MVE136 Random Signals Analysis

Written exam Sunday 26 January 2014 1 – 5 pm

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AIDS: Beta or 2 sheets (4 pages) of hand-written notes (computer print-outs and/or xerox-copies are not allowed), but not both these aids.

GRADES: 12 (40%), 18 (60%) and 24 (80%) points for grade 3, 4 and 5, respectively.

GOOD LUCK!

Task 1. Let A be a random variable with finite mean and finite variance that is independent of another random variable Θ that is uniformly distributed over the interval $[0, 2\pi]$. Consider the discrete time random process $\{X_n, n \in \mathbb{Z}\}$ given by $X_n = A\cos(\Theta n)$. Show that the process X_n is not wide-sense stationary (WSS). (Hint: The formula $\cos(x)\cos(y) = \frac{1}{2}\cos(x+y) + \frac{1}{2}\cos(y-x)$ can be useful.) **(5 points)**

Task 2. A Wiener process (also called Brownian motion) is a zero-mean continuous time Gaussian process $\{W(t), t \geq 0\}$ with autocorrelation function $R_{WW}(s,t) = \min(s,t)$. [Hence the autocorrelation agrees with the autocovariance function of a Poisson processes with arrival rate (/intensity) 1.] Prove that the Wiener process has independent increments, which is to say, that $W(t_4) - W(t_3)$ and $W(t_2) - W(t_1)$ are independent random variables for $0 \leq t_1 \leq t_2 \leq t_3 \leq t_4$. **(5 points)**

Task 3. Find the probability $P(X(t) + X(2t) = 5)$ when $\{X(t), t \geq 0\}$ is a Poisson process with arrival rate (/intensity) $\lambda > 0$. **(5 points)**

Task 4. Consider a so called discrete time random walk process $\{X[n], n \geq 0\}$ on the (both positive and negative) integers that starts at the origin (that is, $X[0] = 0$). At each time instant, the process either increases by 1 with probability p or decreases by 1 with probability $1-p$. Explain why this process is a Markov chain and find its transition probabilities $p_{i,j}$. Are the states of this chain periodic or aperiodic? If they are periocid, what is the period? **(5 points)**

Task 5. A wide sense stationary (WSS) continuous time signal process $Z(t)$ with power spectral density (PSD) $S_{ZZ}(f)$ is sent on a noisy channel with an additive WSS noise process $N(t)$ that is independent of $Z(t)$ and has PSD $S_{NN}(f)$. Hence the recived noise disturbed signal process $X(t)$ is given by $X(t) = Z(t) + N(t)$. We send $X(t)$ through a

so called Wiener filter with output process $Y(t)$ that is designed to minimize the mean square error $E[(Z(t)-Y(t))^2]$. What is the PSD of $Y(t)$? **(5 points)**

Task 6. Explain what weakness of the periodogram that the modified periodogram tries to remove. What is the difference (/are the differences) between the periodogram and the modified periodogram? **(5 points)**