MVE135 Random Processes with Applications Fall 2010

Written Exam Monday 10 January 2011 8.30 am - 12.30 am

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AIDS: Beta.

GRADES: 12 (40%), 18 (60%) and 24 (80%) points for grade 3, 4 and 5, respectively.

Task 1. Find the conditional PDF $f_{X|2<X\leq 4}(x)$ for a continuous random variable X with PDF $f_X(x) = 2\,\mathrm{e}^{-2x}$ for $x \geq 0$ and $f_X(x) = 0$ for $x < 0$. **(5 points)**

Task 2. Let X and Y be jointly Gaussian random variables with given expected values $\mu_X = E[X]$ and $\mu_Y = E[Y]$, given variances $\sigma_X^2 = E[(X-\mu_X)^2]$ and $\sigma_Y^2 = E[(Y-\mu_Y)^2]$ and given correlation coefficient $\rho_{XY} = E[(X-\mu_X)(Y-\mu_Y)]/(\sigma_X \sigma_Y)$. Find a constant c (expressed in terms of the given information) such that X is independent of $Y - cX$. **(5 points)**

Task 3. Let $X(t)$, $t \in \mathbb{R}$, be a WSS random process and define two new processes $Y(t)$, $t \in \mathbb{R}$, and $Z(t)$, $t \in \mathbb{R}$, by $Y(t) = X(t+1)$ and $Z(t) = X(-t)$, respectively. Is it true that the crosscorrelation functions $R_{XY}(t, t+\tau) = E[X(t)Y(t+\tau)]$ and $R_{XZ}(t, t+\tau) = E[X(t)Z(t+\tau)]$ do not depend on t but only on τ? **(5 points)**

Task 4. Exemplify that two random processes X_n, $n \in \mathbb{Z}$, and Y_n, $n \in \mathbb{Z}$, can be quite different to their appearance, but still share the same mean function and autocorrelation function (i.e., $E[X_n] = E[Y_n]$ and $E[X_m X_n] = E[Y_m Y_n]$ for all $m, n \in \mathbb{Z}$). **(5 points)**

Task 5. Write a short essay about the topic of nonparametric spectral estimation [that is, the problem to estimate the power spectral density of a random process from an observed trajetory/realization of the process (but no information other than that about the process)]. **(5 points)**

Task 6. Discuss when a so called Wiener filter can give a perfect result, that is, when the filter can produce an outsignal that is perfectly equal to the wanted signal $Z(t)$ from a noise disturbed insignal $X(t) = Z(t) + N(t)$. **(5 points)**

Good Luck!

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Solutions to Written Exam Monday 10 January 2011

Task 1. $f_{X\,|\,2<X\leq 4}(x) = \frac{d}{dx}\,\mathrm{Pr}\,(X \leq x\,|\,2 < X \leq 4) = \frac{d}{dx}\,\mathrm{Pr}\,(2 < X \leq x)/\mathrm{Pr}\,(2 < X \leq 4) = \frac{d}{dx}\int_2^x f_X(y)\,dy/\mathrm{Pr}\,(2 < X \leq 4) = f_X(x)/\mathrm{Pr}\,(2 < X \leq 4)$ [this is Eq. 3.41 in the book] $= 2\,\mathrm{e}^{-2x}/(\mathrm{e}^{-4} - \mathrm{e}^{-8})$ for $x \in (2,4]$, while $f_X(x) = 0$ for all other values of x.

Task 2. $\mathrm{Cov}\,(X, Y - cX) = \mathrm{Cov}\,(X,Y) - c\,\mathrm{Var}\,(X) = \rho_{XY}\sigma_X\sigma_Y - c\,\sigma_X^2 = 0 \implies c = \rho_{XY}\sigma_Y/\sigma_X$.

Task 3. $R_{XY}(t, t+\tau) = E[X(t)Y(t+\tau)] = E[X(t)X(t+1+\tau)] = R_{XX}(t, t+1+\tau) = R_{XX}(1+\tau)$ does not depend on t while $R_{XZ}(t, t+\tau) = E[X(t)Z(t+\tau)] = E[X(t)X(-t-\tau)] = R_{XX}(t, -t-\tau) = R_{XX}(2t-\tau)$ depends on t in general.

Task 4. Take X_n, $n \in \mathbb{Z}$, to be independent random variables that can take the values -1 and 1 with probabilities $1/2$ each, while Y_n, $n \in \mathbb{Z}$, are independent zero-mean unit-variance normal random variables.

Task 5. See Section 10.4.1 in the book.

Task 6. When $S_{ZZ}(f)S_{NN}(f) = 0$.