EXAM: Matematisk statistik och diskret matematik D (MVE055/MSG810)
Time and place: Wednesday 11 January 2012, kl. 14.00–18.00, V.
Jour: Fredrik Boulund, tel. 0737-706 629.
Aids: Chalmers approved calculater and at most one (double–sided) A4 page of own notes. Tables of appropriate statistical distributions are provided.
Grades: 3: 12 points, 4: 18 points, 5: 24 points. Maximal points : 30.
Motivations: All answers/solutions must be motivated.
Language: There is a Swedish and English version of the questions. You may write your answers in either of these two languages.
IMPORTANT: There are two versions of the exam. One for current students and the other for students from previous years who choose to take the "old style" exam. This is the "old style" exam.

1. (5p) Let A and B be two events.

   a) (2p) Prove the inequality $P(A \cap B) \geq P(A) + P(B) - 1$.

   Assume that $P(A \cap B) = 0.2$ and $P(A) = 0.4$ and A and B are independent.
   b) Calculate $P(B)$.
   c) (2p) Can you say what is $P(A|B)$ without doing any calculations and why or why not? Provide this probability.

2. (5p) Define,

$$f(x) \quad = \quad \begin{cases} Cx(3-x) & \text{if } 0 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

   a) Calculate for what value of C is $f(x)$ a probability density function.
   b) Calculate $P(X \leq 0.5)$.
   c) Calculate $P(X \geq 2.5)$.
   d) Calculate $P(X \in (0.5, 2.5))$.
   e) If $0 \leq \alpha \leq 1$ and $g(x)$, $h(x)$ are continuous probability density functions show that, $d(x) = \alpha g(x) + (1-\alpha)h(x)$ is also a continuous probability density function.

3. (3p) Let $\mathbf{w} = (w_1, w_2, \ldots, w_n)$ be a vector of fixed numbers (weights). For a random sample $(X_1, X_2, \ldots, X_n)$ define the **weighted sum** by,

$$\bar{X}_w \quad = \quad \textstyle\sum_{i=1}^{n} w_i X_i.$$

   Answer the following by recalling that a random sample means that the points are independent and identically distributed.

   a) Find conditions on $\mathbf{w}$ that make $\bar{X}_w$ an unbiased estimator of the mean of the distribution from which the X_is are sampled. If these conditions hold then $\bar{X}_w$ is called a **weighted mean** or **weighted average**.
   b) (2p) Determine the variance of a weighted average (in terms of the weights $\mathbf{w}$) if each X_i has variance σ^2.

4. (3p) Consider a simple gambling game where you place a bet of $n$kr, flip a coin (with probability of heads p) and win $n$kr if it comes up heads and lose the $n$kr if it comes up tails. Consider now the following strategy for playing this game.

   - Start by betting 1kr.
   - Each time you lose, bet twice as much in next coin flip.
   - As soon as you win, quit.

a) Let K be the number of times you bet in a game. What is the distribution of K and derive the density function of K.

b) Let X be the amount of money bet in the last round (when you win). What possible values can X take and why? Derive a formula for X's density function, $P(X = x)$.

c) Calculate, using the definition of the expected value, the expected values of K and X depending on the value of p. What is strange about them? What problem do they reveal about this betting scheme?

5. (5p) You are using a scale to weigh an unknown quantity of gold. It is known that this scales gives measurements which have a normal error with average 0 and standard deviation of 0.03 ounces. You record a sample of 3 measurements and calculate their mean.

   a) Provide the definition of a confidence interval at the $1 - \alpha$ level.

   b) Explain the difference between the following two statements concerning an unknown mean parameter μ:
      - $\mu \in (0.95, 1.05)$ with 95% confidence
      - $\mu \in (0.95, 1.05)$ with 95% probability

   c) What is the probability that the mean of the three measurements is within 0.03 ounces of the actual weight?

   d) How much will this probability change if you increase the number of weightings to 4?

   e) If an ounce of gold costs approximetely 11 500kr would you be worried if you would use this scale to weigh (ONLY ONCE) an ounce of gold for a transaction and why?. And what if you wanted to trade an ounce of silver (approximetly 210kr per ounce)?

6. (2p) A newspaper conducts a phone survey to predict public opinion about a referendum. The referendum will pass if the majority of voters vote "yes". Of the surveyed 850 voters, 400 said that they would vote "yes". Does this result allow the newspaper to predict the outcome of the election at a 95% confidence level or not? You may use the normal approximation for the binomial distribution to answer this *i.e.*
   $\text{Binom}(n, p) \approx \mathcal{N}(np, np(1-p))$.

7. (3p)

   a) Provide the definition of the Moment Generating Function of a random variable X and discuss how this is related to a generating function.

   b) Calculate the mgf of a geometric random variable with parameter p.

   c) Using the mgf calculate the mean and variance of a geometric random variable.

8. (4p) Some time ago Harvard, Dartmouth and Yale admitted only male students. Assume that, at that time 80% of the sons of Harvard men went to Harvard and the rest went to Yale; 40% of the sons of Yale men went to Yale and the rest simplit evenly between Harvard and Dartmouth; and of the sons of Dartmouth men, 70% went to Dartmouth, 20% went to Harvard, and 10% went to Yale. This gives us a Markov chain which models where descendents of the alumnii of these universities take their university degrees.

   a) Write down the transition probability matrix of this Markov chain.

   b) Find the probabability that the grandson of a man from Yale went to Yale.

   c) Assume that the son of a Harvard man always went to Harvard. Write down the transition probability matrix for this modified Markov chain. How do we call a state of the Markov chain like Harvard?

   d) Find the probabability that the grandson of a man from Yale went to Yale in the above changed situation. How would you explain this change?

Lycka till! Good luck!