# MVE135 Random Processes with Applications

# Written Exam Monday 9 January 2012 8.30 am – 12.30 pm

Teacher and Jour: Patrik Albin, telephone 070 6945709.

Aids: Beta.

Grades: 12 (40%), 18 (60%) and 24 (80%) points for grade 3, 4 and 5, respectively.

**Task 1.** Calculate $E(X \,|\, X>0)$ for a random variable $X \sim \mathrm{N}(0,1)$. **(5 points)**

**Task 2.** Find the PDF $f_X(x)$ of $X$ when $(X,Y)$ is a pair of random variables with joint PDF $f_{X,Y}(x,y) = \frac{1}{\sqrt{3}\,\pi}\,\mathrm{e}^{-2\,(x^2-xy+y^2)/3}$ for $x,y\in\mathbb{R}$. **(5 points)**

**Task 3.** Let $X(t)$ be a continuous time WSS random process defined for all real times $t\in\mathbb{R}$. Is the time reversed process $Y(t)=X(-t)$ also WSS? (The answer must be motivated!) **(5 points)**

**Task 4.** Calculate the probability $P(X(3)>X(2)>X(1))$ for a Poisson process $X(t)$ with rate $\lambda$. **(5 points)**.

**Task 5.** For which ferquency $f_0>1$ does the lowpass WSS random process $X(t)$ with PSD $S_{XX}(f)=1$ for $|f|\leq f_0$ and $S_{XX}(f)=0$ otherwise have the same average normalized power $R_{XX}(0)$ as the average normalized power $R_{YY}(0)$ of the bandpass WSS random process $Y(t)$ with PSD $S_{YY}(f)=1$ for $|f-f_0|\leq 1$, $S_{YY}(f)=1$ for $|f+f_0|\leq 1$ and $S_{YY}(f)=0$ otherwise? **(5 points)**

**Task 6.** Let $e[n]$ be discrete time Gaussian noise with zero mean and unit variance. Given a constant $a\in(-1,1)$, how can the Fourier transform (/frequency analysis) techniques of Chapter 11 in the book be employed to establish that the discrete time random process $X[n]=\sum_{k=0}^{\infty} a^k\, e[n-k]$ has autocorrelation function $R_{XX}[n]=a^{|n|}/(1-a^2)$? (The required calculations need not be carried out in full detail - it is sufficient to just outline what should be done.) **(5 points)**

**Good Luck!**

## MVE135 Random Processes with Applications

## Solutions to Written Exam Monday 9 January 2012

**Task 1.** We have $f_{X|X>0}(x) = f_X(x)/P(X>0) = \frac{1}{\sqrt{2\pi}}\,\mathrm{e}^{-x^2/2}/(1/2) = \sqrt{2/\pi}\,\mathrm{e}^{-x^2/2}$ for $x>0$, so that $E(X\,|\,X>0) = \int_{-\infty}^{\infty} x\, f_{X|X>0}(x)\,dx = \int_0^{\infty} x\,\sqrt{2/\pi}\,\mathrm{e}^{-x^2/2}\,dx = \sqrt{2/\pi}$.

**Task 2.** We have $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy = \int_{-\infty}^{\infty} \frac{1}{\sqrt{3}\,\pi}\,\mathrm{e}^{-2\,(x^2-xy+y^2)/3}\,dy = \int_{-\infty}^{\infty} \frac{1}{\sqrt{3}\,\pi}$ $\mathrm{e}^{-2\,(y-x/2)^2)/3-x^2/2}\,dy = \frac{1}{\sqrt{2\pi}}\,\mathrm{e}^{-x^2/2}\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,(\sqrt{3}/2)}\,\mathrm{e}^{-(y-x/2)^2)/(2\,(\sqrt{3}/2)^2)}\,dy = \frac{1}{\sqrt{2\pi}}\,\mathrm{e}^{-x^2/2}$ $\int_{-\infty}^{\infty} f_{\mathrm{N}(x/2,(\sqrt{3}/2)^2)}(y)\,dy = \frac{1}{\sqrt{2\pi}}\,\mathrm{e}^{-x^2/2}$ for $x\in\mathbb{R}$.

**Task 3.** We have $\mu_Y(t) = E(Y(t)) = E(X(-t)) = \mu_X(-t) = \mu_X =$ constant and $R_{YY}(t,t{+}\tau) = E(Y(t)Y(t{+}\tau)) = E(X(-t)X(-(t{+}\tau))) = R_{XX}(-t,-(t{+}\tau)) = R_X(-(t$ $+\tau)-(-t)) = R_X(-\tau) = R_X(\tau)$ a function of $\tau$ only, so that $Y(t)$ is also WSS.

**Task 4.** $P(X(3)>X(2)>X(1)) = P(X(3)-X(2)>0, X(2)-X(1)>0) = P(X(3)-$ $X(2)>0)\,P(X(2)-X(1)>0) = P(X(1)>0)^2 = (1-\mathrm{e}^{-\lambda})^2$.

**Task 5.** As $R_{XX}(0) = \int_{\infty}^{\infty} S_{XX}(f)\,df = 2\,f_0$ and $R_{YY}(0) = \int_{\infty}^{\infty} S_{YY}(f)\,df = 4$ we must have $f_0 = 2$.

**Task 6.** See Example 11.3 in the book.