MVE135 RANDOM PROCESSES WITH APPLICATIONS 2008
HOMEWORK 2

This assignment is optional. It gives two bonus points to the written examination, when the submitted solution collects at least 12 points out of 16.

Posted on September 25. Deadline for submission: October 10, 17:00

From October 7 until the deadline for submission I will be away. In this period you can e-mail me your questions regarding the homework. Note that solutions are to be submitted to SIMA SHAHSAVARI.

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Problem 1. $X(t)$ is a zero-mean stationary Gaussian process with covariance function $C_X(\tau)$.

(a) Compute $P\{X(t) \geq X(s)\}$ when $C_X(t-s) \neq C_X(0)$ and when $C_X(t-s) = C_X(0)$. 2p

(b) Show that for any t and h

$$E[X(t)X^2(t+h)] = 0.$$

Hint. Let $X_1 = X(t)$ and $X_2 = X(t+h)$. Find a constant α such that $X_1 - \alpha X_2$ and X_2 are independent and use the presentation $X_1 = (X_1 - \alpha X_2) + \alpha X_2$. 2p

Problem 2. $X(t)$ is the Poisson process with rate λ. Consider the random process

$$Y(t) = (-1)^{X(t)}, \quad t \geq 0.$$

(a) Find the mean function and the autocorrelation function of $Y(t)$. Is the process wide sense stationary? 2p

(b) Let $Z(t) = AY(t)$, where A is a random variable, independent of $Y(t)$ and taking on values ± 1 with equal probabilities. Find the probability mass function of $Z(t)$. Is Z a wide sense stationary process? 2p

Problem 3. $\{X_n\}$ is a wide sense stationary process with autocorrelation function

$$R_X(\tau) = 16e^{-5|\tau|} \cos 2\pi\tau.$$

Compute the power spectral density and the average power of the process. 4p

Problem 4. Y_n is a WSS process defined as

$$Y_n = \frac{1}{2}Y_{n-1} + W_n,$$

where W_n is the white-noise process of average power 1. Show that the unit impulse response of the system producing Y_n is

$$h_n = \begin{cases} 2^{-n}, & n \geq 0, \\ 0, & n < 0. \end{cases}$$

and compute $R_Y(m)$. Write equations for finding a filter producing the best predictor of Y_n from Y_{n-2} and Y_{n-3} 4p