MVE136 Random Signals Analysis

Written home exam Monday 4 January 2021 2–6 PM

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AIDS: All aids are permitted. (See the Canvas course "Omtentamen 1 Modul: 0111, MVE136" with instructions for this exam for clarifications.)

GRADES: 12 (40%), 18 (60%) and 24 (80%) points for grade 3, 4 and 5, respectively.

MOTIVATIONS: All answers/solutions must be motivated. GOOD LUCK!

Task 1. Is the process $\sin(X(t)^2)$ WSS when $\{X(t)\}_{t\in\mathbb{R}}$ is a WSS Gaussian process?

(5 points)

Task 2. Calculate $\Pr(X(3)=3\,|\,X(1)=1,X(2)=2,X(4)=4,X(5)=5)$ for a Poisson processs $\{X(t)\}_{t\geq 0}$. **(5 points)**

Task 3. Let $\{X(t)\}_{t\geq 0}$ be a unit rate Poisson process and $\{Y(t)\}_{t\in\mathbb{R}}$ a zero-mean WSS Gaussian process with autocorrelation function $R_{YY}(\tau) = \mathrm{e}^{-|\tau|}$ for $\tau \in \mathbb{R}$ that is independent of the Poisson process. Find an expression (that can be readily numerically calculated by Matlab etc.) for $\Pr(X(t) \geq Y(t))$ for $t \geq 0$. **(5 points)**

Task 4. Consider a Markov chain $\{X_n\}_{n=0}^{+\infty}$ with states $\{0, 1, 2\}$, initial probability distribution $\pi(0) = (0\;\;1\;\;0) = (0, 1, 0)$ and transition probability matrix P. Express the probability $\Pr(X(1)=1\,|\,X(2)=2)$ in terms of the elements of P. **(5 points)**

Task 5. Consider a continuous time LTI system with insignal continuous time white noise $N(t)$ and WSS outsignal $\{Y(t)\}_{t\in\mathbb{R}}$ with autocorrelation function $R_{YY}(\tau) = \mathrm{e}^{-|\tau|}$ for $\tau \in \mathbb{R}$. What is the impulse response? **(5 points)**

Task 6. Consider an MA(2)-process $\{X_n\}_{n=-\infty}^{+\infty}$ given by $X_n = e_n - b\,e_{n-1}$, where $b \in (-1, 1)$ is a real number and $\{e_n\}_{n=-\infty}^{+\infty}$ discrete time white noise. Explain how X_n can be considered a limit case of an AR(N)-process as $N \to \infty$. **(5 points)**

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Solutions to written exam 4 January 2021

Task 1. As $X(t)$ is WSS Gaussian it is stationary and then also $\sin(X(t)^2)$ is stationary and therefore WSS.

Task 2. $\Pr = \frac{\Pr(X(1)=1,X(2)=2,X(3)=3,X(4)=4,X(5)=5)}{\Pr(X(1)=1,X(2)=2,X(4)=4,X(5)=5)} = \frac{[\Pr(X(1)=1)]^5}{[\Pr(X(1)=1)]^3\,\Pr(X(2)=2)} = \ldots = \frac{1}{2}$.

Task 3. As $X(t)$ is $\mathrm{Po}(t)$-distributed and $Y(t)$ is $\mathrm{N}(0,1)$-distributed independent of $X(t)$ we have $\Pr(X(t) \geq Y(t)) = \sum_{k=0}^{\infty} \Pr(Y(t) \leq k)\Pr(X(t)=k) = \sum_{k=0}^{\infty} \Phi(k)\,\frac{t^k}{k!}\,\mathrm{e}^{-t}$.

Task 4. $\Pr = \frac{\Pr(X(2)=2\,|\,X(1)=1)\Pr(X(1)=1)}{\Pr(X(2)=2)} = \frac{p_{12}\,\pi(1)_1}{\pi(2)_2}$ where $\pi(1) = (0,1,0)\,P = (p_{10}, p_{11}, p_{1,2})$ so that $\pi(1)_1 = p_{11}$ and $\pi(2)_2 = (\pi(1)\,P)_2 = p_{10}p_{02} + p_{11}p_{12} + p_{12}p_{22}$.

Task 5. As $S_{YY}(f) = \frac{2}{1+(2\pi f)^2} = |H(f)|^2 S_{NN}(f) = |H(f)|^2 N_0/2$ we require $|H(f)|^2 = \frac{4/N_0}{1+(2\pi f)^2}$ which holds for $h(t) = (2/\sqrt{N_0})\,\mathrm{e}^{-t}u(t)$.

Task 6. $e_n = \sum_{k=0}^{N-1} b^k X_{n-k} + b^N e_{n-N} \to \sum_{k=0}^{+\infty} b^k X_{n-k}$ as $N \to \infty$.