Lösningar till tentamen i Matematisk statistik och diskret matematik D2 (MVE055/MSG810).
Den 12 januari 2011. These are sketches of the solutions.

1. Lösning:

   a) They are dependent as $P(A)P(B) = 0.12 \neq 0 = P(A \cap B)$ or alternatively if one of them occurred we know the other one did not.

   b) $P(A \cup B) = 0.3 + 0.4 = 0.7$, $P(A \cap B^C) = P(A) = 0.3$

   c) See book §2.4

   d) $P(C|A) = P(C \cap A)/P(A) = 0.1/0.3 = 1/3$ and $P(C|B) = 0$

2. Lösning:

   a) See book §7.4

   b) If you don't know your data's distribution but know that it has a finite mean and variance and the sample is large enough you can approximate the sample's mean distribution by a normal one and therefore draw inferences about it (*e.g.* the confidence intervals).

   c) Use the formula $\bar{x} \pm t_{\alpha/2}S/\sqrt{n}$. $\bar{x} = 10.22131$, $S = 4.622457$, $\alpha \geq 90\%$ can be considered sensible for such a problem. You are allowed to use the formula because of the CLT, but need to think about it as you just observe the sample and have no information provided about its distribution. You should give him $\bar{x}$+something and this something can be taken as a number of sds, or the CI. The important thing to notice is that one has to take a value greater then the mean and somehow sensibly motivate why this value is related to the problem.

3. Lösning:

   a) $\beta_0 + \beta_1\mu_X$

   b) $\beta_1^2\sigma_X^2$

   c) $\beta_1\sigma_X^2$

4. Lösning:

   a) 0.9

   b) $0.9 \pm 1.65\sqrt{0.9 \cdot 0.1/50} = 0.9 \pm 0.07$

   c) use the fact that $p(1-p) \leq 0.25$ as $p \in [0, 1]$ and then we get the formula $n = \frac{z^2_{\alpha/2}}{4d^2}$ which gives us $n = 1702$

5. Lösning:

   a) Use that the sum of the first n integers is $n(n+1)/2$

   b) $P(X = k) = \sum_{y=1}^{k} \frac{2}{n(n+1)} = \frac{2k}{n(n+1)}$ $k = 1, \ldots, n$ $P(Y = k) = \sum_{x=k}^{n} \frac{2}{n(n+1)} = \frac{2(n-k+1)}{n(n+1)}$ $k = 1, \ldots, n$

   c) No, one can calculate this from definition as $P(X = k, Y = r) \neq P(X = k)P(Y = r)$ for some appropriate k and r or just state that if we observe only X then this gives us information about Y, *i.e.* that it has to be lesser than X (or alternatively observing Y gives us information about X).

   d) Since X and Y are dependent this does not give any information about the covariance between them.

6. Lösning:

   a) $A(x) = \sum_{n=0}^{\infty} x^n a_n$, $A(x) = \frac{1}{1-2x} + \frac{1}{3(1-3x)}$

   b) $\frac{-1/16}{x-1} + \frac{9/32}{x+3} + \frac{25/32}{x-5}$

7. Lösning:

   a) See GS §8.1

   b) Yes as this proportion is the probability that a head will show up and it is also the expectation of the variable $X = 1$ if a head showed up and 0 otherwise. We can tell motivating this by the LLN but we have to remember that it is a probabilistic statement so we only have some measure of certainty on this and not a true/false statement.

8. Lösning:
   See lecture