Solutions to Chapter 4 Exercises (Part 2)

Problem 4.3

Let N = number of packets transmitted until first success.

$$\begin{aligned} \Pr(N=n) &= q^{n-1}(1-q), \qquad n=1,2,3,\ldots. \\ E[N] &= \sum_{n=1}^{\infty} n q^{n-1}(1-q) = (1-q)\sum_{n=1}^{\infty} n q^{n-1} \\ &= (1-q)\frac{d}{dq}\sum_{n=1}^{\infty} q^n = (1-q)\frac{d}{dq}\frac{1}{1-q} = \frac{1}{1-q}. \end{aligned}$$

Problem 4.4

$$\begin{aligned} T &= \text{total transmission time} \\ &= (N-1)T_i + NT_t = N(T_i+T_t) - T_i. \\ E[T] &= E[N](T_i+T_t) - T_i = \frac{T_i+T_t}{1-q} - T_i. \end{aligned}$$

Problem 4.17

$$f_X(x) = 2\,e^{-2x}\,u(x).$$

(a)

$$X \geq 0, Y = 1-X \Rightarrow Y \leq 1.$$

(b)

$$f_Y(y) = \left.\frac{2\,e^{-2x}u(x)}{|-1|}\right|_{x=1-y} = 2\,\exp(-2(1-y))\,u(1-y).$$

Problem 4.23

(a)

$$\begin{aligned}\Pr(Y=0) &= \Pr(X<0)=\frac{1}{2}.\\ \Pr(Y=1) &= \Pr(X>0)=\frac{1}{2}.\end{aligned}$$

(b)

$$\begin{aligned}\Pr(Y=0) &= \Pr(X<0)=1-Q\left(-\frac{1}{2}\right)=Q\left(\frac{1}{2}\right)=0.3085.\\ \Pr(Y=1) &= \Pr(X>0)=Q\left(-\frac{1}{2}\right)=1-Q\left(\frac{1}{2}\right)=0.6915.\end{aligned}$$

Problem 4.28

$$\Phi_Y(\omega)=E\left[e^{j\omega Y}\right]=E\left[e^{j\omega(aX+b)}\right]=e^{j\omega b}E\left[e^{j\omega aX}\right]=e^{j\omega b}\Phi_X(a\omega).$$

Problem 4.36

$$
\begin{aligned}
H_X(z) &= \frac{1}{n}\frac{1-z^n}{1-z} \\
&= \frac{1}{n}\sum_{k=0}^{n-1} z^k
\end{aligned}
$$

We know that

$$
\begin{aligned}
H_X(z) &= \sum_{k=0}^{\infty} P_X(X=k)z^k \\
&= \frac{1}{n}\sum_{k=0}^{n-1} z^k = \sum_{k=0}^{n-1}\frac{1}{n}\, z^k
\end{aligned}
$$

Recognizing that the coeffecient of z^k in the above equation is the $P_X(X=k)$ we get the PMF of the distribution as

$$
P_X(X=k) = \begin{cases} \frac{1}{n} & k = 0,1,2,\ldots n-1 \\ 0 & \text{otherwise} \end{cases}
$$