MVE136 Random Signals Analysis – Test Exam

AIDS: Beta <u>or</u> 2 sheets (4 pages) of hand-written notes (computer print-outs and/or xerox-copies are not allowed), but not both these aids.

GRADES: 12 (40%), 18 (60%) and 24 (80%) points for grade 3, 4 and 5, respectively.

GOOD LUCK!

Task 1. Let (X, Y) be a continuous random variable with PDF $f_{X,Y}(x, y) = \mathrm{e}^{-x-y-xy} \big/ \big(\int_0^\infty (1+z)^{-1} \, \mathrm{e}^{-z} \, dz \big)$ for $x, y \geq 0$ (and 0 otherwise). Find $\mathbf{E}\{X \,|\, Y = y\}$. **(5 points)**

Task 2. Find the probability $Pr(X(1)+X(2)+X(3) > 6)$ for a continuous time WSS Gaussian process $X(t)$ with mean $\mu_X = 1$ and autocorrelation function $R_{XX}(\tau) = \mathrm{e}^{-|\tau|} + 1$ for $\tau \in \mathbb{R}$. **(5 points)**

Task 3. Consider a discrete time Markov chain $X(n)$ with state space E and transition probability matrix P given by

$$E = \{0, 1\} \qquad \text{and} \qquad P = \begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{bmatrix},$$

respectively. What initial distribution $\pi(0)$ of the chain will give a distribution $\pi(2)$ of the value of the chain $X(2)$ at time $n = 2$ given by $\pi(2) = [1/3 \;\; 2/3]$? **(5 points)**

Task 4. The PSD $S_{XX}(f)$ of a continuous time WSS process $X(t)$ has the properties to be real and symmetric (=even). Prove one of these properties. **(5 points)**

Task 5. A WSS discrete-time random process $X(n)$ with PSD $S_{XX}(f)$ is input to two different LTI systems with transfer functions $H_1(f)$ and $H_2(f)$, respectively. Find the cross spectral density $S_{Y_1Y_2}(f)$ between the outputs $Y_1(n)$ and $Y_2(n)$ from the two LTI systems. **(5 points)**

Task 6. Compute the autocorrelation function $r_x[n]$ for $n = 0$ and $n = 1$ when $x[n]$ is an AR(1)-process with parameter $a_1 = 0.7$. You can assume that the input noise has variance $\sigma_e^2 = 1$. **(5 points)**