Written test for examination in MVE135
Random processes with applications, 2007-10-25 Thursday, 14:00 - 18:00, V.
On duty: Rossitza Dodunekova, tel. 772 35 34
Time of visit 15:15 and 17:00.
Allowed material: The handbook *Beta*, Collection of Formulas for MVE135, calculators approved by Chalmers.

There are 30 total points in the examination. One needs 14 points for grade 3 (to pass), 18 points for grade 4, and 24 points for grade 5.

Problem 1. The input X in a binary optical communication system is a random variable with equally likely values 1 and 2. The receiver output Y is a Poisson random variable which parameter is μ, when 1 is transmitted, and ν when 2 is transmitted.

(a) Compute $E[Y|X]$ and $E[Y]$. 1.5p

(b) Given that the receiver output is equal to 2, find the conditional probability that 1 was sent. 1.5p

Problem 2. A multiplexer combines N digital television signals into a common transmission line. Signal n generates X_n bits every 33 milliseconds, where X_n is a Gaussian random variable with mean m/N and variance $\sigma^2/\sqrt{N}$. Suppose that the multiplexer accepts a maximum total of T bits from the combined sources every 33 ms, and that any bits in excess of T are discarded. Let the signals be independent and assume that $T = m + t\sigma$, where $t > 0$ is a fixed number. Let Y_{Disc} be the number of bits discarded per 33-ms period, i.e.,

$$Y_{Disc} = \begin{cases} X - T, & X > T \\ 0, & X \leq T. \end{cases}$$

Compute $E[Y_{Disc}]$. What is the result when $t \to \infty$? 3p

Problem 3. Suppose Z_1 and Z_2 are independent standard normal random variables. Define $X_1 = Z_1, \quad X_2 = 3/5 Z_1 + 4/5 Z_2$. Compute $f_{X_2|X_1}(x_2|x_1)$, the conditional PDF of X_2, given $X_1 = x_1$. 3p

Problem 4. Messages arrive in a multiplexer according to a Poisson process with mean $\lambda = 10$ messages/second. Use the CLT to estimate the probability that more then 650 messages arrive in one minute. 3p

Problem 5. Let X_1, X_2, ... be iid random variables with expected value m and variance σ^2, and consider the discrete time process $\{Z_n, \, n \geq 1\}$ with

$$Z_n = \frac{X_1 + X_2 + \, ... \, + X_n}{n}.$$

(a) Find the autocovariance function of Z_n. 3p

(b) Why is this process Markovian? Assume that X_1 is continuous with CDF $F(x)$ and PDF $f(x)$ and compute

$$F_{Z_n|Z_{n-1}}(x|\, Z_{n-1} = y) = P(Z_n \le x|\, Z_{n-1} = y) \quad \text{and} \quad f_{Z_n|Z_{n-1}}(x|\, Z_{n-1} = y).$$

3p

Problem 6. Consider the short term integration of $X(t)$

$$Y(t) = \frac{1}{T}\int_{t-T}^{t} X(u)du,$$

where $X(t)$ is the white noise process with PSD $S_X(f) = N_0/2$.

(a) Compute $S_Y(f)$, the PSD of $Y(t)$. 3p

(b) Compute the average power of $Y(t)$. 3p

Problem 7. $\{X_n\}$ is a WSS process with autocorrelation function

$$R_X(k) = 4(1/2)^{|k|}, k = 0, \pm1, \pm2, \ldots.$$

Find the optimum linear filter for estimating X_n from the observations X_{n-1} and X_{n-3} and compute the mean-square estimation error. 3p

Problem 8. The spectrum of a stationary stochastic process is to be estimated from the following data:

$$x[n] = \{0,6,\ -0,7,\ 0,2,\ 0,3\}.$$

Due to the small sample support, a simple AR(1)-model is exploited:

$$x[n] + a_1 x[n-1] = e[n].$$

Determine estimates of the AR-parameter a_1 and the white noise variance σ_e^2. Based on these, give a parametric estimate of the spectrum $P_X(e^{j\omega})$. 3p