Written test for the examination
"Random Processes with Applications", 2008-10-23, 14:00 - 18:00, a house on Hörsalsvägen.
On duty: Rossitza Dodunekova, 772 3534.
Times of visits: 15:00 and 17:00.
Allowed material: The handbook *Beta*, *Collection of formulas for MVE135*, calculators approved by Chalmers.

There are 30 total points in the examination. One needs 14 points for grade 3 (to pass), 18 points for grade 4, and 24 points for grade 5.

Problem 1. A communication channel accepts an arbitrary voltage input V and outputs a voltage

$$Y = V + N$$

where N is a Gaussian random variable with mean zero and variance one, independent of the input value. Suppose that the channel is used to transmit binary information as follows:

to transmit 0:	input -1
to transmit 1:	input 1.

The receiver decides a 0 was sent if the voltage is negative and a 1 otherwise. Find the probability of the receiver making an error if both inputs are equally probable. 4p

Problem 2. The number of bytes in a message is described by a random variable N with $P(N = n) = (1 - p)p^n, \quad n \geq 0$. The messages are broken into packets of length M bytes. Let Q be the number of full packets in a message and R be the number of bytes left over.

(a) Compute the joint probability mass function of Q and R and the marginal probability mass functions of Q and R. 2p

(b) What is the expected value of Q? Are Q and R independent? 2p

Problem 3. The random variables X and Y are jointly Gaussian with expectation 0, variance 1, and correlation coefficient $\frac{1}{4}$. Find the distribution of $Z = X - aY$, where a is some non-zero constant. For which value of a is the variance of Z equal to 1? For this value, compute $E[Z|Z < 1]$. 4p

Problem 4. $N(t)$ is the Poisson process with parameter λ. Show that its autocovariance function is $C_{NN}(t_1, t_2) = \lambda \min(t_1, t_2)$ and compute the autocovariance function of the process $e^{-t/2}N(e^t)$. 5p

Problem 5. The input to a linear time invariant system with impulse response

$$h(t) = \begin{cases} 8\delta(t) + 1, & \text{if } 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

is the random process

$$X(t) = \sin(2\pi t + \Theta), \quad -\infty < t < \infty,$$

where Θ is a random variable uniformly distributed over $[0, 2\pi)$. Give a formula for the output process $Y(t)$ and compute the mean function of this process. 4p

Problem 6. Let $X(t)$ be a wide sense stationary process with autocorrelation function $R_{XX}(\tau)$. A new process is formed by multiplying $X(t)$ by a carrier to produce

$$Y(t) = X(t)\cos(\omega_0 t + \Theta),$$

where ω_0 is a fixed frequency and Θ is a random variable, which is independent of the process $X(t)$ and uniformly distributed over $[0, 2\pi)$. Compute the power spectral density and the average power of $Y(t)$. 4p

Problem 7. $Y[n]$ is an AR(1) process defined as

$$Y[n] = \frac{1}{2}Y[n-1] + e[n],$$

where $e[n]$ is the white-noise process of average power σ^2.

(a) Compute $R_{YY}(m)$, the autocorrelation function of $Y[n]$. 2p

(b) Let $\sigma^2 = 3$. Find the unite impulse response of the filter producing the best predictor of $Y[n]$ from $Y[n-2]$ and $Y[n-3]$. 2p

(c) Give a formula for the estimation error. 1p