EXAM: Matematisk statistik och diskret matematik D (MVE055/MSG810)
Time and place: Tuesday 28 October 2014, morning, V.
Jour: Alexey Lindo, tel. 0763278070
Aids: Chalmers approved calculator and at most one (double–sided) A4 page of own notes.
Grades: Maximal points : 10. You must score at least 3 points on this exam. For the final grade your score here will be combined with scores from the VLE tests on scale 3: 12 points, 4: 18 points, 5: 24 points.
Motivations: All answers/solutions must be motivated.
Language: You may write your answers in either English or Swedish.

1. (2p) Let X be a random variable, prove that for every $a > 0$,

   $$\mathbb{P}(|X| \geq a) \leq \frac{\mathbb{E}(X^2)}{a^2}.$$

   Hint: Modify the proof of Markov's inequality.

2. (4p)

   a) Provide the definition of a moment generating function.

   b) Compute the moment generating function for a Poisson random variable with parameter $\lambda > 0$.

   c) Skewness is a measure of asymmetry of a probability distribution.

   Definition 1. *Skewness of a random variable X, denoted by γ_1, is defined as $\gamma_1 = \mathbb{E}\Big(\big(\frac{X-\mu}{\sigma}\big)^3\Big)$, where μ is the mean and σ is the standard deviation of X.*

   Calculate the skewness of a Poisson random variable with parameter $\lambda > 0$.
   Hint: Show that $\gamma_1 = \frac{\mathbb{E}(X^3) - 3\mu\sigma^2 - \mu^3}{\sigma^3}$.

3. (2p) Suppose that $X_1, \ldots, X_n$ are independent and identically distributed random variables, having a normal distribution with unknown mean μ and unknown variance σ^2. Explain how confidence interval for the mean can be constructed.

4. (2p) Let $(X_n)_{n \geq 0}$ be a Markov chain with state space $S = \{1, 2, 3\}$ and matrix of transition probabilities

   $$\begin{pmatrix} 3/7 & 3/7 & 1/7 \\ 1/11 & 2/11 & 8/11 \\ 1/11 & 4/11 & 6/11 \end{pmatrix}.$$

   Assume that $\mathbb{P}(X_0 = 1) = 1$. Consider

   $$Y_n = \begin{cases} 1 & \text{if } X_n = 1, \\ 2 & \text{if } X_n \neq 1. \end{cases}$$

   Show that sequence of random variables $(Y_n)_{n \geq 0}$ forms a Markov chain and find its transition matrix.

Lycka till! Good luck!