E}(c^X) = 1$.
4. (3p) Determine the sequence generated by each of the following functions:
 - a) $f(x) = (x - 2)^3$;
 - b) $f(x) = \frac{x^3}{1-x^2}$;
 - c) $f(x) = \frac{1}{1-x} + 1 - x + 3x^3$.
5. (3p) Let X_1, \dots, X_n be independent and identically distributed random variables with the density function

$$f(x|\theta) = (\theta + 1)x^\theta, \quad 0 \leq x \leq 1.$$

- a) Find the method of moments estimate of θ .
- b) Find the maximum likelihood estimate of θ .

Lycka till! Good luck!