MVE136 Random Signals Analysis

Written home exam Monday 17 August 2020 2 PM–6 PM

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AIDS: All aids are permitted. (See the Canvas course "MVE136 Re-Exam MVE136" with instructions for this reexam for clarifications.)

GRADES: 12 (40%), 18 (60%) and 24 (80%) points for grade 3, 4 and 5, respectively.

MOTIVATIONS: All answers/solutions must be motivated. GOOD LUCK!

Task 1. Calculate $\Pr(X(1)Y(2) > 0)$ when $X(t)$ and $Y(t)$, $t \in \mathbb{R}$, are independent zero-mean WSS Gaussian processes with PSD's $S_{XX}(f) = S_{YY}(f) = \mathrm{e}^{-|f|}$. **(5 points)**

Task 2. Let $X(t)$ and $Y(t)$, $t \geq 0$, be independent Poission processes with intensity (/rate) 1. Find $\Pr(X(1) = Y(2))$. **(5 points)**

Task 3. Let $X(n)$, $n = 0, 1, 2, \ldots$, be a Markov chain with possible values 0 and 1, transition matrix $P = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$ and starting distribution $\pi(0) = (1 \;\; 0)$. Find the expected value $E(T)$ of the random time

$$T = \min\{n > 0 : X(m) = 1 \text{ for some } 0 < m < n \text{ and } X(n) = 0\}$$

it takes for the chain to make a journey from 0 to 1 and back to 0. [HINT: A geometric discrete random variable ξ with PMF $\Pr(\xi = k) = p\,(1-p)^{k-1}$ for $k = 1, 2, \ldots$ has expected value $E(\xi) = 1/p$.] **(5 points)**

Task 4. Show by an example that two WSS random processes $X(t)$ and $Y(t)$ need not be jointly WSS, i.e., that $R_{XY}(t, t+\tau)$ may depend on t. **(5 points)**

Task 5. A WSS process $X(t)$ with PSD $S_{XX}(f)$ is sent on a noisy channel where an independent zero-mean WSS noise process $N(t)$ with PSD $S_{NN}(f)$ is added. The received signal $Y(t) = X(t) + N(t)$ is processed through a Wiener filter LTI system with transfer function $H(f) = S_{XX}(f)/(S_{XX}(f) + S_{NN}(f))$ and outsignal $Z(t)$ that minimizes the mean-square distance $D = E((X(t) - Z(t))^2)$ between the sent signal and the processed recived signal. It is possible to have $D = 0$: What exactly is required for this to happen? **(5 points)**

Task 6. How can one do parametric spectral estimation for an AR(1) process? **(5 points)**

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Solutions to written exam 17 August 2020

Task 1. $\Pr(X(1)Y(2) > 0) = \Pr(X(1) > 0)\Pr(Y(2) > 0) + \Pr(X(1) < 0)\Pr(Y(2) < 0)$ $= \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$ since $X(1)$ and $Y(2)$ are independent zero-mean normaldistributed with variance $\int_{-\infty}^{\infty} \mathrm{e}^{-|f|}\, df > 0$.

Task 2. $\Pr(X(1) = Y(2)) = \sum_{k=0}^{\infty} \Pr(X(1) = k)\Pr(Y(2) = k) = \sum_{k=0}^{\infty} \frac{1^k}{k!}\,\mathrm{e}^{-1}\,\frac{2^k}{k!}\,\mathrm{e}^{-2} = \sum_{k=0}^{\infty} \frac{2^k}{(k!)\cdot(k!)}\,\mathrm{e}^{-3}$.

Task 3. $E(T) = 1/(1/2) + 1/(1/2) = 4$.

Task 4. For $Y(t) = X(-t)$ we have $R_{XY}(t, t+\tau) = R_{XX}(2t+\tau)$ which depends on t unless $R_{XX}(\tau)$ is constant.

Task 5. That $S_{XX}(f) = 0$ when $S_{NN}(f) \neq 0$.

Task 6. See, e.g., Patrik Albin's lecture notes.