Marco Longfils and Petter Mostad
Applied Mathematics and Statistics
Chalmers and GU

MVE055 / MSG810 Matematisk statistik och diskret matematik

Exam 19 December 2017, 14:00 - 18:00

Allowed aids: Chalmers-approved calculator
and one (two-sided) A4 sheet of paper with your own notes.
Total number of points: 30. To pass, at least 12 points are needed.
Note: All answers should be motivated.

1. **(5 points)** Consider the following matrices

$$P_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad P_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \quad P_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

   (a) Are P_1, P_2, P_3 the transition matrix of a Markov chain? Motivate your answer for each matrix.

   (b) For those matrices among P_1, P_2, P_3 that are transition matrix, describe the corresponding Markov chain in terms of transient /absorbing states.

   (c) For those matrices among P_1, P_2, P_3 that are transition matrix, compute the distribution of the chain after two steps if the initial distribution is

$$\pi = [0.2, 0.1, 0.45, 0.25].$$

2. **(5 points)** Assume you know that, for events A and B, that $P(A) = 0.6$ and that $P(B) = 0.5$

   (a) Is it possible that $P(A \cap B) = 0.05$?

   (b) Assuming that A and B are independent events, prove that A^C and B^C are independent, where A^C and B^C are the complementary events of A and B, respectively.

3. **(5 points)** A scientist would like to study the time at which the next earthquake happens in the world. Denote by X the waiting time until an earthquake happens in the northern emisphere and by Y the waiting time for a sismic event in the southern emisphere. Assume X, Y are exponential random variables and earthquakes occurr at a rate of one per year and two per year in the northern and southern emisphere respectively. Assume X and Y are independent of each other.

   (a) Find the cumulative distribution function of the random variable Z which models the waiting time to the next earthquake in the world.

(b) Compute the probability that there won't be any earthquakes in the next year.

4. **(5 points)** Let X be a discrete random variable with moment generating function $m_X(t)$ and characteristic function $\phi_X(t)$.

   (a) Compute the moment generating function $m_Y(t)$ of the random variable $Y = a + bX$ as a function of $m_X(t)$.

   (b) Could it be that $\phi_X(t) = 2$, that is the characteristic function of X is the constant 2?

   (c) Find $P(X = 5)$ for the discrete random variable X such that $\phi_X(t) = 1$ for all t.

5. **(7 points)** Alex has measured the weight of the contents of 13 packs of potato chips of brand P. He has found an average of 197 and a sample standard deviation of 5.6. He assumes the weights of such contents are actually normally distributed with expectation μ and variance σ^2.

   (a) Compute a 95% confidence interval for σ^2.

   (b) Anna has measured the contents of 7 packs of potato chips of brand Q. She has found an average of 203 and a sample standard deviation of 4.1. She assumes these weights are normally distributed; she also assumes the variance of this distribution is the same as for brand P, i.e., σ^2. Using all available information, compute an estimate for σ^2, and also a 95% confidence interval.

   (c) Compute a 95% confidence interval for the difference in the expected weights of the contents of brands Q and P.

   (d) Formulate and compute a hypothesis test of whether the expected weight of the contents of brand Q packs is greater than the expected weights of the contents of brand P packs. Explicitly state all the choices you need to make along the way.

6. **(3 points)** Assume Z is a standard normal random variable, $Z \sim Normal(0, 1)$. Find approximately the value of z such that $P(z \leq Z \leq 1) = 0.7413$.