Lösningar till tentamen i Matematisk statistik och diskret matematik D2 (MVE055/MSG810).
Den 11 januari 2012. These are sketches of the solutions.

1. Lösning:
   a) $1 \geq P(A \sum B) = P(A) + P(B) - P(A \cap B)$,
   $P(A \cap B) = P(A) + P(B) - P(A \cup B) \geq P(A) + P(B) - 1$
   b) $0.2/0.4 = 0.5$
   c) It is 0.4 due to the independence between A and B.

2. Lösning:
   a) $\int_0^3 x(3-x)\mathrm{d}x = 9/2$ so $C = 2/9$
   b) $P(X \leq 0.5) = (2/9)\int_0^{0.5} x(3-x)\mathrm{d}x \approx 0.074$
   c) $P(X \geq 2.5) = (2/9)\int_{2.5}^3 x(3-x)\mathrm{d}x \approx 0.074$
   d) $P(X \in (0.5, 2.5)) = 1 - (P(X \leq 0.5) + P(X \geq 2.5)) \approx 0.852$
   e) as $h(x), g(x) \geq 0$ and $\alpha \in [0,1]$ then $d(x) \geq 0$ and now we check whether $d(x)$ integrates to 1
   $\int_{-\infty}^{\infty} d(x)\mathrm{d}x = \int_{-\infty}^{\infty} \alpha g(x) + (1-\alpha)h(x)\mathrm{d}x = \alpha \int_{-\infty}^{\infty} g(x)\mathrm{d}x + (1-\alpha)\int_{-\infty}^{\infty} h(x)\mathrm{d}x =$
   $\alpha + 1 - \alpha = 1$

3. Lösning:
   a) $\sum_{i=1}^{n} w_i = 1$
   b) $\sigma^1 \sum_{i=1}^{n} w_i^2$

4. Lösning:
   a) Geometric distribution, $P(K = k) = (1-p)^k p$ for $k = 0, 1, 2, \ldots$
   b) $P(X = 2^k) = (1-p)^{k-1} p$ for $k = 0, 1, 2, \ldots$
   c) $\mathrm{E}[K] = 1/p$. If $p > 0.5$ $\mathrm{E}[X] = 2p(2-2p)/(2p-1)$ and if $p \leq 0.5$ $\mathrm{E}[X] = \infty$. This means that if our chances of winning are lesser than 0.5 even though we expect to win the game we need an infinite amount of money to do this.

5. Lösning:
   a) see lectures on CIs
   b) see lectures on CIs
   c) $\Psi(\sqrt{(}3)) - 1 \approx 0.917$
   d) $\Psi(\sqrt{(}3)) - 1 \approx 0.954$
   e) The 95% confidence interval is $(0.9412oz, 1.0588oz)$ and this means a possible error of $\pm$0.0588oz which amounts to $\pm$676.20kr, for silver this would amount to $\pm$12.35kr per ounce.

6. Lösning:
   The confidence interval for p is approximately 0.471 ± 0.0334. As 0.5 is contained in this interval the newspaper cannot predict the result.

7. Lösning:

   a) $M_X(t) = \mathcal{E}[e^{tX}]$, it is the generating function of the sequance of the moments of the random variable

   b) $pe^t/(1-(1-p)e^t)$ or $(1-p)e^t/(1-pe^t)$ depending what you took for p

   c) $\mathrm{E}[X] = 1/p$ (or $1/(1-p)$), $\mathrm{Var}[X] = (1-p)/p^2$ (or $p/(1-p)^2$)

8. Lösning:

   a)

$$\mathbf{P} = \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.1 & 0.7 \end{bmatrix}$$

   where the first row (column) refers to Harvard, second to Yale and third to Dartmouth

   b) $[1, 0, 0]\mathbf{P}^2 = [0.42, 0.25, 0.33]$ so the probability is 0.42

   c)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.1 & 0.7 \end{bmatrix}$$

   and we call such a state an absorbing state

   d) $[0, 1, 0]\mathbf{P}^2 = [0.48, 0.19, 0.33]$ so the probability is 0.19. There is a chance that the son of a Yale man goes to Harvard and so the grandson is then “lost” for Yale and so this probability is smaller.