RANDOM PROCESSES WITH APPLICATIONS 2009
HOMEWORK 2

This assignment is optional. It gives two bonus points to the written examination, when the submitted solution collects 12 points or more out of a total of 16 points.

Day assigned: September 28
Due date: October 5, 15:15

Problem 1. Let X be the number of active speakers in a group of M independent speakers, each one of which is active with probability p. Suppose that a voice transmission system can transmit up to $N < M$ voice signals at a time, and that when X exceeds N, $X - N$ randomly selected signals are discarded. Give a formula for computing the expected value of the discarded voices. Estimate the probability that voices are not discarded if $M = 45$, $p = 1/3$, $N = 16$.
(3)

Problem 2. The joint PDF of the random variables Z_1 and Z_2 is given by

$$f_{Z_1, Z_2}(z_1, z_2) = \frac{1}{\sqrt{2}\pi} e^{-(z_1^2 - \sqrt{2} z_1 z_2 + z_2^2)}.$$

Compute $Cov\,(Z_1 - Z_2/\sqrt{2},\ Z_2)$. (1)

Problem 3. Let $N(t)$ be a Poisson process with parameter α. Suppose each time an event occurs, independently a coin is flipped and the outcome (heads or tails) is recorded. Let $N_1(t)$ and $N_2(t)$ denote the number of heads and tails recorded up to time t, respectively. Assume that p is the probability of heads.

(a) Compute the conditional probability

$$P\{N_1(t) = m,\ N_2(t) = n - m \,|\, N(t) = n\},$$

where n and m are non-negative integers satisfying $0 \le m \le n$. 2p.

(b) Compute the joint PMF of $N_1(t)$ and $N_2(t)$. 2p.

(c) Compute the PMF of $N_1(t)$ and the PMF of $N_2(t)$. Are $N_1(t)$ and $N_2(t)$ independent?
2p

Problem 4. Consider the random process

$$Y(t) = (-1)^{X(t)}$$

where $X(t)$ is a Poisson process with rate λ.

(a) Find the mean-value time function and the autocorrelation function of $Y(t)$. Is the process WSS?
(3)

(b) Consider the process $Z(t) = AY(t)$, where A is a random variable, independent of $Y(t)$ and with equally likely values ± 1. Is $Z(t)$ WSS? Find the power spectral density of $Z(t)$.
(3)