EXAM: Matematisk statistik och diskret matematik D (MVE055/MSG810)
Time and place: Tuesday 22 October 2013, em, V.
Jour: Alexey Lindo, tel. 772 82 94
Aids: Chalmers approved calculator and at most one (double–sided) A4 page of own notes. Tables of appropriate statistical distributions are provided.
Grades: Maximal points : 10. You must score at least 3 points on this exam. For the final grade your score here will be combined with scores from the VLE tests on scale 3: 12 points, 4: 18 points, 5: 24 points.
Motivations: All answers/solutions must be motivated.
Language: You may write your answers in either english or swedish.

1. (2p) Suppose that 3 white and 3 black balls are distributed in two urns in such a way that each urn contains 3 balls. We say that the system is in state i if the first urn contains i white balls, $i = 0, 1, 2, 3$. At each stage, 1 ball is drawn from each urn and the ball drawn from the first urn is placed in the second, and conversely with the ball from the second urn. Let X_n denote the state of the system after the nth stage, show that $\{X_n, n \geq 0\}$ is a markov chain and compute the transition probabilities.

2. (3p) Find generating functions of the following sequences. [For example, in the case of the sequence 1, 1, 1, ..., the answer required is $\frac{1}{1-x}$, not $\sum_{i=0}^{\infty} x^i$ or simply $1 + x + x^2 + \cdots$]

   a) $\langle \binom{8}{0}, \binom{8}{1}, \binom{8}{2}, \ldots, \binom{8}{8}, 0, 0, \ldots \rangle$
   b) $\langle \binom{8}{1}, 2\binom{8}{2}, 3\binom{8}{3}, \ldots, 8\binom{8}{8}, 0, 0, \ldots \rangle$
   c) $\langle 0, 0, 0, 55, 55, 55, \ldots \rangle$
   d) $\langle 3, 10, 38, \ldots \rangle = \langle 2^n + 3^n + 5^n \rangle$, $n = 0, 1, 2, \ldots$.

3. (3p) Show that the least squares estimates of the slope and intercept of a line may be expressed as:
$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$
   and
$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$
   Use this expression to show that the line fit by the method of least squares passes through the point $(\bar{x}, \bar{y})$.

4. (2p) Suppose that X is a discrete random variable with $P(X = 1) = \theta$ and $P(X = 0) = 1 - \theta$. Three independent observations of X are made: $x_1 = 0, x_2 = 0, x_3 = 1$.

   a) Find the method of moments estimate of θ.
   b) Find the maximum likelihood estimate of θ.

Lycka till! Good luck!