RANDOM PROSECCES WITH APPLICATIONS 2007

Optional home work 1

Day assigned: **September 14**

Assignment deadline: **12:00, September 21**

Problem 1. A multichannel microwave link is to provide telephone communication to a remote community having 12 subscribers, each of whom uses the link 20% of the time during pick hours. How many channels are needed to make the link available during pick hours to:

(a) Eighty percent of the subscribers all of the time? (1)

(b) All of the subscribers 80% of the time? (1)

(c) All of the subscribers 95% of the time? (1)

Problem 2. A power supply has five intermittent loads connected to it and each load, when in operation, draws a power of 10 W. Each load is in operation only one-quarter of the time and operates independently of all other loads.

(a) Find the mean value of the power required by the loads. (1)

(b) Find the variance of the power required by the loads. (1)

(c) If the power supply can provide only 40 W, find the probability that it will be overloaded. (1)

Problem 3. A common method for detecting a signal in a presence of noise is to establish a threshold value and compare the value of any observation with this threshold. If the threshold is exceeded, it is decided that a signal is present. Sometimes, of course, noise alone will exceed the threshold and this is known as a "false alarm". Usually, it is desired to make the probability of a false alarm very small. At the same time, we would like any observation that does contain a signal plus the noise to exceed the threshold with a large probability. This is the probability of detection and it should be as close to 1 as possible. Suppose we have Gaussian noise with zero mean and a variance of 1 V^2 and we set a threshold level of 5 V.

(a) Find the probability of false alarm. (1)

(b) If a signal having a value of 8 V is observed in the presence of this noise, find the probability of detection. (1)

(c) When noise only is present, find the conditional mean value of the noise that exceeds the threshold. (1)

(d) Repeat part (c) when both the specified signal and noise are present. (1)

Problem 4. Marbles rolling on a flat surface have components of velocity in orthogonal directions that are independent Gaussian random variables with zero mean and a standard deviation of 4 ft/s.

(a) Find the most probable speed of the marbles. (1)

(b) Find the mean value of the speed. (1)

(c) What is the probability of finding a marble with a speed greater than 10 ft/s? (1)

Problem 5. A sinusoidal signal has the form

$$X(t) = cos(100t + \theta)$$

where θ is a random variable that is uniformly distributed between 0 and 2π. Another sinusoidal signal has the form

$$Y(t) = cos(100t + \psi)$$

where ψ is independent of θ and is also uniformly distributed between 0 and 2π. The sum of these two sinusoids, $Z(t) = X(t) + Y(t)$ can be expressed in terms of its magnitude and phase as

$$Z(t) = Acos(100t + \Phi).$$

(a) Find the probability that $A > 1$. (1)

(b) Find the probability that $A \leq 0.5$. (1)